Comparative Study on Voltage Stability Assessment using Different Indices

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Abstract— The problem of voltage instability has been increasing day by day because of the increased demand of power. It is important to analyze the power system behavior with respect to voltage stability. This paper shows the performance of the stability indices and their comparison. The indices whose analysis are performed in the paper are line stability index, fast voltage stability index, line stability factor, novel voltage stability index. The indices are tested on IEEE 30-Bus standard test system. The simulation of the IEEE 30-Bus system is implemented using PSAT toolbox. With the help of the above mentioned indices the most critical lines and the weakest buses can be identified. Lines having index value equal to one or close to one are considered as the most critical lines.

Key word: Voltage stability, voltage collapse, voltage stability indices, power system analysis toolbox (PSAT)

I. INTRODUCTION

Voltage instability is relatively recent and challenging problem in a power system. Day by day it is gaining importance as the trend of operating power system close to their maximum limits increases. Present day power systems are being operated closer to their stability limits due to economic and environmental constraints.

Power system stability can be stated as the ability of the power system that enables it to maintain in state of equilibrium under normal operating condition and to regain an acceptable state of equilibrium after being subjected to a disturbance. Voltage stability is the ability of the power system to maintain voltage up to their limit, so that when the load admittance is increased, load power will increase, thus both power and voltage are controllable [1].

To maintain power system secure is therefore very important. Voltage instability is a major concern and it has been given much more attention by power system researchers and planners in recent years, and is being regarded as one of the major causes of power system insecurity. “Voltage collapse is the process by which the voltage falls to a very low value as a result of an avalanche of events accompanying voltage instability” [2].

Voltage collapse may occur in the power system due to loss of voltage stability in the system. Therefore a proper attention should be given to the voltage stability analysis in order to identify critical buses in a power system. As a result many techniques have been proposed to identify critical buses and lines. Voltage stability index for a stressed power station from a reduced system model can be derived [3]. The index can identify how far a system is from its point of collapse. Line stability factors which could identify critical lines developed by [4]. How the singularity in the jacobian matrix can be avoided by reforming power flow and point of voltage instability by employing PV curve can be obtained [5].

In this paper, voltage stability indices i.e. line stability index and fast voltage stability index are analyzed. The capability of these indices to identify critical lines and buses can be done by keeping buses at different loading conditions. The load change is increased from the base load until load flow fails to converge.

II. THEORETICAL ANALYSIS OF VOLTAGE STABILITY INDEX

Voltage stability analysis can be performed on the system by analyzing the stability indices referred to a line. Two such indices are mentioned in this paper, they are line stability index and fast voltage stability index.

A. Line Stability Index:

M. Moghavemmi et al. [6] derived the line stability index which is based on the concept of power transmission on the single line. According to the Moghavemmi discriminator of the voltage equation should be set to zero or greater than zero to maintain stability. In order to derive the index a single line transmission network can be used, fig.1 illustrates the network.

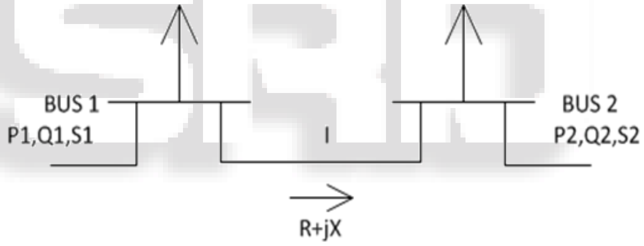


Fig. 1:

The power flow at the receiving end can be expressed as

$$S_2 = \frac{|V1|\,|V2|}{|Z|}\angle(\Theta - \delta 1 + \delta 2) - \frac{|V2|^2}{|Z|}\angle(\Theta) \qquad (1)$$

From this power equation, real and reactive power can be separated as

$$P_2 = \frac{|V1|\,|V2|}{|Z|}Cos(\Theta - \delta 1 + \delta 2) - \frac{|V2|^2}{|Z|}cos(\Theta) \qquad (2)$$

$$Q_2 = \frac{|V1|\,|V2|}{|Z|}Sin(\Theta - \delta 1 + \delta 2) - \frac{|V2|^2}{|Z|}Sin(\Theta) \qquad (3)$$

Where Θ is the angle of line impedance and δ is the angle difference between the supply voltage and the receiving voltage. Putting $\delta = \delta 1 - \delta 2$ into (3) and solving it for V_2 yields quadratic equation of:

$$|V_2|^2 \text{ Sin } \Theta - |V_1 V_2| \,[\text{Sin }(\Theta - \delta)] + |Z| \text{ Sin }(\Theta) Q_2 \qquad (4)$$

Using (5), the root of V_2 is obtained and yields to (6):

