A Review of Analysis of Concrete Beam by Fracture Mechanics Approach

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Abstract— Petersson (1981) had conducted a three-point bend test on notched concrete beam in order to determine the fracture behaviour of the concrete in tension. In this paper, in order to understand actual behaviour of the concrete beam, relevant theory is studied from literature. A finite element analysis is done subsequently to simulate behaviour of the beam. For this, a two dimensional prototype model of the experimentally tested beam is modelled and analysed in a finite element based software. The load-displacement response of the beam is obtained and compared with the experimental results. In order to carry out mesh refinement study the prototype model is discretized into three different mesh sizes as a coarse, medium and a fine mesh. Further, the influence of shape of stress-displacement relations on the load-displacement response is also discussed. The absolute stress-displacement curve for concrete given by Petersson is approximated by 1-segment, 2-segment and a 4-segment curve. Three different approximations for the shape of stress-displacement relations are used to evaluate the sensitivity of the degree of the curve.

Key words: Fracture Mechanics, Finite Element Method, Tension Softening

I. INTRODUCTION

Concrete is a heterogeneous anisotropic non-linear inelastic composite material, full of flaws that may initiate crack growth when it is subjected to stress. Concrete is strong in compression but weak in tension. That is, concrete can withstand considerable amount of compressive loads but it cannot take much of tensile stresses. When tensile loads are applied, concrete undergoes fracture easily. The reason behind this phenomenon is, the aggregates in concrete are capable of taking compressive stresses so that concrete withstands compressive loading. But during tensile loading cracks are formed which separates the cement particles which hold the aggregates together. This separation of cement particles causes the entire structure to fail as the crack propagates. Failure of concrete typically involves growth of large cracking zones and the formation of large cracks before the maximum load is reached. This fact, and several properties of concrete, points towards the use of fracture mechanics.

The need to apply fracture mechanics, results from the fact that classical mechanics of materials techniques are inadequate to handle cases in which severe discontinuities, such as cracks, exists in a material. For example, in a tension field, the stress at the tip of a crack tends to infinity if the material is assumed to be elastic. Since no material can sustain infinite stress, a region of inelastic behaviour must therefore surround the crack tip. Classical techniques cannot handle such complex phenomena. The discipline of fracture mechanics was developed to provide techniques for predicting crack propagation behaviour.

Fortunately, the Finite Element Method (FEM) is sufficiently general that it can model continuum mechanical phenomena as well as discrete phenomena such as cracks and interfaces. Engineers performing finite element analysis of reinforced concrete structures over the past years have gradually begun to recognize the importance of discrete mechanical behaviour of concrete. Methods of determining the ultimate strength of concrete based on linear elasticity or plasticity is widely developed. However, the possibilities for analysis in the serviceability limit state, e.g. deflection and crack width estimations, are lacking. For structures with a complex geometry the Finite Element Method, must be used, which should be able to model the cracking of the concrete.

Since concrete is a complicated material in order to model such material within the finite element framework, a proper material model should be used. The model should be capable of representing both the elastic and plastic behaviour of concrete in compression as well as in tension. There are constitutive models suitable for brittle material such as concrete in which cracking is important phenomena. These models are intended for unreinforced as well as reinforced concrete structures.

Crack formation and crack growth plays an important role in the performance of the concrete members. Several different models have been proposed to characterize mode-I crack propagation in concrete. The fictitious crack model proposed by Hillerborg et al (1976) and the crack band theory developed by Bazant & Oh (1983) are particularly well suited for a finite element analysis. Petersson (1981) have done experimental investigations to determine the fracture properties of concrete and he applied the finite element method to the fictitious crack model by using the substructure method and superposition method and obtained the load-displacement response of concrete under a three point bend test.

II. LITERATURE SURVEY

Kaplan (1961) seems to have been the first to have performed physical experiments regarding the fracture mechanics of concrete structures. He applied the Griffith (1920) fracture theory (LEFM) to evaluate experiments on concrete beams with crack simulating notches. Kaplan concluded, with some reservations, that the Griffith concept of a critical potential energy release rate or critical stress intensity factor being a condition for crack propagation is applicable to concrete. His reservations seem to have been justified, since more recently it has been demonstrated that LEFM is not applicable to typical concrete structures.

