Study of Fracture in Concrete using Fractals

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Abstract— The characteristics of the fractured surface in a concrete varies from one to the other surface. In order to characterize such surfaces, in this paper the fractal dimension of fracture surface in concrete is determined by box counting method. For evaluating and assessment of fractal dimension various literature available, that are most common studied in this paper. The slit island method and box counting method is discussed in literature survey. From laboratory testing of concrete beam the flexural strength were found and correlates with fractal dimension, which is calculated by software is discussed.

Key words: Fractal Dimension, Slit Island, Box Counting

I. Introduction

It is observed that the concrete structures are full of cracks. Failure of concrete structures typically involves stable growth of large cracking zones and the formation of large fractures before the maximum load is reached. The design is not based on fracture mechanics on up to last decade of 19th century; even though the basic fracture mechanics theory has been available since the middle of 19th century. So why has not fracture mechanics been introduced into concrete design? Have concrete engineers been guilty of ignorance? Not at all. The forms of fracture mechanics which were available until recently were applicable only to homogeneous brittle materials such as glass, or to homogeneous brittle-ductile metals. The question of applicability of these classical theories to concrete was explored long ago the idea of using the stress intensity factor appeared already in the early 1950's (e.g., Bresler and Wollack, 1952) and serious investigations started in the 1960's(e.g., Kaplan). But the answer was, at that time, negative (e.g., Kesler,Naus and Lott, 1971). As is now understood, the reason was that in concrete structures one must take into account strain-softening due to distributed cracking, localization of cracking into larger fractures prior to failure, and bridging stresses at the fracture front. A form of fracture mechanics that can be applied to such structures has been developed only during the last three decade.

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From above discussion it is clear that concrete is a widely used and an important material for structures, the might be of different type, such as highways, airports, bridges and ocean structures material that is required to withstand a large number of cycles of repeated loading. The present trend of designing such structures against the distress due to fatigue loading is largely empirical, gained by many years of experience. As long as the designer is dealing with structures made of similar materials to those for which the relationships were derived, the performance can be reasonably well predicted. However, as conditions change, a need exists for a rational approach. [11]

For the analysis of such fracture surfaces of fractured concrete the concept of the fractal introduced by many as described in next chapter 2. The process of the fracture in concrete is described in next sub chapter.

II. Literature Survey

V. Saouma et al. (1990) shows fractal geometry is used to characterize the roughness of cracked concrete surfaces through a specially built profilometer apparatus. Profilometer is used to calculate fractal dimension. And the fractal dimension is subsequently correlated to the fracture toughness and direction of crack propagation

Longer profiles (24 in.) showed a decrease in (D), which either represents the actual fractal dimension since the profiles cover most of the crack surface, or again, it is the limitations of the hardware/software precision that used a min box size (MBS) of 0.125 rather than 0.045 used with the analysis of 8 in. profiles.

Using very short profiles, (1.0 in.) with 240 readings per inch and much smaller MBS (0.0055), showed an increase in D to 1.17. [3]

Saouma E. et al. [1994] correlates the fractal dimensions and fracture properties in concrete are reported. For calculating fractal dimension wedge splitting tests is conducted. The specimen of length 1.52 m and having 7.62 maximum size of aggregate is used. The test is performed to determine first fracture toughness K_{IC} and fracture energy G_F. Subsequently, one of the split parts was mapped using a profilometer to provide detailed one-dimensional profiles. Finally, the fractal dimension of the profiles was determined by a specially developed computer program. Results are correlated with different properties of fractured concrete.

characteristics	Parallel direction			Perpendicular direction			
	0.75 in	1.5 in	3 in	0.75 in	1.5 in	3 in	1.5 in
Profile length	8.0	8.0	8.0	8.0	24	24	1
Min.box size	0.045	0.04	0.045	0.045	0.125	0.125	0.0055
Readings in inch	240	120	120	348	40	40	240
Fractal dimension (D)	1.13	1.10	1.11	1.12	1.09	1.07	1.18
	1.11	1.09	1.11	1.09	1.07	1.09	1.22
	1.12	1.09	1.11	1.18	1.07	1.08	1.15
	1.10						
	1.10						
	1.10						
	1.12						
	1.12						
	1.10						
	1.10						
	1.10						

Table 2.1 Profiles' geometry and fractal dimensions.

