Effect of Medium to High Grade of Concrete and Depth on Moment-Curvature Relationship for Reinforced Concrete Beams Section

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Abstract— This paper deals with the development of computer program to calculate the moment-curvature relationship of the reinforced concrete rectangular beam. The moment-curvature relationship helps in understanding the strength characteristics and ductility properties of the beam. The moment-curvature relation at the hinge is required in the nonlinear analysis. The characteristics of the beams under flexure can be detected by the moment-curvature relationship. The influence of some material and geometric parameters like the grade of concrete and depth of the beam section respectively in singly and doubly reinforced beam are examined and discussed.

Key words: Moment, Curvature, Nonlinear analysis, Reinforced Concrete Beams, Ductility, High strength Concrete

I. INTRODUCTION

In the present days, the limit state design has created a need for a better understanding of the nonlinear analysis. Almost all problems which are concerned with the inelastic deformation properties of reinforced concrete cross sections need the understanding of relationship between Moment and Curvature.

A structure is composed of many structural materials which forms a member by combination of each material section. Hence the nonlinear behavior of the section causes the nonlinear behavior of the structure [1]. The nonlinear analysis of the Reinforced Concrete beams can be conducted with the moment-curvature relationship as most of the deformations in the beams are associated with flexure.

The main requirement in finding the moment-curvature relationship of a flexural member is the knowledge about its variables. In the design of structures strength and ductility are the important factors. Over the past two decades, the development of high-strength concrete has been evolved. With the increase in the research and the increase in the usage of the high strength concrete, importance is given to strength aspects along with the ductility characteristics [2]. The moment-curvature relation helps in finding the strength and ductility of the beam section.

II. STRESS-STRAIN RELATIONSHIP

It is very important to study the stress-strain relationship to find the moment-curvature relationship as the forces in the beam section totally depend upon the stress at the extreme compression fiber. The relation between the Stress-Strain of a material is called the Stress-Strain curve of that material. The stress-strain curves for concrete and cold worked steel given in IS: 456- 2000 are used to calculate the stress for the required strain.

The design compressive stress can be obtained for any required strain by the following equation [3]

$$Fc = \{0.447Fck\,[2(\varepsilon/0.002) - (\varepsilon/0.002)^2]\} \qquad (1.1)$$

Where,

Fc= Compressive stress (N/mm^2)
Fck= Characteristic strength of concrete (N/mm^2)
ε = Strain

III. MOMENT-CURVATURE RELATIONSHIP

The moment-curvature relationship helps in studying the flexural behavior of the non-linear material like reinforced concrete. The capacity of a beam section to resist the bending moment is known as moment of resistance of that section. Curvature is the angle change in the slope of elastic curve per unit length [3].

The non-linear analysis of the RC beams can be conducted with the moment-curvature relation constructed by the section analysis as most of the deformations in the beams arise from strains associated with flexure. Moment-curvature relationship helps in predicting the behavior of reinforced concrete member under flexure and also in fixing the plastic hinge positions in the members [4].

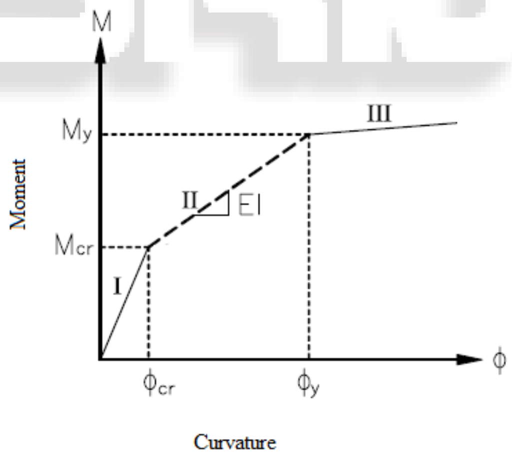

Fig. 1: Generalized moment-curvature relationship of RC sections. (Hyo-Gyoung 2002).

ϕ_{cr} = Curvature at cracking

ϕ_y = Curvature at yielding

The generalized curve of moment-curvature for the RC section shown in fig. 1 can be divided into three stages. First stage is the elastic stage where the moment carrying capacity increases with curvature without cracking. Second stage is the cracking stage in which the tension steel yields to form the cracks. Third stage is the ultimate stage where the curvature goes on increasing with least change in the moment carrying capacity [5].

IV. PROCEDURE TO FIND MOMENT-CURVATURE RELATIONSHIP

The following procedure has been adopted to find the moment-curvature relation of a rectangular RC beam used in this study.

