A Dynamic Analysis of Overhung Rotor System with Bearing Defect

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Abstract—In this paper an analytical model is proposed to study behavior of overhung rotor-defective bearing system. Overhung rotor is considered with unbalanced mass and bearing is taken as cylindrical roller bearing having surface defect. To analyze all system component's effect at one node, finite element method is used to predict exact system vibrations. Euler-Bernoulli's beam element is used to discretize shaft. Gyroscopic effect of overhung rotor is also taken into account. Governing equation of motion has been modified according to our system. Hertz contact stress theory is used for every roller-race contact to calculate overall nonlinear bearing force. Governing differential equation is solved by newmark-beta time integration method. Nonlinear matrix equation which is generated at each time-step in newmark's method is solved by Broyden's quasi-newton method. Results at defective bearing is obtained and plotted in time and frequency domain. Higher level Poincare maps are drawn to view system's minimum stability time. Whole analysis work is done in software package MATLAB. The current study provides a powerful tool for design and health monitoring of overhung rotor systems.

Key words: Overhung Rotor System, Bearing Defect

I. INTRODUCTION

With increasing application of overhung rotors like pump impellers, turbines; it is vital to know their vibration behavior for design and maintenance. Bearing defects and unbalanced mass in rotors play major role in chaotic vibrations and then breakdown. Continues monitoring of vibrations can detect bearing defects before major breakdowns and thus it can reduce downtime. Vibration analyzers used today uses their diagnostic software to know whether there is a defect in bearing or in other components. These software use predefined graphs of frequency components to compare diagnosed data with ideal one. So one has to predict vibration behavior analytically for every new configurations. The exact vibration prediction considering the effect of all components of the system is practically impossible. However, with ignoring certain effects like shear deformation and rotational inertia, Finite Element Method provides more realistic approach to know system's vibration response.

A software package ANSYS provides large variety of element types and methods to analyze system with FEA. However, the rotor dynamic solver in ANSYS only solves linear bearing force systems which have constant bearing stiffness. Actually, and in this study, rolling element restoring force varies nonlinearly with its deflection. So, governing equation of motion of current system becomes nonlinear which has to be solved by custom FEA code. That is why, a MATLAB code has been made to analyze this system.

II. LITERATURE REVIEW

A lot of research work has been published on the analysis of rotor-bearing system behavior. An Sung Lee [1] used transient response analysis technique of a rotor system by the generalized finite element modelling method of a rotor-bearing system considering a base-transferred shock force along with the state-space Newmark method of a direct time integration scheme based on the average velocity concept. By Experiments he showed that the transient responses of the rotor are sensitive to the duration times of the shocks. Hamdi Taplak [2] analysed turbine rotor supported by roller bearings with finite element method and determined Campbell diagrams and unbalanced response over the length. He analysed two rotors with distributed mass of shaft consist of two bearings. He took node responses, Campbell diagram and whirl orbits as output. Jiawei Xiang [3] analyzed rotor-bearing system with gyroscopic effects with wavelet based finite element method. He proposed a finite element model of rotor-bearing systems based on wavelet-based shaft element. Sauer G. and Wolf M. [4] worked on the gyroscopic effect of rotating disc-shaft systems. They presented truncation method to reduce order of the model and thus computation time.

On the other hand, research works have been done also for single node vibration prediction and bearing defect detection. We can say, major contribution in this field is given by S.P. Harsha. He developed an analytical model [5] to predict non-linear dynamic responses in a rotor bearing system due to surface waviness. He also studied [6] the effect of speed of balanced rotor on nonlinear vibrations associated with ball bearings. He studied [7] the nonlinear dynamic response of a balanced rotor Supported by rolling element bearings due to radial internal clearance effect. Ahmad Rafsanjani [8] developed an analytical model to study the nonlinear dynamic behavior of rolling element bearing systems including surface defects. Various surface defects due to local imperfections on raceways and rolling elements are introduced to the proposed model. Hamdi Taplak [9] studied an Experimental analysis on fault detection for a direct coupled rotor-bearing system. He proved that the 17 rotating machineries can have one or more vibration sources.