As experimental work a series of series of wedge splitting (WS) tests was performed on specimens 0.91 m and 1.52 m long as shown in figure 4, 40.64 cm thick, and with maximum aggregate sizes up to 7.62 cm. The effects of both aggregate size and the type of aggregate were studied.

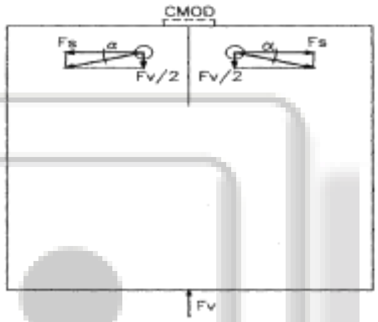

Fig. 2.1: Frontal View of Wedge-Splitting-Test Specimen Sowing Forces Applied to Specimen.

A fully automated profilometer which scans the surface by means of a motorized computer-controlled placement of a linear variable differential transformer (LVDT), was used to determine crack profile with an average resolution of 0.0254 mm. the results obtained from profilometer is shown in table 2.2

Koch curve type	Generation	Fixed grids		Fixed grids		Theoretical value
		D	Grid size	*D*	Grid size	
Triadic	5th	1.37	15	1.3	500	1.262
Triadic	6th	1.3	64	1.3	500	
Triadic	8th	1.2	67	1.2	500	
Quadratic	4th	1.4	17	1.4	500	1.5
Quadratic	5th	1.18	93	1.4	500	
Modified	5th	1.17	93	1.19	500	1.16
Modified	6th	1.4	401	1.18	500	

Table 2.2 Fractal Dimension D versus Profile Orientations.

Comparing the fractal dimension D obtained from 0.91 m and 1.52 m long specimens, there is a clear indication that the longer the specimen size, the lower the fractal dimension everywhere on the fracture surface. This may result from the correlation between D and K_{IC} shows bigger specimens yield slightly higher fracture toughness and smaller D values than smaller specimens. They find no correlation between D and the aggregate size. This confirms two earlier findings: (1) Fracture toughness values are independent from aggregate sizes (Saouma et al., 1991); and (2) the size-effect law (Bazant 1984) is not dependent on aggregate sizes as reported in Saouma et al. (1991).

The conclusion they have made that the fractal dimension of casted specimen has a narrow range from 1.06 to 1.12, The fractal dimension of cracked concrete is insensitive to the orientation of the roughness profile, the fractal dimension near the centre of surface is slightly larger than those computed on the sides of the same surface, For the concrete studied there, fracture toughness increases with a decrease in fractal dimension in concrete.

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Mohsen Issa et al. [2007] addresses the potential application of fractal geometry to characterize the fracture surface and to determine whether there is any correlation between fracture properties and the roughness of the fracture surface. For finding Fracture properties of concrete and properties of roughness and toughness wedge-splitting specimens were prepared, dimensions varying from 420 x 420 x 50 mm to 1680 x 1680 x 200 mm with four different maximum aggregate sizes of 9.5, 19, 38, and 76 mm, were analyzed using a modified slit-island technique.

A. Slit-Island Method:

Following this method, the rough surface was coated with a layer of a second material and carefully ground and polished parallel to a reference plane. After removing the polished material the ‘‘islands’’ of the base materials appear in a ‘‘sea’’ of the coating material. For each contour line of different elevation over the fracture surface, the perimeter, P, and the area, A, were determined and then plotted in a log–log scale. The fractal dimension, D, was obtained from the relation.

$$D = D' + 1 \quad \ldots\ldots\ldots\ldots (1)$$

Where *D*' is determined from a power function of the relationship between the area, A, and perimeter, P, of the islands

$$A = P^{(2/D')} \quad \ldots\ldots\ldots\ldots (2)$$

B. Modified Slit-Island Method:

In this method the fracture surface was placed in dyed water, therefore when viewing from the top, higher regions on the fracture surface appeared lighter than deeper regions. A photo was taken from the top and analyzed by an image analyzer. At each threshold level (intensity of light) increment, new islands appear and their perimeter and area were measured. Then Equations (1) and (2) are used to compute the fractal dimension.

