# Development of Rainfall Intensity-Duration-Frequency Relationships from Daily Rainfall Data for the Major Cities in Bangladesh based on Scaling Properties

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*Abstract*—The rainfall Intensity-Duration-Frequency (IDF) relationship is one of the most commonly used tools for various engineering projects against floods. It is important in water resources engineering, either for planning, designing and operating of water resource projects. For estimation of runoff especially for urban areas short duration rainfalls are necessary. Developing countries like Bangladesh short duration rainfall is scarce and data available is mostly for daily rainfall data. In such cases, design rainfall is estimated by approximation and thus leading to frequent failure of drainage network. The purpose of this paper is to develop an empirical formula to estimate design rainfall intensity for seven major cities based on simple scaling properties. This method allows for the estimation of the design value of rainfall of selected return period and durations shorter than a day by using only the daily data. The assumption of simple scaling implies in direct relationships between the moments of different orders and the rainfall durations, which can be used to derive IDF relation¬ships from larger durations.

***Key words:*** Scaling Exponent, Simple Scaling, IDF Relationship, Daily Rainfall, Short Duration Rainfall

## I. Introduction

The rainfall Intensity-Duration-Frequency (IDF) curves express the relation between the intensity, duration and return period of the rainfall. The IDF relationship is one of the most commonly used hydrologic tools for engineers to design storm sewers, culverts, retention basins, and other drainage structures of storm water management systems in urban areas against floods.

The establishment of IDF relationships was done as early as by Sherman (1905) then Bernard (1932). Since then, many sets of relationships have been constructed for several parts of the world: Hershfiled (1961) developed various rainfall contour maps to provide the design rain depths for various return periods and durations. Bell (1969) and Chen (1983) derived the IDF formulae for the United States, Kouthyari and Garde (1992) presented a relationship between rainfall intensity and duration for India.

In order to construct of IDF relationships, a historical series of the maximum rainfall intensities at a higher time resolution is required. Developing countries like Bangladesh short duration rainfalls is scarce and data available is mostly for daily rainfall data. For the regions where data at higher time resolution are scarce, simple scaling properties is used to develop IDF characteristics of short-duration events from daily rainfall. In this regard recently, research has focused on the mathematical representation of rainfall both in time and space, in which development of scaling invariance models is used to derive short-duration rainfall intensity-frequency relations from daily data (Gupta and Waymire, 1990; Burlando and Rosso, 1996; Menabde et al., 1999; De Michele et al., 2002, Yu, et al., 2004; Nhat et al. 2006). Scaling invariance occurs when the connections among the statistical descriptors of a given phenomenon at different scales are constant and defined by a scale factor. The statistical descriptors can be scaled either by a single factor (simple scaling) or by a more complex function of the scale (multiscaling).

Earlier, Matin et al. (1984) developed IDF curve for North-East region of Bangladesh. Recently, Rasel et al. (2015) developed IDF for for North-West region Bangladesh, Chowdhury et al. (2007), Rashid et al. (2012) developed IDF for Sylhet city, Afrin et al. (2015) developed IDF for Dhaka city considering climate change. Among them only Afrin et al. (2015) used scaling properties.

The purpose of this paper is to develop an empirical formula to estimate design rainfall intensity for seven divisional cities (Dhaka, Chittagong, Barishal, Khulna, Rjshahi, Rangpur and Sylhet City) based on simple scaling properties. The benefit of using the principles of scaling is that it reduces the amount of parameters required to compute the quantiles. In this paper, the simple scaling theory has been applied to the IDF characteristics of a short duration rainfall. This method allows for the estimation of the design values of rainfall of selected return period and duration shorter than a day by using only the daily data. Afrin et al. (2015) used annual maximum monthly, daily and 3-hourly rainfall data series and found scaling exponent very close. In this study daily rainfall data is used and up to 10 days rainfalls have been derived to determine scaling exponent.