III. SYSTEM CONFIGURATION

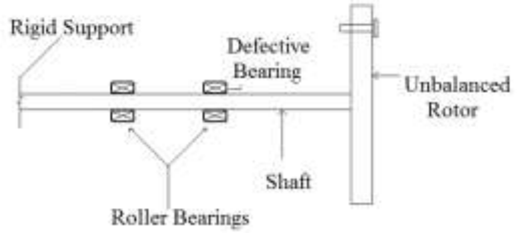

Fig. 1: System Model to Analyze

In this study, above configuration of two bearings and a rotor system is analyzed. A single row cylindrical roller bearing NU305 having 11 rolling elements is used for study. An inner race of bearing having localized surface defect - a spall of 0.5 mm of depth and 0.5 mm of width. Imbalanced mass is taken of 10 grams in rotor. Two bearings having a distance of 0.33 meter from each other.

IV. MODELING OF THE SYSTEM

A. Governing Equation of Proposed Rotor System

The general form of the equations of motion for a multi-degree of freedom system is written as:

$$[M]\ddot{X} + [C]\dot{X} + [K]X = F(t)$$

In our case, bearing stiffness is not a constant value and bearing restoring force is not linearly depends on deformation. It should not be added to global stiffness matrix K. So governing equation for our system is modified as,

$$[M]\ddot{X} + [C]\dot{X} + [K]X + Bf(X,t) = F(t) \qquad (3.1)$$

In above equation, $Bf(X, t)$ is bearing force vector, which depends on displacement vector and time. M, K, and C are mass, stiffness and damping matrices which are calculated with finite element method taking Euler-Bernoulli's beam element.

B. FEA Matrices Calculation

An element consist of two nodes with four degrees of freedom at each node – two for displacements in x and y directions and other two for angular displacement in x and y planes. So degrees of freedom vector for an element is given by:

$$\delta^{(e)} = \{x_1 \; y_1 \; \varphi_1 \; \Theta_1 \; x_2 \; y_2 \; \varphi_2 \; \Theta_2\}^T$$

Where 1 and 2 notations are for left and right nodes. Note that above matrix is column [8x1] vector.

1) Element Stiffness Matrix $K^{(e)}$

$$\frac{EI}{L^3}\begin{bmatrix} 12 & 0 & 0 & 6L & -12 & 0 & 0 & 6L \\ 0 & 12 & -6L & 0 & 0 & -12 & -6L & 0 \\ 0 & -6L & 4L^2 & 0 & 0 & 6L & 2L^2 & 0 \\ 6L & 0 & 0 & 4L^2 & -6L & 0 & 0 & 2L^2 \\ -12 & 0 & 0 & -6L & 12 & 0 & 0 & -6L \\ 0 & -12 & 6L & 0 & 0 & 12 & 6L & 0 \\ 0 & -6L & 2L^2 & 0 & 0 & 6L & 4L^2 & 0 \\ 6L & 0 & 0 & 2L^2 & -6L & 0 & 0 & 4L^2 \end{bmatrix}$$

In which L is length of an element, E is young modulus and I is area moment of inertia which is $\pi r^4/4$. r is radius of an element.

2) Element Mass Matrix $M^{(e)}$

$$\frac{m}{420} \times \begin{bmatrix} 156 & 0 & 0 & 22L & 54 & 0 & 0 & -13L \\ 0 & 156 & -22L & 0 & 0 & 54 & 13L & 0 \\ 0 & -22L & 4L^2 & 0 & 0 & -13L & -3L^2 & 0 \\ 22L & 0 & 0 & 4L^2 & 13L & 0 & 0 & -3L^2 \\ 54 & 0 & 0 & 13L & 156 & 0 & 0 & -22L \\ 0 & 54 & -13L & 0 & 0 & 156 & 22L & 0 \\ 0 & 13L & -3L^2 & 0 & 0 & 22L & 4L^2 & 0 \\ -13L & 0 & 0 & -3L^2 & -22L & 0 & 0 & 4L^2 \end{bmatrix}$$

3) Disk Matrices

Disk (rotor) angular displacement causes gyroscopic effect forces which resists angular velocity. So it accounted as damping effect here and disc damping matrix is given as:

$$C_d = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & J_p\omega \\ 0 & 0 & -J_p\omega & 0 \end{bmatrix}$$

Where ω is driving frequency which is 2πn/60. J_p is polar mass moment of inertia of disc (rotor) which is ½($m_d R^2$). R is radius of disc.