In the experimental procedure nondestructive modified slit-island technique was used to characterize the fracture surfaces of concrete specimens. The configuration and dimensions of three different size wedge-splitting concrete specimens are shown in Figure 2.2 and Table 2.3. The mix proportions and 28-day compressive strengths of four different concrete made with four maximum aggregate sizes are listed in Table 2.4. The fracture surface of the specimen generated as a result of wedge-splitting test was sprayed with a very thin coating of white paint in order to have uniform color. The specimen was placed in a separate water tank and was left to soak for a day or two so that it did not produce bubbles during the measurement procedure. The fractured specimens were then placed in a tank filled with non staining dyed water. The concentration of the dye was adjusted so that the highest and lowest points on the fracture surface were clearly visible.

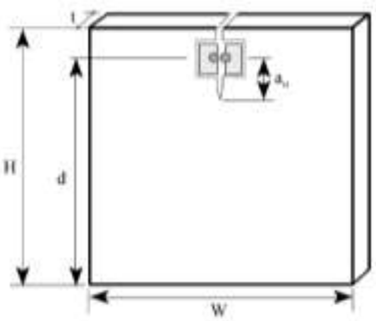


Fig. 2.2: Configuration of wedge-splitting specimen.

Specimen name	Total depth, H (mm)	Depth, d (mm)	Width, W (mm)	Thickness, t (mm)	Initial crack length, a_0 (mm)
S1	1680	1400	1680	200	210
S3	840	700	840	100	105
S4	420	350	420	50	53

Table 2.3 Specimen dimensions

The entire setup, i.e., the tank and the camera, was placed in a black box in order to avoid any artificial effects due to disturbance of light

Pictures were taken using macro-lenses. Black and white pictures were then scanned and processed using an image analyzer. For comparison, two different techniques (area–perimeter and wave number–spectral energy density relationships) were used for fractal analysis. The image of the fractured surface was filtered to remove high spatial frequency noise.

Fig. 2.3: Comparison of fracture surfaces of a typical concrete specimen: (a) photographic view of the original fracture surface, (b) computer simulated fracture surface reconstructed on the basis of the image analysis.

The fractured surfaces of three different size wedge-splitting specimens with dimensions varying from 420 x 420 x 50 mm to 1680 x 1680 x 200 mm (width x total depth x thickness) and made with four different maximum aggregate size of 9.5, 19, 38 and 76 mm were submitted to fractal analysis. The photographic view of the original fracture surface is shown in Figure 2.3. The results obtained from two different techniques were compared. The results of the area– perimeter measurements are plotted on a log–log scale and best fitted with linear regression.

The computation of the fractal dimension for specimens is demonstrated in Figure 2.11 The variation of fractal dimension with maximum aggregate size and specimen size is presented in Table 2.4 and Figure 2.5

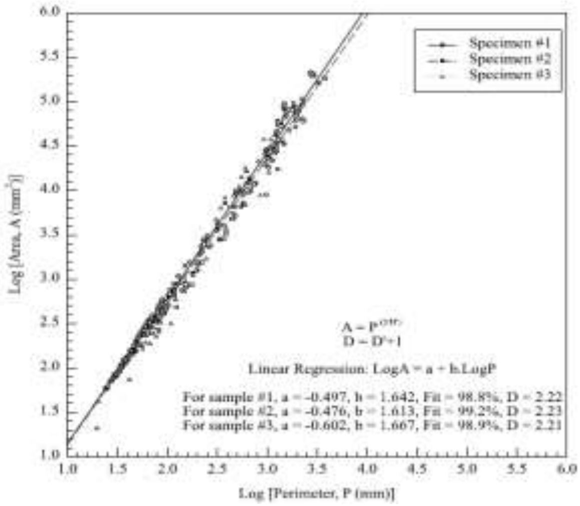


Fig. 2.4: Area–perimeter relations for specimens with maximum aggregate size 19 mm