## II. Study locations


Fig. 1: Seven major cities in Bangladesh

Bangladesh has a tropical monsoon climate characterized by wide seasonal variations in rainfall. About 80% rain falls during the monsoon season in Bangladesh. Heavy rainfall is characteristic of Bangladesh causing it to flood every year. Water logging is one of the major problems of the urban areas. Thus water logging in Dhaka and Chittagong City is not a new problem but the frequency of this problem is increasing day by day. This study carried out for the seven major cities Dhaka, Chittagong, Barishal, Khulna, Rjshahi, Rangpur and Sylhet City in Bangladesh. These cities are the seven divisional headquarters of Dhaka, Chittagong, Barishal, Khulna, Rjshahi, Rangpur and Sylhet division shown in Fig. 1.

## III. METHODOLOGY

### A. Generalized IDF relationships

Generalized IDF relationships are according to Koutsoyiannis et al. [1998] in the form

$$\mathrm{i} = \frac{\mathrm{w}}{(\mathrm{d}^{\nu}+\theta)^{\eta}}, \qquad (1)$$

where i is the rainfall intensity of duration d, and w, ν, θ, and η are non-negative coefficients. Koutsoyannis et al. (1998) also show that the errors resulting from imposing ν=1 in equation (1) are much smaller than the typical parameter and quantile estimation errors from limited size samples of data and considering ν≠1 as a model over-parameterization. Hence, Koutsoyannis et al. (1998) suggested for a given return period, the general IDF relationships as

$$\mathrm{i} = \frac{\mathrm{w}}{(\mathrm{d}+\theta)^{\eta}}. \qquad (2)$$

The coefficients *w*, θ, and η are not independent on the return period and this dependence cannot be arbitrary. The IDF curves for different return periods cannot intersect each other. This restriction, the range of variation of parameters w, θ, and η are limited.

If {w1, θ1, η1} and {w2, θ2, η2} denote the parameter sets for return periods T1 and T2 respectively, with T2<T1, Koutsoyannis et al. (1998) suggest the following restrictions

$$\theta_1 = \theta_2 = \theta \geq 0; 0 < \eta_1 = \eta_2 = \eta < 1; \mathrm{w}_1 > \mathrm{w}_2 > 0 \quad (3)$$

In these restrictions, the only parameter that can consistently increase with increasing return periods is *w* and these arguments justify the formulation of the following general model for IDF relationships:

$$\mathrm{i} = \frac{\mathrm{a(T)}}{\mathrm{b(d)}}, \qquad (4)$$

which exhibits the great advantage of expressing separable relations between *i* and *T*, and between *i* and *d*. In equation (4), $b(d)=(d+\theta)^{\eta}$ with θ>0 and 0<η<1, whereas *a*(*T*) is completely defined by the probability distribution function of the maximum rainfall intensities.

### B. The Scaling Property Defined

Let a continuous precipitation R(t), which represents rainfall rate measured at a point at time t, and introduce a mean rainfall rate over a time duration d:

$$R_{\mathrm{d}}(t) = \frac{1}{\mathrm{d}}\int_{1-\mathrm{d}/2}^{1+\mathrm{d}/2} \mathrm{R}(\tau)\mathrm{d}\tau, \qquad (5)$$

where R(τ) is a time continuous stochastic process rainfall intensity and d is duration. The annual maximum mean rainfall intensity over the time duration d, which is defined as a maximum value of a moving average (5) of width d of a continuous rainfall process in a given year.

The random variable $I_d$ has a cumulative probability function $F_d(i)$, which is given by

$$\Pr(I_d \leq i) = \mathrm{F}_d(i) = 1 - \frac{1}{T_{(i)}}, \qquad (6)$$

where *T* represents the return period.

The scaling property of rainfall intensity *I* for a duration *d* can be expressed by the following relationship (e.g., Menabde, 1999; Yu, 2004)

$$I_d{}^{dist} = \left(\frac{\mathrm{d}}{\mathrm{D}}\right)^{-\eta} \mathrm{I_D} \qquad (7)$$

where the sign of equality refers to identical probability distributions in both sides of the equation, η is the scaling exponent which will be same as defined in (2). This behavior is denoted as 'simple scaling in the strict sense'(Gupta and Waymire,1990).