Similarly, disc mass matrix is given by:

$$M_d = \begin{bmatrix} m_d & 0 & 0 & 0 \\ 0 & m_d & 0 & 0 \\ 0 & 0 & J_d & 0 \\ 0 & 0 & 0 & J_d \end{bmatrix}$$

Where J_d is mass moment of inertia of disc which is

$$J_d = 2\left[\frac{mR^2}{8} + \frac{m}{6}\left(\frac{t}{2}\right)^2\right]$$

In which t is width of disc.

4) External Force Vector

For 4 degrees of freedom with N nodes, force vector is of 4N x 1 matrix. Forces applied externally at nodes are added to its respective place in vector. For an overhung rotor with unbalanced mass,

$$F_{(4N-3,1)} = m\,g + um\,\omega^2\,r\cos(\omega\,t)$$
$$F_{(4N-2,1)} = m\,g + um\,\omega^2\,r\sin(\omega\,t)$$

Where N is number of nodes and um is unbalanced mass. r is radius of center of unbalanced mass. Equation represents sum of gravitational and centrifugal forces. Note that rotor is assumed to be placed at last node. Above vector is different for each time step because it includes t.

5) Assembly of Matrices

To get overall system matrices, element matrices are added to global system matrices to their respective degree of freedom (Dof). For example, Disc matrices are added to last four Dof in global matrices as it is considered as lumped mass at last node. Total number of Dofs of system is 4 times its number of nodes. Initially, global matrices are taken as zeros of Dof x Dof matrix. Penalty approach has been used to give boundary conditions so at first node and at its respective Dofs in global matrices, Very high (Usually 10^4 times of maximum) values of stiffness are added as first node is considered as fixed.

C. Bearing Force Calculation:

In this calculation, rolling element is assumed as a nonlinear spring according to Hertz contact theory and the effect of loading zone has been taken into account. Total bearing force is calculated by vector addition of forces exerted by all of those springs (Rollers). For a cylindrical contact, Hertz theory gives following force-deflection relation:

$$F = K\,D^e \qquad (3.3)$$

In above equation F is force exerted by two cylindrical contact deformation of D. The exponent - e describes the nonlinear load-deflection relation. The value of e for the ball bearing is 1.5 and for the cylindrical roller bearing is 1.1. K is the effective stiffness constant for the contact between the rolling element and the bearing race which is calculated by geometric properties of bearing. In present study, it is taken as 4.68 x 10^8 N/m. To calculate overall bearing force, force exerted by all of the rollers in loading zone have to be calculated. Consider the rolling element bearing shown in Fig.2. In this model, the inner race of the bearing is assumed to have two degrees of freedom. The contact forces are summed over each of the rolling elements to give overall forces on the shaft and bearing housing. The overall contact deformation for the j^{th} rolling element, δ_j, is given by

$$\delta_j = x\cos\theta_j + y\sin\theta_j - \gamma$$

Where x and y are deflections of bearing inner race in two axis perpendicular to axis of shaft and γ is the internal radial clearance.

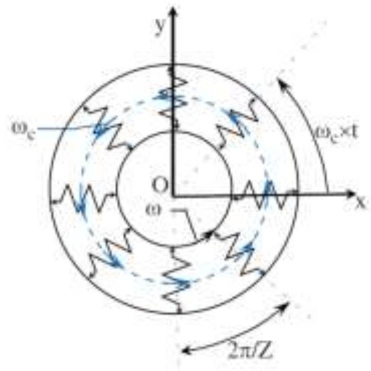

Fig. 2: Rolling element angular position in bearing

The total restoring forces in x and y direction on the shaft and bearing housing is given by:

$$f_x = K\sum_{j=1}^{z} \lambda_j\, \delta_j^{1.1}\cos\theta_j$$

$$f_y = K\sum_{j=1}^{z} \lambda_j\, \delta_j^{1.1}\sin\theta_j$$

Where λ_j is the loading zone parameter for the j_{th} rolling element which is given by:

$$\lambda_j = \begin{cases} 1 & \delta_j > 0 \\ 0 & \delta_j \le 0 \end{cases}$$

And θ_j is the angular position of the j^{th} rolling element which is:

$$\theta_j = \frac{2\pi(j-1)}{Z} + \omega_c t + \theta_0$$

Where z is number of rolling elements, θ_0 is initial angular position of first rolling element and ω_c is cage frequency which can be calculated as:

$$\omega_c = \frac{\omega}{4\pi}\left(1 - \frac{d}{D}\cos\alpha\right)$$

Now if there is a defect on inner race of bearing, it will produce a location of increased clearance, and roller/race contact at the defect is actually unloaded temporarily over a very short time period. Thus every time roller passes to that region of inner race, small impulse of deflection will be produced at roller pass inner race frequency ω_{rpi}. Which is given by:

$$\omega_{rpi} = \frac{z(\omega-\omega_c)}{2\pi} = \frac{Z\omega}{4\pi}\left(1 + \frac{d}{D}\cos\alpha\right)$$