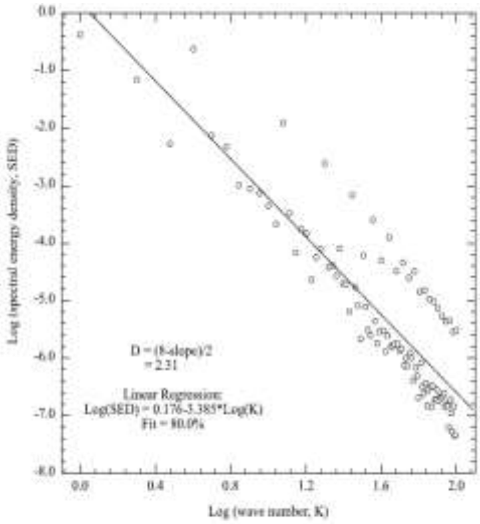


Fig. 2.5: Relationship between spectral energy density and wave number for a typical S3 specimen.

Specimen designation	Maximum aggregate size, mm	Fractal dimension (D)	
		Image pro	Impact pro
S1 (1680 x 1680 x 200 mm)	9.5	2.19	2.28
	19	2.22	2.31
	38	2.25	2.35
	76	2.29	2.39
S3 (840 x 840 x 100 mm)	9.5	2.15	2.26
	19	2.19	2.29
	38	2.21	2.32
	76	2.24	2.34
S4 (420 x 420 x 50 mm)	9.5	2.15	2.25
	19	2.17	2.27
	38	2.20	2.30

Table 2.4 Comparison of average fractal dimensions computed by different software

Fractal dimension results from the three different size specimens were best fitted with a logarithmic equation as shown in Figure 2.4. From Table 2.5 it is clear that fractal dimension increases with maximum aggregate size. In specimens with larger size aggregates, bridging and other forms of crack face interaction take place and cracks form a more tortuous path. For the same maximum aggregate size, the fractal dimension also increases with specimen size. [7]

III. EXPERIMENTAL WORK [23, 24, 25]

In this study nondestructive technique of image analysis system is used to characterize the fracture surface of concrete surface. The configuration and dimensions of the different size of concrete specimens are shown in Figure3.1 and Table 3.1. The mix proportions and 28-day compressive strengths of different concretes are listed in Table 3.2. The specimens were designated based on the specimen size and the design strength i.e 28 days cube strength of concrete. The fracture surface of the specimen generated by testing beam flexural strength and further procedure of image analysis is carried out.

Specimen name	Depth D in mm	Width B in mm	Length L in mm
DA	100	100	550
VB	150	150	550
PM	35	150	550

Table 3.1: Specimen dimensions

The specimen were prepared in laboratory of size given above and design is prepared according to IS standards and mixing procedure adopted as per Indian stander of mixing concrete and cured in curing tank for 28 day. The procedure of mixing is based on weight mix type and mixed in mixer for standard time and revolution as given in IS standards. The coarse aggregate used is angular crushed stone type and fine aggregate use as zone II passing throw 4.75 mm sieve size. Cement used as binding material of PPC type 53 Grade cement.

Specimen	Mix proportion	Concrete comp. strength
DA	1:1.25:2.2	M30
DB	1:1.25:2.2	M30
VB	1:1.50:3.0	M20
PM	1:2	M40

Table 3.2 Mix proportion and compressive strength of concrete in different specimen

The size of coarse aggregate used in concrete is same in all specimen type. The maximum size of aggregate is taken as 12 mm i.e aggregate passing throw 12 mm sieve size. The specimens are tested on UTM for flexural test and fractured surface obtained after testing are as shown in figure 3.2. After testing of specimen the fractal analysis of fractured surface is carried out by mean of software. The technique is used in this software is based analysis of gray scale image.

Fig. 3.2(a): The fractured beams of DA type specimen.

Fig. 3.2(b) The fractured Beams of VB type specimen.

Further for more clarification on fractal dimension analysis the plain mortar specimens also checkout for fractal dimension analysis. The mortar panel is prepared for analysis of ferrocement. There is in the lab a project on ferrocement is also in progress so we have taken the plain mortar specimen for the fractal calculation. Plain mortar panels are specified by PM (as name given in table above), the panels are tested for the flexure.