[1] Set the initial strain at the extreme compression fiber with a small value like 0.001
[2] Assume the depth of neutral axis from the extreme compression fiber.
[3] With the initial strain, the strains at the centroid of the compression and tension reinforcement are calculated
[4] The stresses corresponding to the strains are found out by utilizing the stress-strain curves given in the IS-456: 2000.
[5] The compressive force is found by the numerical integration of the equation of stress over the compressive region.
[6] The tensile force is calculated by multiplying the area of steel on tension side with the stress in the reinforcement.
[7] If the tensile force is equal to the compressive force then the considered neutral axis depth is accurate or else the depth of neutral axis is changed and steps are repeated.
[8] The whole procedure is repeated with different values of the initial strains.
[9] The moment v/s curvature curve for different initial strains is plotted.

A computer program has been developed to generate the moment-curvature relationship for a rectangular beam section according to the procedure given above.

V. BEAM DETAILS

The reinforced concrete section is designed and the details are given in the table 1 for singly and doubly reinforced sections. The moment-curvature relationship for singly and doubly reinforced beam sections is calculated with different grade of concrete and the depth of the beam section.

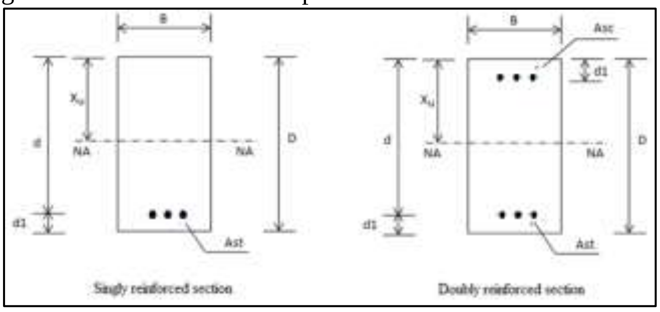

Fig. 2: Dimensions of Section

Ast – Area of steel in tension
Asc – Area of steel in compression

Variable	Singly reinforced	Doubly Reinforced	Units
B	250	250	Mm
D	450	350	Mm
d1	25	25	mm
d	425	325	mm
Fck	20/30/40/50	20	N/mm2
Fy	415	415	N/mm2
Ast	940.4982	1124.1168	mm2
Asc	-	351.13809	mm2

