Reconstruction of Compressive Sensing Signal using Orthogonal Matching Pursuit Algorithm

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Abstract— This paper represents the reconstruction of sampled signal in CS by using OMP algorithm. We have used the concept of compressive sensing for sub Nyquist sampling of sparse signal. Compressive sensing reconstruction methods have complex algorithms of l1 optimisation to reconstruct a signal sampled at sub nyquist rate. But out of those algorithm OMP algorithm is fast and computationally efficient. To prove the concept of CS implementation, we have simulated OMP algorithm for recovery of sparse signal of length 256 with sparsity 8.

Key words: CS, OMP, BP,ADC

I. INTRODUCTION

Compressive sensing is a technique which can reduce the number of measurements require to acquire the signal information. This technique not only allows the signal acquisition in condensed form but also enables the full reconstruction of the original signal from the compressed signal even if the original signal is sampled at well below the Nyquist rate. Hence the concept of CS allows low rate ADC to be used instead of high rate ADC for sampling of the actual signal.

CS technique works particularly with sparse signals. In most of the applications where information is sparsely spread over the signal. Such signals have sparse Representation in some predefined basis(domain) in which signal is actually represented by only few significant (or non zero) coefficients and rest of the major no. of coefficients are zero(or very near to zero).

A. Sparse And Compressible Signal

Sparse means, for a signal of length N, we can represent it with K very much less than N non zero coefficients. By compressible we mean that a signal; can be well approximated by K non zero coefficients only. CS allows for the recovery of sparse signals sampled at a sub Nyquist rate directly in the sparse domain or after an invertible linear transformation.

The sampling of the signal is done in continuous time domain or space domain by multiplying the signal by a pseudorandom matrix φ .This is the basic idea behind the CS concept, where signal is acquired directly in condensed form, rather than sampling it at very high rate and then compressing the sampled data. To make this possible CS relies on two principles: Sparsity and Incoherence.

B. Sparsity

Sparsity expresses the idea that the information rate of a continuos time signal may be much smaller than the actual bandwidth of the signal. All sparse/compressible signals can be expressed in proper basis ψ.

Consider a length-N discrete-time signal x which can be interpreted x as an(N×1) column vector. The signal x is sparsely representable if there exists a sparsity basis i that provides a K-sparse representation of x; that is

$$x = \sum_{i=1}^{N} \theta i(t)\psi i(t) \qquad (1)$$

Where x is a linear combination of K basis vectors of and the weighting coefficients θi . The basis vectors can be combinedly written in matrix form ∅ = [ψ1ψ2.....ψN]

And we can write the sparse signal in matrix form

$$x=\psi\theta \qquad (2)$$

where θ is an N×1 column vector with K non zero elements.

When we transform a signal to a new basis resultant coefficient and basis is useful for reducing data storage and data transmission. In cases where small number of coefficients can be stored instead of all, thus sparse signals are compressible too.

C. Incoherence

CS greatly relies on incoherence between the sensing basis φ and the representation basis ψ which form the ortho basis pair and has minimum coherence between them. Incoherence extends the duality between time and frequency and expresses the idea that objects having a sparse representation in ψ must be spread out in the domain in which they are acquired just as a dirac or a spike in the time domain is spread out in the frequency domain.The coherence between and φ and ψ is given by

$$\mu(\varphi,\psi)= \sqrt{N}\max(\varphi_k,\psi_j) \qquad (3)$$

where has values between 1 and $\sqrt{N}$. When μ(φ,ψ)=1,then we have maximum should be random enough so that incoherence is high. Because random matrices are largely incoherent with any fixed basis ψ. The sparse signal can be compressed by sensing matrix φ to obtain condensed form signal y

$$y=\varphi x \qquad (4)$$

where y is (M×1) column matrix, φ is (M×N) matrix and x is (K×1) column matrix.

Since M is less than N, recovery of the signal x from the measurements y is ill posed in general. However, CS theory tells us that when the matrix φψ has the restricted isometry property then it is possible to recover the K largest coefficients provided M=O(KlogN/K) measurements are taken. When the RIP/incoherence holds, the signal x can be recovered exactly from y by solving an l1 norm minimization problem.

$$\min \| x \|_{l1} , x \in R^n$$

These optimization problem can be solved with traditional linear programming techniques. Some algorithms such as Matching pursuit and Orthogonal matching pursuit (OMP) can recover the signal x from the measurements y.