D. Ngo and A.C. Scordelis (1967) performed a numerical simulation of discrete cracks in concrete. At the same time, Rashid (1968) successfully applied a smeared crack model for concrete. Discrete crack simulation aims at the initiation and propagation of dominating cracks, whereas the smeared crack model was based on the observation, that the heterogeneity of concrete leads to the formation of many, small cracks which, only in a later stage, nucleates to form one larger, dominant crack. The smeared crack model captures the deterioration process by smearing the effect of micro-cracks, that is, a reduction in stiffness, over a given volume.

A. Hillerborg, M. Modeer and P. E. Petersson (1976) presented a method in which fracture mechanics is introduced into finite element analysis by means of a mathematical model. For this they have made assumption that the stresses are assumed to act across a crack as long as it is narrowly opened. The basic idea of their proposed model is similar to the Barenblatt model which assumes that there is a plastic zone near the crack, and the stresses are assumed to vary with the deformation. Whereas in the proposed model, when the crack opens the stresses are not assume to fall to zero at once, but decreases with the increase in crack width as the crack propagates. The special feature of the proposed method was that it explains not only growth of existing crack but also the formation of new cracks, as it was assumed that cracks start forming when the tensile stress reaches tensile strength of the material. Finally concluded that the proposed method of combining fracture mechanics with finite element analysis seems to yield realistic results regarding crack formation and propagation as well as regarding failure even if a coarse element mesh is used which opens up the possibilities of studying complicated problems with a limited amount of computer work.

Zedenek P. Bazant and B. H. Oh (1983) extended Hillerborg, Modeer and Peterssons work to a smeared crack band model. They have developed a fracture theory for heterogeneous aggregate material which exhibits a gradual strain softening due to micro-cracking. They have considered only mode I fracture behaviour which is modelled as a blunt smeared crack band. They derived simple Tri-axial stress-strain relations which model the strain-softening and describe the effect of gradual micro-cracking in the crack band. They have shown that material fracture properties are characterized by only three parameters, fracture energy, uniaxial strength and width of crack band which is width of fracture process zone and, the strain softening modulus is the function of these parameters. A method for determining the fracture energy from measured complete stress strain relations was also given. Finally they have given a simple formula to predict from tensile strength and aggregate size the fracture energy, as well as the strain softening modulus.

Jenq and Shah (1985) proposed a two parameter fracture model to characterize the fracture process of concrete and to include nonlinear slow crack growth i.e fracture process zone, prior to peak load. The critical stress intensity factor and the critical crack tip opening displacement were the two fracture criteria. Critical stress intensity factor was calculated at the tip of the effective crack, and the critical effective crack extension was obtained by the elastic critical crack tip opening displacement. To verify the validity of the proposed model, notched beams were tested in a three-point loading arrangement. Three different beam sizes and five different mix-proportions were employed. Based on tests results, it was concluded that the stress intensity factor and the crack tip opening displacement were size independent. These two parameters, for any given geometry, can be calculated using linear elastic fracture mechanics. By Using this model, it was demonstrated that the uniaxial tensile strength of unnotched concrete specimens was size independent, but the modulus of rupture was dependent on the beam size. Using the proposed model, it is possible to calculate the maximum load for mode I failure of a given structure with an arbitrary geometry. This was demonstrated by calculating the peak loads for the uniaxial tension and flexural beams. Unlike the fictitious crack model the two- parameter fracture model by Jenq and Shah does not require any post peak strain softening constitutive law.

Karihaloo and Nallathambi (1989) presented an effective crack model by assuming that the effect of various non linear energy consuming processes taking place in the fracture zone can be represented by a supplementary traction free crack. They presented an improved expression for the calculation of effective notch depth in three-point bend specimens used for the determination of the fracture toughness of plain concrete. They have calculated the total mid span deflection as the superposition of the mid span deflection of an unnotched beam in three-pont bending and the additional deflection due to presence of the notch. It was achieved by using the exact elasticity solution for the mid span deflection of the unnotched beam and, by calculating the additional mid span deflection due to the presence of the notch from the corresponding expression for the stress intensity factor of a three-point bend specimen. The predictions of the improved effective crack model were shown to be in good agreement with those of the two parameter model by Jenq and Shah.