Table 1: Beam Details

VI. RESULTS AND DISCUSSIONS

A. Effect of Grade of Concrete on Moment-Curvature Relationship:

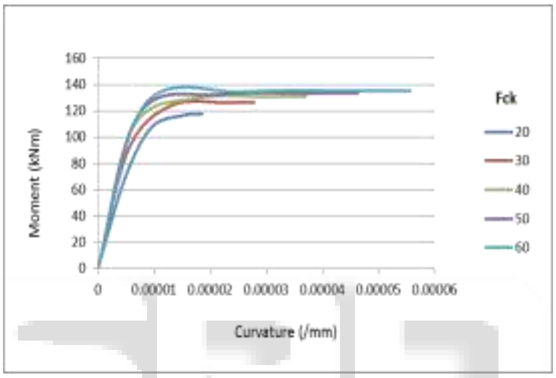

Fig. 2: Moment-Curvature relationship of singly reinforced beam with varying Fck

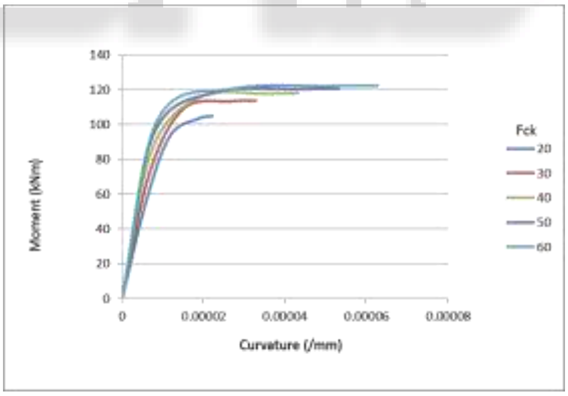

Fig 3: Moment-Curvature relationship of doubly reinforced beam with varying Fck

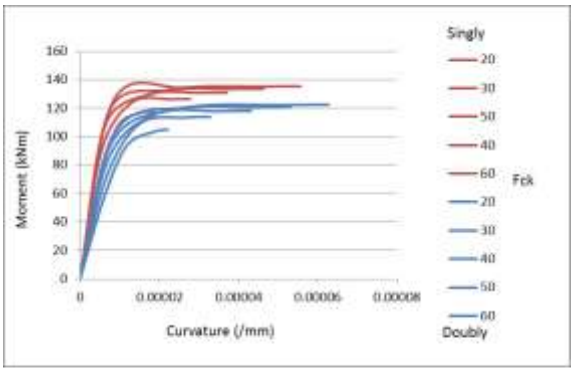

Fig. 4: Combined Moment-Curvature with varying Fck

The variation in the moment-curvature for varying grade of concrete from medium strength concrete (M20) to high strength (M60) in case of singly reinforced beam sections and doubly reinforced beam sections are plotted in fig 2 and fig 3 respectively. Fig 4 shows the combined moment-curvature relationship curves for both the sections with increasing initial strain at the extreme compression fiber from 0.001 to 0.0035. It can be observed that, as the grade of concrete increases from 20 to 60, the moment carrying capacity moderately increases about 15.09 % for singly reinforced section and 16.61 % for doubly reinforced sections.

With the increase in the grade of concrete, the concrete becomes stiffer and bendability of the beam decreases which reduces the radius of curvature of the beam which in turn increases the section curvature as it is inversely proportional to the radius of curvature.

	Fck	Ductility	% increase
SR	20	3.886028	-
DR		3.709618	-
SR	30	5.056638	23.15
DR		4.919091	24.58
SR	40	5.853594	13.61
DR		5.93359	17.09
SR	50	6.273302	6.69
DR		6.652352	10.80
SR	60	6.554953	4.29
DR		7.082909	6.07

Table 2: Ductility of the beam section with varying Fck
SR = Singly reinforced
DR= Doubly reinforced

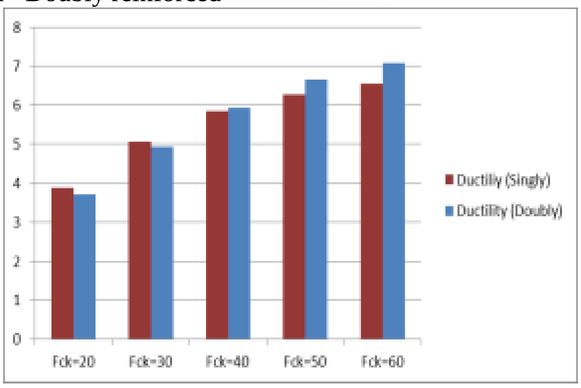

Fig. 5: Ductility with varying Fck

Fck	Initial strain	Singly reinforced		Doubly reinforced	
		Moment	Curvature	Moment	Curvature
20	0.000	0	0	0	0
	0.001	68.21615	4.77E-06	56.65562	5.98E-06
	0.002	107.4956	9.55E-06	93.58101	1.18E-05
	0.003	116.6934	1.54E-05	103.2992	1.85E-05
	0.0035	117.6162	1.85E-05	104.9926	2.22E-05
30	0.000	0	0	0	0
	0.001	91.21761	5.5E-06	71.84822	6.68E-06
	0.002	124.2850	1.32E-05	109.3131	1.51E-05
	0.003	126.2691	2.29E-05	113.4166	2.67E-05
	0.0035	126.4961	2.78E-05	113.8642	3.28E-05
40	0.000	0	0	0	0
	0.001	108.0434	6.34E-06	85.25636	7.28E-06
	0.002	129.6747	1.75E-05	116.0552	1.93E-05
	0.003	130.7658	3.06E-05	117.822	3.51E-05
	0.0035	130.9361	3.71E-05	118.1774	4.32E-05
50	0.000	0	0	0	0
	0.001	118.2568	7.39E-06	95.91915	7.99E-06
	0.002	132.591	2.18E-05	118.9677	2.36E-05
	0.003	133.4639	3.82E-05	120.4678	4.33E-05
	0.0035	133.6001	4.64E-05	120.7344	5.31E-05
60	0.000	0	0	0	0
	0.001	125.5685	8.49E-06	103.4474	8.86E-06
	0.002	134.5351	2.62E-05	120.8349	2.76E-05
	0.003	135.2625	4.58E-05	122.1868	5.1E-05
	0.0035	135.376	5.56E-05	122.4411	6.28E-05

Table 3: Moment and curvature at different initial strains and Fck

The ductility of the singly reinforced and the doubly reinforced concrete beam sections are shown in the fig 5. It shows gradual increase in ductility with the increase in the grade of concrete. Initially the percentage increase of the ductility in the singly reinforced section is less than that of the doubly reinforced section up to M40 grade then the percentage increase of ductility for M50 and M60 is more for doubly reinforced section than the singly reinforced section as depicted in the table. But the magnitude of ductility is more in singly reinforced sections than doubly reinforced sections for M20 and M30. Further increase in the grade of concrete up to high strength (M60), the magnitude of ductility is more for doubly reinforced section compared to the singly reinforced sections. The stiffness which is the resistance to the bending of the beam decreases along with the increase in the strength of concrete.