This impulse is taken as a half sine wave in this study. It is actually a reduction in depth of indentation because already compressed roller and races will somewhat expand while passing through defect. So it reduces the restoring force between bearing races and rollers. Amplitude of impulse is maximum when roller center is at middle of the defect.

$$\rho = 0.5\,d - \sqrt{(0.5\,d) - (0.5\,L)}$$

Where ρ is maximum penetration of roller into defect pit (when roller is at the middle of pit). d is rolling element diameter. L is length of defect.

The half sine wave impulse function is given by:

$$\psi = \begin{cases} \rho\sin\left(\frac{\pi}{L}\,dl\right) & If\ dl < L \\ 0 & If\ dl \ge L \end{cases}$$

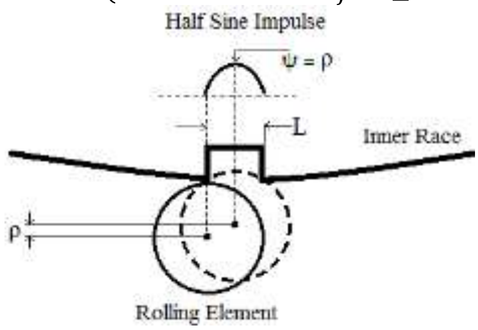

Fig. 3: Impulse function modeling

Where,

$$\begin{aligned} dl &= \theta\ x\ R_i \\ &= mod\,(\theta_{ij}, 2\pi)\ x\ R_i \\ &= mod\{(\omega t + \phi_d - \theta_j), 2\pi\}\ x\ R_i \\ &= mod\left[\left\{\omega t + \phi_d - \left(\frac{2\pi(j-1)}{Z} + \omega_c t + \theta_0\right)\right\}, 2\pi\right] \\ &\quad x\ R_I \end{aligned}$$

Here θ_{ij} is difference between angle covered by defect on inner race and j_{th} roller. Note that angular velocity of defect on inner race is higher than angular velocity of cage (or j_{th} roller). θ_j is the angular distance travelled by j_{th} rolling element till now and $\omega t + \phi_d$ is the angular distance travelled by defect till now. mod(x,y) is a modulo function which is remainder after dividing x by y. θ is the current angular difference between j_{th} rolling element and defect center. Φ_d and θ_0 are initial angular position of defect and first roller from X-axis respectively. R_i is radius of inner race. Distance between roller center and defect center is - dl and calculated by equation: arc length = angle multiplied by radius. Impulse functions will give positive value if this dl is less than defect length or else 0. That means it will only give value when rolling element is passing through defective region and this value is current depth of penetration of roller into defect. Fig.4 explains how we find "dl".

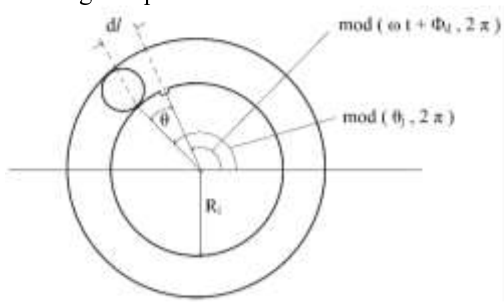

Fig. 4: Linear distance of defect from rollers

So, the total restoring forces of bearing including inner race defect in X and Y directions are:

$$f_x = K\sum_{j=1}^{z} \lambda_j\left(x\cos\theta_j + y\sin\theta_j - \gamma - \psi\right)^{1.1}\cos\theta_j$$

$$f_y = K\sum_{j=1}^{z} \lambda_j\left(x\cos\theta_j + y\sin\theta_j - \gamma - \psi\right)^{1.1}\sin\theta_j \quad (3.2)$$