J. P. Ulfkjaer, S. Krenk and R. Brincker (1992) have presented an analytical model for fictitious crack propagation in concrete beam proposed by A. Hillerborg et al. For this they have combined the two simple models, the fracture is modelled by a fictitious crack in an elastic layer around the mid-section of the beam and outside the elastic layer the deformations are modelled by the Timoshenko beam theory. In the proposed method, the failure of a simply supported beam loaded in three point bending was modelled by assuming development of a single fictitious crack at the mid-section of the beam. A simple model for calculation of load-displacement curves of notched and un-notched concrete beams in three point bending was presented. Finally they have concluded that, the point on the load-displacement curve where the fictitious crack starts to develop, and the point where the real crack starts to grow will always correspond to the same bending moment, the points lying on each side of the peak point.

III. Objective

Crack formation and crack growth plays an important role in the performance of the concrete members. Several different models have been proposed to characterize mode-I crack propagation in concrete. The fictitious crack model proposed by Hillerborg et al (1976) and the crack band theory developed by Bazant & Oh (1983) are particularly well suited for a finite element analysis. Petersson (1981) have done experimental investigations to determine the fracture properties of concrete and he applied the finite element method to the fictitious crack model by using the substructure method and superposition method and obtained the load-displacement response of concrete under a three point bend test. In the present study we are going to use a finite element based software as a numerical tool to analyse the concrete fracture. The fracture parameters which were determined experimentally are used as material properties for the three point bend test and the load-displacement response for the notched beam is compared with the experimental results.

In order to simulate the fracture behaviour of concrete beam using finite element method, first a mesh refinement study is carried out to understand the effect of mesh sensitivity, and then the influence of various tension softening models on the load-displacement response of the beam is studied.

IV. COMPUTATIONAL MODEL

In this work, a two-dimensional computational model for the three point bend test on notched plain concrete beam is modelled. The geometry for the proposed model is shown in Figure 1 and the material properties of the beam is given in Table 1.

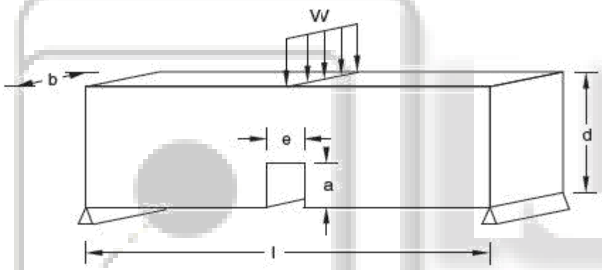


Fig. 1: Proposed model for the plain concrete notched beam

Where,

l - length of beam (2m), d - beam depth (0.2m)

b - beam width (0.05m), a - notch depth (0.01m)

e - notch width (0.04m)

Youngs modulus (E)	30 GPa
Poissons ratio (μ)	0.2
Density (ρ)	2400 kg/m^3
Fracture energy (G_f)	124 N/m
Failure stress (f_t)	3.33 MPa

Table 1: Material properties

Because of the symmetry, only half of the beam is modelled. In order to study the influence of mesh refinement on the results the model is discretized into three different mesh sizes as a coarse, medium and a fine mesh. Figure 2 shows the half two-dimensional prototype model with all three meshes.