B. Effect of Depth of Beam on Moment-Curvature Relationship:

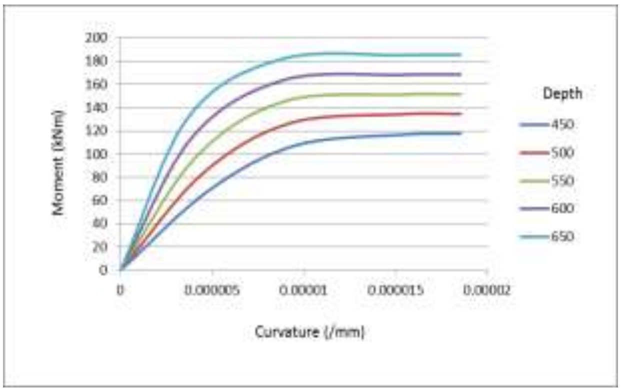

Fig. 6: Moment-curvature of a beam section with varying depth.

The moment-curvature relationship of a rectangular beam section with varying depth of cross section is studied and is as shown in fig 6. The depth is increased by equal intervals of 50 mm from 450 to 650 mm and graph is plotted. With the increase in depth of the beam section from 450 to 650, the moment carrying capacity increases by 57.71 % whereas the curvature of the section remains more or less the same.

Depth	Ductility	% increase
450	3.88603	-
500	4.18181	7.0730156
550	4.46383	6.3179619
600	4.73385	5.7041056
650	4.99329	5.1957214

Table 4: Ductility of beam section with varying depth

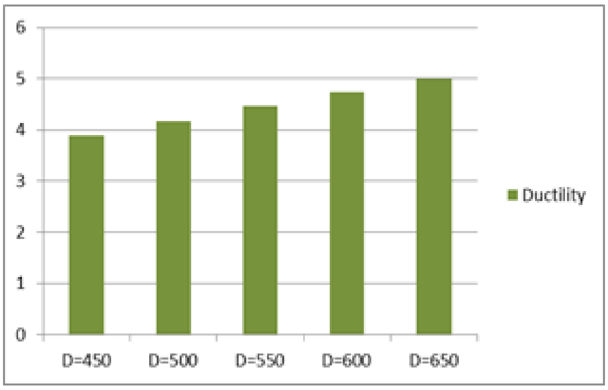

Fig. 5: Ductility with Varying Depth

Depth	Initial strain	Beam section	
		Moment	Curvature
450	0	0	0
	0.001	68.21615	4.77E-06
	0.002	107.4956	9.55E-06
	0.003	116.6934	1.54E-05
	0.0035	117.6162	1.85E-05
500	0	0	0
	0.001	82.68027	4.44E-06
	0.002	126.5031	9.22E-06
	0.003	134.247	1.53E-05
	0.0035	134.5875	1.85E-05
550	0	0	0
	0.001	98.21135	4.16E-06
	0.002	145.6182	8.98E-06
	0.003	151.2183	1.53E-05
	0.0035	151.5588	1.85E-05
600	0	0	0
	0.001	114.7631	3.92E-06
	0.002	163.6944	8.88E-06
	0.003	168.1896	1.53E-05
	0.0035	168.5300	1.85E-05
650	0	0	0
	0.001	132.2947	3.71E-06
	0.002	181.8445	8.79E-06
	0.003	185.1609	1.53E-05
	0.0035	185.5013	1.85E-05

Table 5: Moment and curvature at different initial strains and depths

The ductility of the beam section with the varying depth of cross section of beam increases with the increase in the beam depth. Though the magnitude of ductility is seen to be increasing, the percentage increase in the ductility falls down gradually with the increase in the depth.

VII. CONCLUSIONS

A computer program is developed to find moment-curvature relationship and the strength and the ductility properties are studied for varying grade of concrete and depth of beam. It has been found that, in genral that the moment carrying capacity increases moderately with the increase in the grade of concrete Further, curvature also increases with the grade of concrete and the singly reinforced beam section show higher increase compared to doubly reinforced section. For medium grade of concrete the curvature is more in singly reinforced beams but for higher grade of concrete, the doubly reinforced sections tends to show more ductility. The increasing depth increases the moment carrying capacity moderately keeping the curvature unchanged.

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