The fractured surfaces of panels is obtained from test are as shown in figure 3.3.The panels are prepared by mortar of mix 1:3. Fine aggregate use in the mix is taken from zone II, fine aggregate passing throw 4.75 mm sieve size.


Fig. 3.3: The fractured surface of plain mortar panel.

IV. COMPUTATIONAL TECHNIQUE

An image is represented by a set of pixels, each having corresponding intensity value ranging from 0 to 255. The intensity variation of pixel varies from 255 high intensity represent white colour pixel to 0 low level intensity black colour pixels. The software analyzes the given image in input and calculate fractal dimension of rough fractured surface. Fracture analysis system software calculates the pixel intensity of grey scale image. The program require a gray scale image i.e bitmap bmp type image, which then calculates roughness fracture in concrete in terms of fractal dimension. The fracture surface of concrete and mortar is as shown in figure 4.1 and 4.2.

The fractal analysis system software calculate the fractal dimension system by slope of the line plotted between the no boxes intersected the fracture surface to no boxes provided on the image called as box counting method. The fracture analysis software window is shown in figure 4.3


Fig. 4.1: fractured surface of beam DA 2


Fig. 4.2: Fractured surface of mortar panel PM 1

The images shown above are the input form of the computer program (image analysis software) input. The program read the image roughness and shows the capacity of the image. Fractal also called as correlation dimension, simply the fractal dimension is capacity dimension of fractals. The mathematical form of fractal was seen earlier section of literature survey.

In result program gives the values of slope and correlation for graph plotted for the rough surface of concrete specimen. For nearest and best fitting value the correlation is taken and from that software shows the slope value, slope of line shown on graph is fractal dimension or capacity dimension. The input and output window of software is as shown in figure 4.3(a and b) and 4.4(a and b).

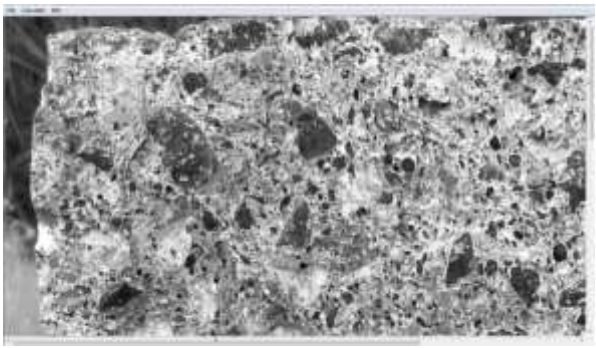


Fig. 4.3(a): Input given to software (fracture surface of concrete)

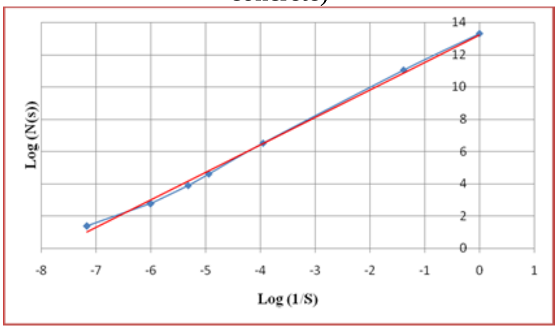


Fig. 4.3(b): Output result from software

Similarly for cement mortar the fractal dimension calculation is done in the software window, which shows the relative less fractal dimension as compare to concrete specimen because of absence of coarse aggregate. As the size of coarse aggregate is increases the fractal dimension is also increases. [7] Figure 3.7 shows the analysis of plane mortar in software in which we can see the difference in the value of fractal dimension value. [16]


Fig. 4.4(a): Input given to software (fracture surface of cement mortar)