The relationship between the *q*th moments of rainfall intensity can be obtained after raising both sides of Eq. (7) to the power *q* and taking the ensemble's average (Menabde, et al., 1999; Yu, et al., 2004)

$$E\left[I_d^q\right] = \left(\frac{d}{D}\right)^{-\eta q} E\left[I_D^q\right] \qquad (8)$$

or

$$d^{\eta q} E\left[I_d^q\right] = D^{\eta q} E\left[I_D^q\right] \qquad (9)$$

The only functional form of equation (9) is

$$E\left[I_d^q\right] = f(\mathrm{q}) d^{-\eta q} \qquad (10)$$

where *f*(*q*) is a function of *q* (Menabde et al. 1999). This expression reflects the property of 'simple scaling in the wide sense', meaning that equation (10) is implied by equation (7) but not vice-versa. In the case of 'multiscaling', the exponent of *d*, as in equation (10), would have to be replaced by a non-linear function *K*(*q*). *ηq* represents the scaling exponent of order *q*. The scaling exponent can be estimated from the slope of the linear regression relationships between the log-transformed values of the moments and scale parameters for various orders of moments. The case when the relationship between the scaling exponents and the order of moments is linear is referred to as 'wide sense simple scaling' (Gupta and Waymire, 1990).

The scaling property can also be found for the parameters of a cumulative distribution function (CDF) (Menabde, et al., 1999; Yu, et al., 2004);

$$\mathrm{F}_d(i) = \mathrm{F}_D\left[\left(\frac{d}{D}\right)^{\eta} i\right] \qquad (11)$$

For many parametric forms, equation (11) may be expressed in terms of a standard variant,

$$\mathrm{F}_d(i) = F\left[\frac{\mathrm{i}-\mu_\mathrm{d}}{\sigma_\mathrm{d}}\right]. \qquad (12)$$

where *F* is a function independent of *d*. From equation (11), we get

$$\mu_\mathrm{d} = \left(\frac{d}{D}\right)^{-\eta} \mu_D \qquad (13)$$

and

$$\sigma_\mathrm{d} = \left(\frac{d}{D}\right)^{-\eta} \sigma_\mathrm{D} \qquad (14)$$

By substituting expressions (12), (13), and (14) into equation (6) and inverting it with respect to *i*,

$$i_{d,T} = \frac{\mu_\mathrm{D}\mathrm{D}^{\eta} + \sigma_\mathrm{D}\mathrm{D}^{\eta}\mathrm{F}^{-1}(1-\frac{1}{\mathrm{T}})}{\mathrm{d}^{\eta}}. \qquad (15)$$

By equaling equation (15) to the general model for IDF relationships, as in equation (4), it is easy to verify that

$$a(T) = \mu_D D^{\eta} + \sigma_D D^{\eta} F^{-1}(1 - \tfrac{1}{T}) \quad (16)$$

$$b(d) = d^{\eta} \text{ with } \theta = 0, \quad (17)$$

and

$$i_{d,T} = \frac{\mu + \sigma F^{-1}(1-\frac{1}{T})}{d^{\eta}} \quad (18)$$

where $\mu = \mu_D D^{\eta}$ and $\sigma = \sigma_D D^{\eta}$ are constants. The μ and σ represent respectively the location and scale parameters of the probability distribution fitted to the rainfall intensities of duration *D.*

The simple scaling property, as formalized by equation (10), can be verified by replacing the population moments by the corresponding sample moments about the origin. To check the validity of equation (11), one needs to specify a probability distribution for the annual maximum rainfall intensities. The most frequently used probability distributions, namely the Generalized Extreme Value (GEV) and the EV1 (Gumbel) are examples of functional forms that are compatible with expression (12) and appropriate for the empirical verification of equation (11).

## IV. Application of Simple Scaling Properties and Results

The rainfall data for analysis have been collected from Bangladesh Water Development Board (BWDB). It is daily precipitation data since 1970. The simple scaling model to IDF estimation, as prescribed by equation (18), can be verified through the use of sub-daily rainfall data. However, because sub-daily rainfall data are not available at BWDB, the studies concerning the scale invariance properties performed by using daily data.