D. Solution of Nonlinear Equation of Motion

Equation (3.2) is not just nonlinear equation but it includes if else conditions which makes it discontinues function. So an implicit numerical time integration technique so called Newmark-Beta technique is used. It assumes constant acceleration between two instances of time and uses in Taylor's expansion. Newmark truncated Taylor series expansion for displacement and velocity, and gave us following equations:

$$X_{t+\Delta t} = X_t + \Delta t\, X'_t + \left(\frac{1}{2} - \beta\right) \Delta t^2\, X''_t + \beta\, \Delta t^2\, X''_{t+\Delta t} \qquad (3.3)$$

$$X'_{t+\Delta t} = X'_t + (1-\gamma)\Delta t\, X''_t + \gamma\, \Delta t\, X''_{t+\Delta t} \qquad (3.4)$$

In which values of β and γ are 1/4 and 1/2 respectively for constant average acceleration. These equations of displacement $X_{t+\Delta t}$ and velocity $X'_{t+\Delta t}$ is substituted in (3.1) to find next time step's acceleration $X''_{t+\Delta t}$. Substituted governing equation is then solved by non-linear equation solving technique. In this study. A Broyden's quasi-newton method is proposed to solve nonlinear system of equation. The algorithm of Newmark's method is given below:

1) Get constant values - M, K, C, F_0, X_0, X'_0 , T.
2) Find initial acceleration using (3.1)
3) Time t = 0. Choose proper value of Δt.
4) Put values of $X_{t+\Delta t}$ and $X'_{t+\Delta t}$ in governing equation and solve equation for time $t + \Delta t$ to get value of $X''_{t+\Delta t}$. (solve with quasi-newton method)
5) Find values of $X_{t+\Delta t}$ and $X'_{t+\Delta t}$ using above equation.
6) $t = t + \Delta t$
7) Break – if current time t exceeds maximum time limit T, else go to step 4.

A MATLAB code has been made according to above algorithm to get values of X, X' and X''.

V. Results and Discussion

Results are plotted and compared here for single node and system analysis. In both cases, same configuration and inner race defects have been taken. However single node analysis doesn't know what is overhung rotor but still most of bearing defect analysis done with single node because it takes minimum computations and thus time. Using 40 nodes with 10^{-5} accuracy for 5 seconds at 1500 RPM, FEA code and single node analysis gives following results at defective bearing node:

A. Response in Vertical Axis:

Displacement verses time plots are given here for single node and multiple nodes based analysis.

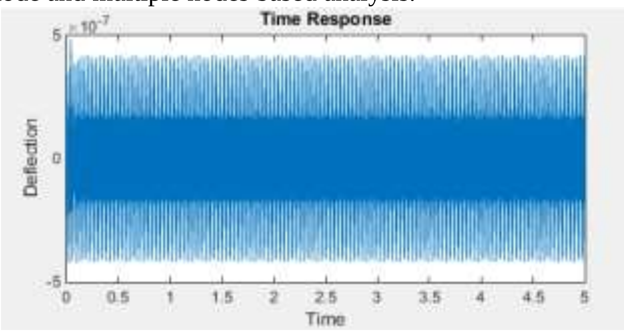

Fig. 5: Response with Single Node Analysis

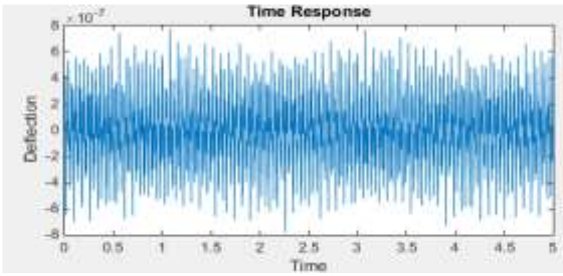

Fig. 6: Response with FEA Considering Entire System

Fig.5 represents stable vibrations with single node. While fig.6 shows vibrations taking whole system into account. Graphs are changed because of gyroscopic effect, shaft stiffness and another healthy bearing influence.

B. Power Spectrum:

The only limitation with defect detection and diagnosis is we cannot predict bearing housing vibrations but we often measure at stationary housing. Above graphs of response are of shaft or inner race of bearing and not bearing housing. So real response what we are measuring at housing would be different. However, one solution is to compare their frequency components because they will be nearly same for both cases.