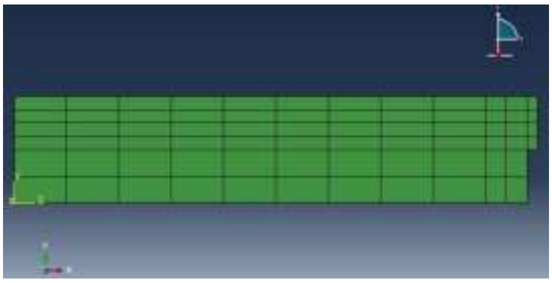


Fig. 2(a): Coarse mesh

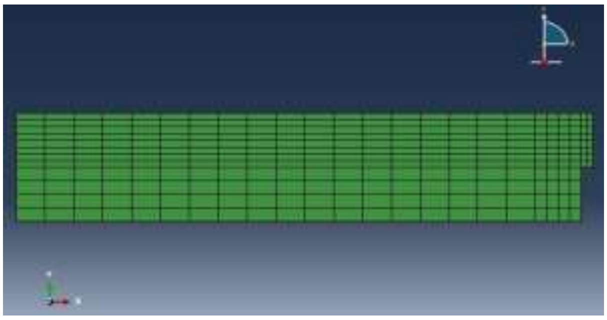


Fig. 2(b): Medium mesh

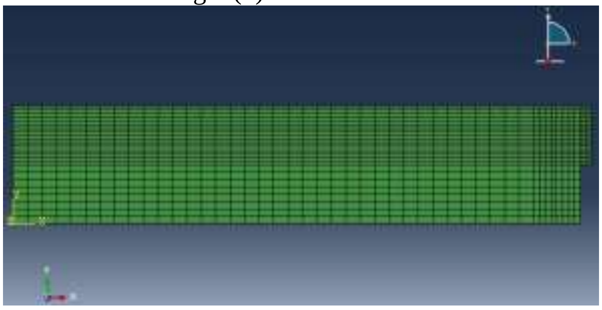


Fig. 2(c): Fine mesh

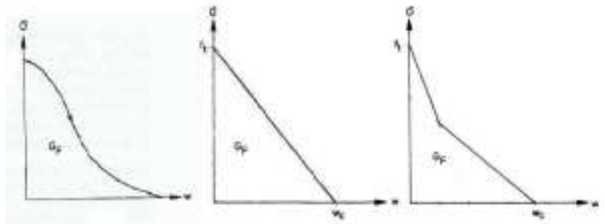
Fig. 2: Finite element meshes of half of notched beam.

A. *Loading Condition*

As in the three-point bend test of the notched beam, a displacement control testing machine was used in order to obtain stable load-displacement curve, in our analysis also the beam is loaded by prescribing a vertical displacement at the centre until it reaches a value of 0.0015m.

B. *Tension Softening Relation*

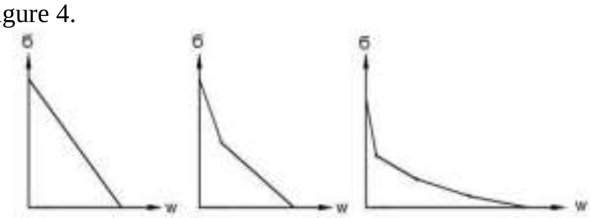
Petersson had given the absolute stress-displacement curve for the concrete and for the analytical calculations they have approximated the curve to a single-line and a two-line relation as shown in Figure 3.



a) Absolute b) Single-line c) Two-line

Fig. 3: Stress-Displacement curves (σ-w) for concrete (Petersson 1981).

From the experimentally observed stress-displacement response, stress and displacement values were extracted to define the tension softening behaviour of the beam, as given in Table 2. In our prototype model we have used three tension softening models to evaluate the sensitivity of the degree of the stress-displacement curve, as 1-segment, 2-segment and 4-segment model as shown in Figure 4.



(a) 1-Segment (b) 2-Segment (c) 4-Segment

Fig. 4: Tension softening models

(σ - w) relation	Yield stress (MPa)	Displacement (m)
TS 1 – Segment	3.33 x 10^6	0
	3.33 x 10^4	7.447 x 10^{-5}
TS 2 – Segment	3.33 x 10^6	0
	0.833 x 10^6	2.5 x 10^{-5}
	3.33 x 10^4	1.98 x 10^{-4}
TS 4 – Segment	3.33 x 10^6	0
	0.833 x 10^6	2.20 x 10^{-5}
	0.416 x 10^6	3.50 x 10^{-5}
	0.208 x 10^6	4.60 x 10^{-5}
	3.33 x 10^4	6.86 x 10^{-4}

Table 2: Stress and displacement values used in tension softening models

V. RESULTS AND DISCUSSION

To validate the modelling strategy employed in the prototype model, simulated results of the prototype model are compared to the experimentally observed results.