D. Recovery of Sparse Signal

The sparse signal is generally reconstructed using Basis pursuit. In this method convex optimization is used to represent a signal in an over complete dictionary D, which minimizes the l1 norm of the coefficients in the representation. This algorithm gives more accurate representation of the signal, but actually it is more computationally intensive and significantly slower than other recovery methods. Out of other method OMP is better than the others from computational complexity point of view.

OMP algorithm requires two inputs, measurement vector y and sampling matrix φ and generates the approximation ˆx of the original signal. Since the original signal is K sparse, so we need to find the columns of φ out of N which is the best correlated with the remaining part of y. Tropp and Gilberrt proved that OMP can be used to recover a sparse signal with high probability using compressive measurements. OMP is an efficient method for the CS recovery, especially when sparsity K is low.

Fig. 1: OMP Reconstruction

II. PROCEDURE FOR OMP RECONSTRUCTION

1) Initialise the residual R_o = y, the index set S=∅,and the iteration $_{\text{counter t=1.}}$
2) Find the index S_t which solves the sample optimisation problem S_t=argmax$\left| < R_{t-1},\phi_j > \right|$
   j=1….N

where ϕ_j is the j column of the φ

3) Update the index set s_t=S_{t+1}∪{ st }
4) Perform modified Gram-Schmidt using the s_t column of φ and $q_{1….t}$
5) In order to determine q_{t+1}.
6) Calculate the new residual according to R_t=R_{t-1}-q_{t+1}.q'_{t+1}.
7) Increment t and return to step b if t is less than the assumed sparsity m of the signal.
8) Find the signal values at the indices in S by solving the least squares problem.

Where φ consists of K relevant columns of φ.

The OMP reconstruction can be divided into two stages. In first stage m indices of the non-zero entries of the signal x is determined in steps a to f. In second stage values of x at those m indices are determined using the minimization technique in step g. Determining the values of the reconstructed signal involves solving the minimization problem as given in step g. This equation is generally solved using the Moore Penrose pseudo inverse of matrix ˜φ. here we take an example of sparse signal and apply concept of compressive sensing and show the stimulation result in MATLAB obtained for recovered signal using OMP algorithm for reconstruction. We simply took a signal vector x of length of 256 with all zero entries except 8 non zero elements for the sparsity of 8. Then we added random noise in our signal vector to make it noisy. Random noise is generated using MATLAB function with variance of 0 to 10. Now this noisy sparse signal is randomly sampled by using sensing matrix φ which is again a random Bernoulli matrix having 1 and 1 entries. Matrix size of φ is 64×256.The compressed signal y and sensing matrix φ is then given to the OMP algorithm for the reconstruction of the original signal.

A flowchart of the OMP algorithm has been shown in fig.