The main objective of the study was to develop IDF relationships for short duration(less than 1-day) from daily rainfall data. As sub-daily rainfall data are not available, it is necessary to show that downscaling would be possible from daily data to hourly. Afrin et al. (2015) developed IDF for Dhaka city considering climate change by using Bangladesh Meteorological Department (BMD) rainfall data. They used monthly, daily and 3-hourly rainfall data by applying simple scaling theory and scaling exponent (η) found very close to each others. According to Afrin et al. (2015) the scaling exponent found by downscaling from monthly to daily and daily to 3-hourly are almost equal. So in this study, the scaling exponent (η) has been estimated by using 1 day to 10 days rainfall data series and it is used for short duration (less than 1-day) rainfall. From daily rainfall data 2-days to 10-days rainfall data have been estimated. The scaling properties of extreme rainfall data were investigated by computing the moment of each duration and then by estimating scaling exponent (η) from the slope of the linear regression relationship between log-transformed moments of extreme values and log-transformed duration for various orders of moments. Results show that a linear relationship exists between scaling exponents and orders of moment, which implies that the property of wide sense simple scaling of rainfall intensity exists.

The verification of the simple scaling model can be done by using the sample moments in equation (10), Figure 2, 4, 6, 8, 10, 12 and 14 illustrates the association of log $E[I_d^q]$ with log(*d*), for moment orders *q*=1, 2, 3, 4 and 5 for the city of Dhaka, Chittagong, Rjshahi, Khulna, Barishal, Sylhet and Rangpur respectively. Note that for all orders *q*, there are well defined scale relationships for durations from 1-day to 10-day rainfall.

By representing the exponent of *d* in equation (10) by *K*(*q*) = -η*q* and using the coefficients of the regressions between log $E[I_d^q]$ and log (*d*), it is clear from Figures 3, 5, 7, 9, 11, 13, and 15 the relations between *K*(*q*) and *q* are linear. The linearity of these relations is an argument to confirm the hypothesis of simple scale invariance in a wide sense, as formalized by equation (10).

Taking as an example of Dhaka city, where the coefficient of the linear regression between *K*(*q*) and *q* gives the following estimates of the scale factor: η = 0.703.

The derivation of IDF estimates from 24-hour rainfall can be accomplished by using equation (18), with η= 0.703 and with the estimates of μ and σ, for *D*=24 hours. Gumbel distributions fits well the rainfall intensities of 24 hours with parameters $\mu_{24}$=5.85 mm/h and $\sigma_{24}$= 2.82 mm/h and by the Gumbel inverse function in equation (18) the IDF equation for the Dhaka city,

$$i_{d,T} = \frac{60.04 + 28.98F^{-1}(1-\frac{1}{T})}{d^{0.703}}. \quad (19)$$

Similarly for the others cities, the parameters of the equation (18) has been presented in table 1.

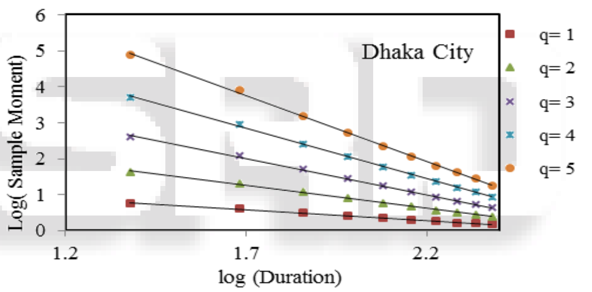


Fig. 2: relationship between sample moment of order q and duration

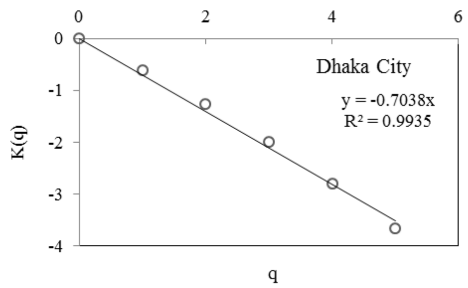


Fig. 3: relationship between K(q) and sample moment order q

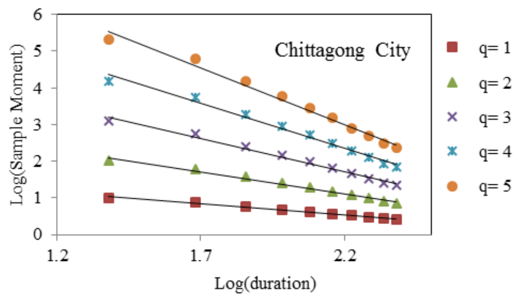


Fig. 4: relationship between sample moment of order q and duration

 