A. Mesh Refinement Study

For the mesh refinement study linear stress-displacement relation is considered and compared with the Peterssons single-line approximation model i.e Petersson 1-seg as shown in Figure 5.

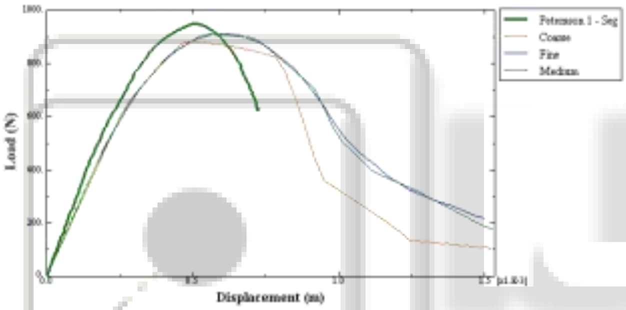


Fig. 5: Comparison of load-displacement response of prototype model with experimental results for mesh refinement

These curves shows that the load-displacement response for all three meshes are in close agreement with the experimental results. The medium mesh and fine mesh response is quite similar. Here we can conclude that the load-displacement response can be efficiently simulated using medium mesh as it matches with the experimental response, thus we don't need to refine the mesh further. As it will affect the economy and computational time therefore medium mesh is sufficient to get satisfactory results.

B. Influence of Tension Softening

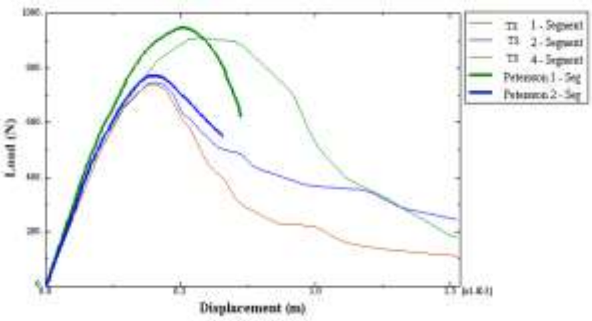


Fig. 6: Comparison of load-displacement response of prototype model with the experimental results for tension softening study

The comparison of the peak load obtained for the prototype model with the experimental is given in table 3.

(σ - w) relation	Experimental	Prototype model	Discrepancy
TS 1 – Segment	946 N	908 N	4%
TS 2 – Segment	771 N	746 N	3%
Difference	18.5 %	17.8%	-

Table 3: Comparison of peak load obtained using prototype model with experimental

Thus it can be concluded that the shape of stress-displacement relation has a significant influence on the results. We have used three different approximations for the experimentally observed stress-displacement relation to evaluate sensitivity of the degree of the stress-displacement curve. Form the results it can concluded that improving the approximation from 1-segment curve to a 2-segment curve causes a significant drop in the peak load of approximately 18% and the shape of the load-displacement curve changes significantly.

VI. CONCLUSION

From the results of mesh refinement study and the influence of tension softening models following conclusions can be drawn:

- By using finite element method we can model the actual beam by discretizing into a medium mesh, as it gives satisfactory results.
- We have compared the results by using different tension softening models for tensile behaviour of concrete, the results obtained are in close agreement with the experimental one.
- From the results of tension softening models it can be concluded that the shape of the stress-displacement curve have significant influence on the peak load.
- Further it can be seen that, approximating the stress-displacement relation from 2-segment curve to a 4-segment curve, does not changes the results significantly which indicates that the 2-segment approximation of the stress-displacement relation is sufficient.
- From the results it is clear that 2-segment and 4-segment softening models gives peak responses that agree well with the experimental observations of Petersson(1981). The small discrepancy between results is because a relatively blunt notch is used in the prototype model of beam, while a much sharper cast notch was used in experiment.

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