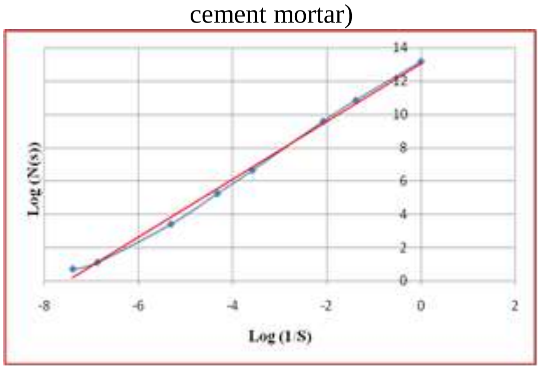
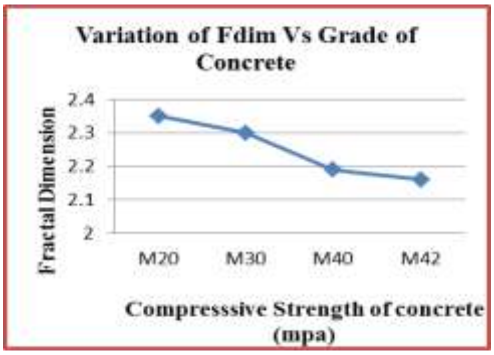


Fig. 3.7(b) Output result from software of cement mortar panel.

V. RESULTS AND DISCUSSION

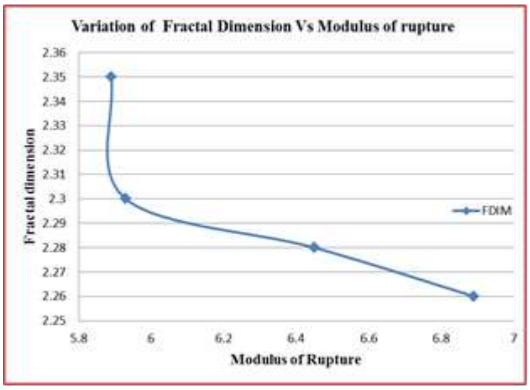
The fractal dimension of mortar specimens PM A and PM B is found in the range of 2.16 to 2.18. Fractal dimension of concrete surface decreases as the strength of concrete increases because of the crack propagates throw the aggregate if the as strength of concrete increases. The variation of fractal dimension is as shown in graph 5.1.



Graph 5.1 Variation of Fractal Dimension Vs Grade of Concrete

Graph shows decrement in the value of fractal dimension with strength, this phenomenon is because tougher the specimen lesser the roughness of fracture surface. From plain mortar specimens with same properties of mortar but difference in size of specimen the fractal dimension is slightly increases with increase in size of specimen. This is attributed to that fact specimen becomes tougher the fracture crack path becomes more tortuous with increase in size. An increase in fracture toughness with an increase in specimen size is most probably associated with the ratio of process zone and specimen size. The greater this ratio, smaller the fracture resistance. Since the process zone is relatively large in a small specimen, its fracture characteristic is lower. As the specimen becomes bigger, the relative size of the process zone becomes smaller, and the fracture characteristics rise.

From test result we have got the modulus of rupture values which are correlated with fractal dimension are as shown in graph 5.2. The relation between Fractal dimension and modulus of rupture of concrete beam is shows same trend as seen fractal dimension Vs. grade of concrete as shown below.



Graph 5.2 Variation of Fractal Dimension Vs. Modulus of rupture.

From above graph the fractal dimension decreases as with increase in fracture modulus. This shows that rougher the surface lower the fractal dimension and higher fracture toughness. Now from fractal dimension one can say that we can predict the fracture properties of concrete.

Now we can drive a relation between fractal dimension, grade of concrete and modulus by an equation as shown below, which is obtained by correlation analysis.

$$f_{cr} = 60.14 - 22.17 \times D - 0.1068\ f_{ck}$$

Where,

f_{cr} = Modulus of rupture in mpa,
f_{ck} = Cube compressive strength of concrete in mpa,
D = Fractal dimension of concrete fractured surface.

VI. CONCLUSIONS

Over 15 Beams of different sizes and strength of concrete were tested for flexural test. Fractographic analysis was then performed on the fracture surfaces. Based on the experimental results, the following conclusions can be drawn

- Concrete surfaces exhibit fractal characteristics over the range of the scales studied.
- The Fractal Analysis system software is used for characterization fracture surface of concrete.
- A good correlation exists between Fractal dimension and grade of concrete.
- For higher grade of concrete fractal dimension concrete surface found lesser value.
- Fractal dimension of plain mortar of two different size found the nearly same values fractal dimension i.e no size effect on fractal dimension.
- A good correlation is found between modulus of rupture concrete beam with fractal dimension.

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