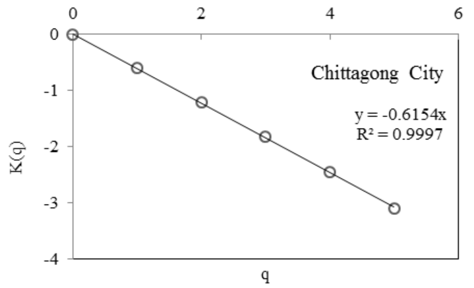


Fig. 5 relationship between K(q) and sample moment order

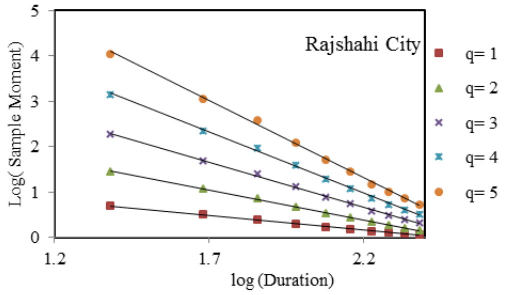


Fig. 6 relationship between sample moment of order q and duration

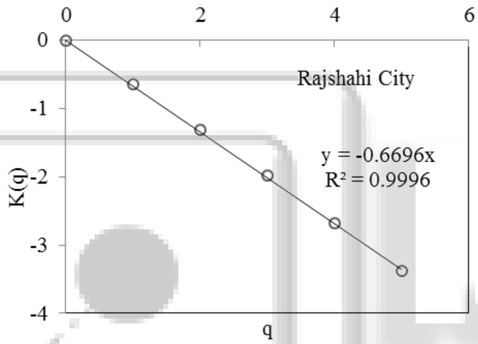


Fig. 7 relationship between K(q) and sample moment order q

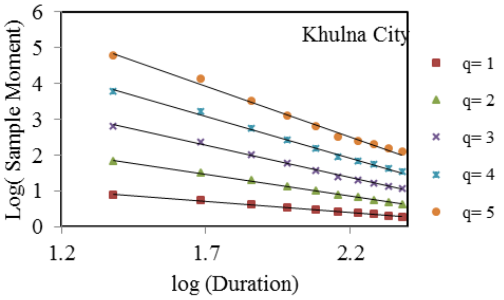


Fig. 8 relationship between sample moment of order q and duration

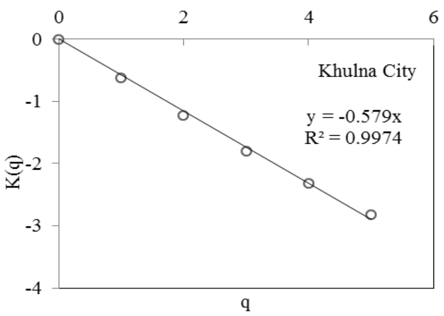


Fig. 9 relationship between K(q) and sample moment order q

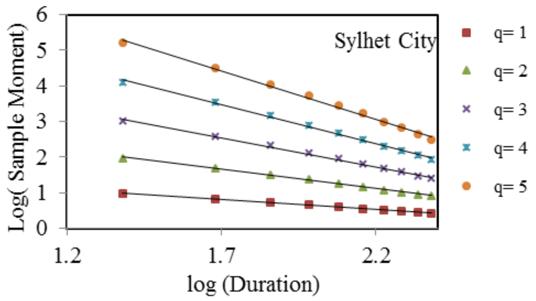


Fig. 10 relationship between sample moment of order q and duration

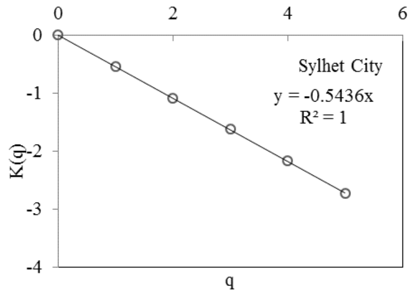


Fig. 11 relationship between K(q) and sample moment order q

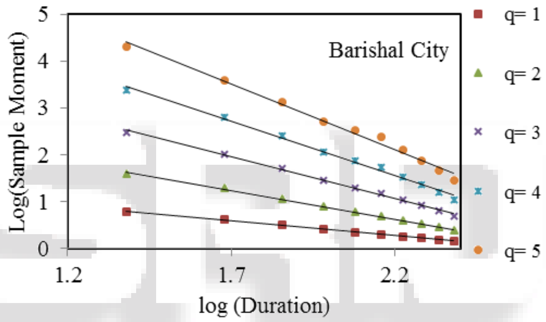


Fig. 12 relationship between sample moment of order q and duration

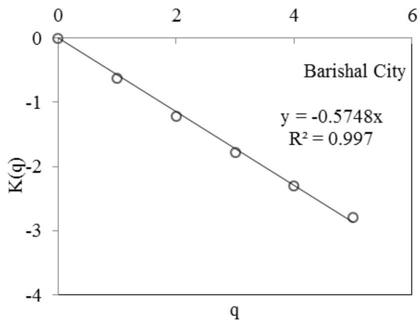


Fig. 13 relationship between K(q) and sample moment order q

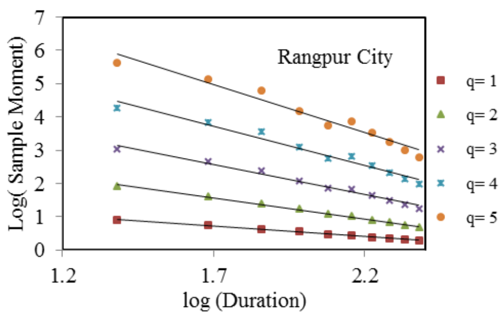


Fig. 14 relationship between sample moment of order q and duration

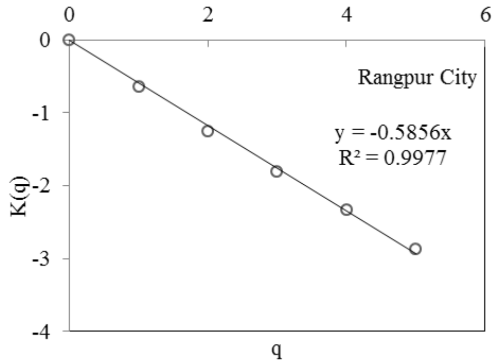


Fig. 15 relationship between K(q) and sample moment order q

| City | μ | σ | η | Remarks |
|---|---|---|---|---|
| Dhaka | 60.04 | 28.98 | 0.703 | |
| Chittagong | 70.28 | 22.11 | 0.615 | |
| Rajshahi | 42.54 | 16.96 | 0.669 | |
| Khulna | 48.05 | 13.82 | 0.579 | |
| Barishal | 35.41 | 12.30 | 0.574 | |
| Sylhet | 51.85 | 15.52 | 0.543 | |
| Rangpur | 50.22 | 23.65 | 0.585 | |

Table 1: Exponent and parameters of IDF equ. (18) in different cities

## V. CONCLUSION

In practical applications, most hydrological studies require short duration rainfall data and generally in developing countries, such short duration data are not available. This study has been conducted to the formulation and construction of IDF relationships using daily rainfall data by simple scaling method. The value of scaling exponent from 1-day to 10-days rainfall data has been validated and thus it could be used for short duration. Hence efforts have been made to develop short duration data from daily data. Results of this study are of significant practical importance because statistical rainfall inferences can be made from a higher aggregation model (i.e. observed daily data) to a finer resolution model (i.e. less than 1 day). Recently, BWDB start to install automated rain gauges stations in different locations in Bangladesh. In future, short duration (15 minutes) rainfall will be available and the findings from this study can be further updated.

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