A displacement response data in time domain is thus converted in frequency domain using fast fourier transform algorithm which is efficient method of a discrete fourier transform. Data obtained from fft is plotted against square of complex component called power density. Power spectrum shows how signals power distributed throughout frequency domain. Plots for power density versus frequency for singlenode and system analysis are given below:

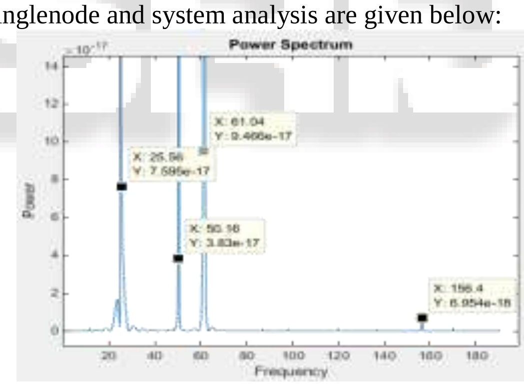

Fig. 7: Power Spectrum taking System

Fig.7 indicates shaft rotating frequency (25 Hz) and roller pass inner race frequency (156 Hz). Other picks are because of other system components effect like rotor-system natural frequencies and varying compliance frequencies.

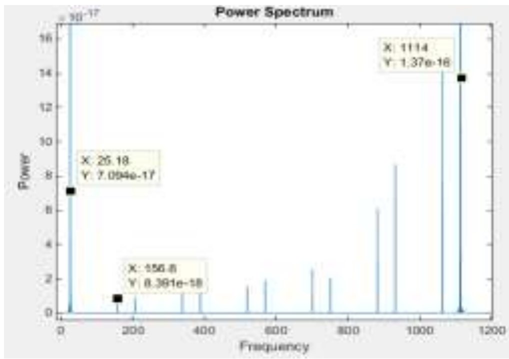

Fig. 8: Power Spectrum with Single Node

Fig.8 indicates shaft rotating frequency (25 Hz), defect frequency (156 Hz) which indicates inner race defect and natural frequency of the system (1114 Hz). Note that other system components doesn't change plot as it is single node analysis.

VI. CONCLUSION

Finite element analysis gives more realistic values than single node analysis one. However it costs more calculation time. Current study provides a method to analyze system with FEA with minimum calculations. Other system components may change the frequency response of system especially in overhung rotors. The current study gives designers a powerful tool for prediction of the trends of instability in rolling element bearing-overhung rotor systems in the presence of local surface defects. The proposed model can be used for design, predictive maintenance and also condition monitoring of machines.

REFERENCES

[1] An Sung Lee, Byung Ok Kim, Yeong-Chun Kim, "A finite element transient response analysis method of a rotor-bearing system to base shock excitations using the state-space Newmark scheme and comparisons with experiments", journal of sound and vibration, vol.297, pp.595-615, November 2006.

[2] Hamdi Taplak and Mehmet Parlak, "Evaluation of gas turbine rotor dynamic analysis using the finite element method", Measurement, vol.5, pp.1089-1097, June 2012.

[3] JiaweiXiang, DongdiChen, XuefengChen, Zhengjia, "A novel wavelet-based finite element method for the analysis of rotor-bearing systems", Finite Elements in Analysis and Design, vol.45, pp.908-916, October 2009.

[4] Sauer, G., and Wolf, M., "Finite Element Analysis of Gyroscopic Effects", Finite Elements in Analysis and Design, vol.5, pp.131-140, July 1989.

[5] S.P. Harsha, K. Sandeep, R. Prakash, "Non-linear dynamic behaviors of rolling element bearings due to surface waviness", Journal of Sound and Vibration, vol.272, pp.557-580, May 2004.

[6] S.P. Harsha, K. Sandeep, R. Prakash, 'The effect of speed of balanced rotor on nonlinear vibrations associated with ball bearings', International Journal of Mechanical Sciences, vol.45, pp.725-740, April 2003.

[7] S.P. Harsha, "Non-linear dynamic response of a balanced rotor supported on rolling element bearings", Mechanism and Machine Theory, vol.41, pp.688-706, June 2006.

[8] Ahmad Rafsanjani, Saeed Abbasion, Hamid Moeenfard, "Nonlinear dynamic modeling of surface defects in rolling element bearing systems", Journal of Sound and Vibration, vol.319, pp.1150-1174, January 2009.

[9] Hamdi Taplak, Selcuk Erkaya, Ibrahim Uzmay, "Experimental analysis on fault detection for a direct coupled rotor-bearing system", Measurement, vol.46, pp.336-344, January-2013.

[10] J.S.Rao., Rotor Dynamics, 1996.

[11] P Seshu, Finite Element Analysis, 2003