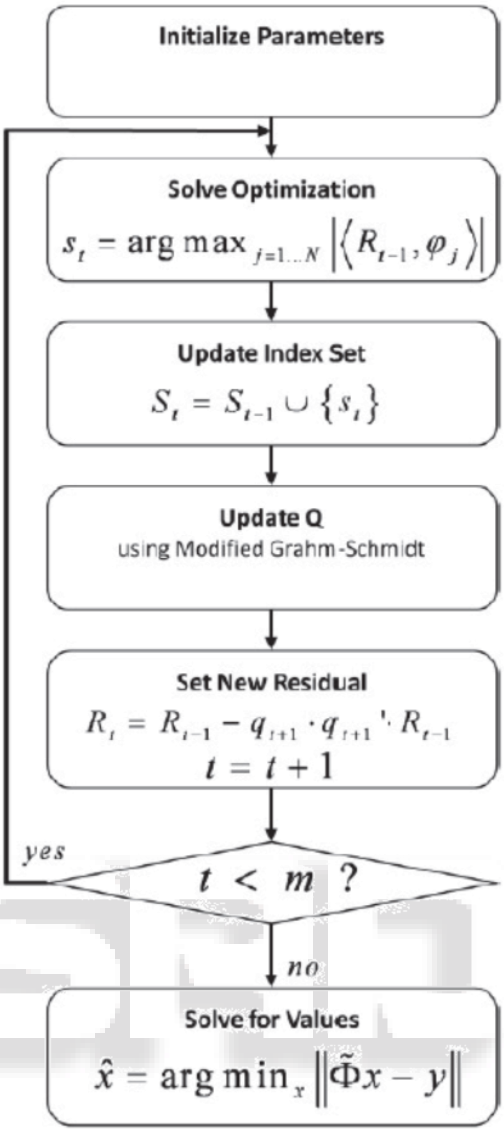

Fig. 3: OMP Flow Chart

Now we have to obtained the reduced size matrix from the sensing matrix so as to solve minimization problem. To get such a matrix we correlate our input vector y with each column of φ and find best matched column by calculating the inner product with each column. This selected column and its index is stored in another matrix. then we used modified Gram Schmidt orthogonalisation to obtain the orthonormal vectors of each selected column of φ̃. These orthonormal vectors are then used to calculate the residual vector which in turn will be used for correlation in the next iteration. This process goes on the till iteration values reaches the no. of sparsity. After all the iteration we will get matrix φ̃ with 64×8 which is now easier to compute and can be processed in hardware. Next we have to solve the minimization problem to obtain x^.This can be solved by taking the pseudo inverse of φ̃ and multiply with y. The OMP will give the non-zero values of the original signal and the location of those non zero elements is given by index vector. Rest of the places we put zero and the original signal vector is recovered and plotted in fig.

III. SIMULATION RESULTS

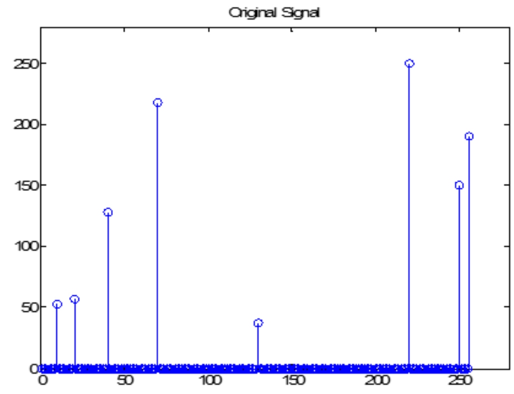

Fig. 3: Original Signal

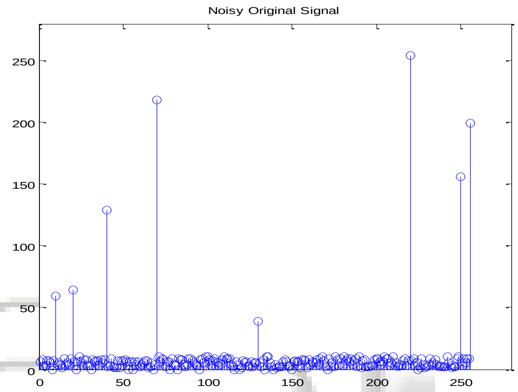

Fig. 4: Noisy Original Signal

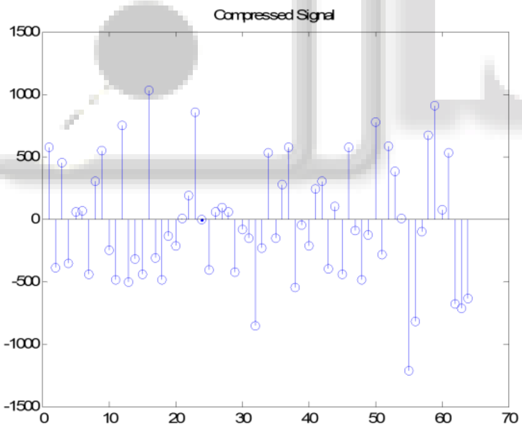

Fig. 5: Compressed Signal

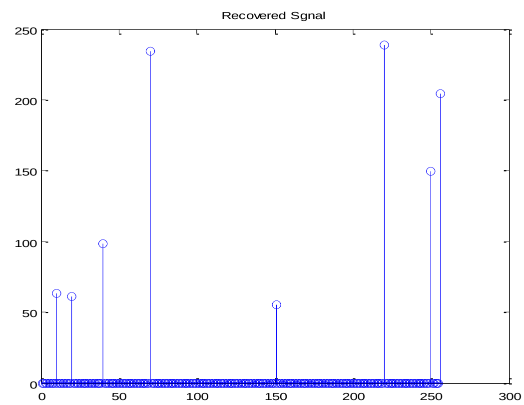

Fig. 6: Recovered Signal

IV. CONCLUSION

In this paper we have brought the concept of Compressive sensing for sub Nyquist sampling of sparse signal and recovery algorithm. It is capable of reconstructing a sparse signal of length N=256, M=64 and sparsity of K=8.OMP algorithm can be efficiently implemented in hardware to reconstruct the original signal from fewer samples.The above concept of CS can also be used in radar application.

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