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad (5)$$

$$V_2 = \frac{|V1|\text{Sin}(\Theta-\delta) \pm \sqrt{[|V1|\text{Sin}(\Theta-\delta)]^2 - 4|Z||Q_2 Sin(\Theta)}}{2\,Sin\,(\Theta)} \qquad (6)$$

In order to maintain a stable system, V_2 needs to satisfy the stability criterion where

$$b^2 - 4ac \geq 0$$

Thus, V_2 is expressed as

$$[|V_1|\,\text{Sin}(\Theta - \delta)]^2 - 4\,|Z||Q_2|\text{Sin }(\Theta) \geq 0 \qquad (7)$$

And for |Z| sin(Θ)= X,

$$\frac{4\,X\,|Q_2|}{[|V1|Sin(\Theta-\delta)]^2} \le 1.00 \qquad (8)$$

Thus, the L_{mn} index can be defined as

$$L_{mn}=\frac{4\,X\,|Q_2|}{[|V1|Sin(\Theta-\delta)]^2} \qquad (9)$$

B. Fast Voltage Stability Index:

Fast voltage stability index proposed by I.Musirin et al. [6] is based on the concept of power flow through the single line. For a typical transmission line as shown earlier,

$$I = \frac{V_1\angle\delta_1 - V_2\angle\delta_2}{|Z|\angle\Theta} \qquad (10)$$

$$I = \left(\frac{S_2}{V_2}\right)^* = \frac{P^2 - jQ_2}{V_2\angle\delta} \qquad (11)$$

Taking (10) = (11), $\boldsymbol{\delta} = \boldsymbol{\delta}_1 - \boldsymbol{\delta}_2$ and rearranging the equations will lead to

$$P_2 - j\,Q_2 = \frac{-|V1|^2\angle-\Theta + |V1V2|\angle\delta-\Theta}{|Z|} \qquad (12)$$

From (12) the reactive power of receiving bus form the quadratic equation for the V_2 is evaluated as

$$|V_2|^2 - |V_1V_2|\left[\frac{R}{X}\sin\delta + \cos\delta\right] + \left[\frac{R^2}{X} + X\right]Q_2 = 0 \qquad (13)$$

$$\frac{\left[\frac{R}{X}\sin\delta + \cos\delta\right]|V1| \pm \sqrt{\left[\left[\frac{R}{X}\sin\delta + \cos\delta\right]|V_1|^2\right] - 4\left[\frac{R^2}{X} + x\right]Q_2}}{2} \qquad (14)$$

Where R is the line resistance and X is the line reactance. In order to obtain the real root of V_2, the determinant must be set to be greater or equal to '0' to fulfill the stability criterion.

$$\frac{4\,|Z|^2 Q_2 X}{|V1|^2(R\sin\delta + X\cos\delta)^2} \le 1.00 \qquad (15)$$

Since the angle difference is normally small and can be neglected, therefore R sin $\boldsymbol{\delta}$~ 0 and X cos $\boldsymbol{\delta}$~ X. Thus

$$\frac{4\,|Z|^2 Q_2}{|V1|^2 X} \le 1.00 \qquad (16)$$

Finally, the line stability index FVSI is formulated as

$$FVSI = \frac{4\,|Z|^2 Q_2}{|V1|^2 X} \qquad (17)$$

C. Line Stability Factor:

LQP is derived by A. Mohamed at el [6] using same power flow concept between two buses in a transmission system as L_{mn} and FVSI. Utilizing (10) and (11) may also produce real and reactive power as (18) and (19) respectively.

$$P_2 = [(V_1 \cos\boldsymbol{\delta} - V_2)\frac{R}{R^2+X^2} - V1\sin\boldsymbol{\delta}\frac{X}{R^2+X^2}]\,V_2 \qquad (18)$$

$$Q_2 = [(V_1 \cos\boldsymbol{\delta} - V_2)\frac{X}{R^2+X^2} + V1\sin\boldsymbol{\delta}\frac{R}{R^2+X^2}]\,V_2 \qquad (19)$$

By considering losses in the system and taking R/X << 1, (18) and (19) can be simplified to

$$P_2 = \frac{V1V2\sin\delta}{X} \qquad (20)$$

$$Q_2 = \frac{V1V2\cos\delta - V^2}{X} \qquad (21)$$

Rearranging (20) and (21) to satisfy $Sin^2\,\delta$ + $COS^2\,\delta$ =1 yields quadratic equation V_2

$$V_2^4 + (2Q_2X + V_1^2)\,V_2^2 + Q_2^2X^2 + P_2^2X^2 = 0 \qquad (22)$$

Thus, applying stability criterion where the determinant of V_2^2 needs to be greater or equal to zero formed

$$(2Q_2X + V_1^2)^2 - 4\,(Q_2^2X^2 + P_2^2X^2) \ge 0 \qquad (23)$$

$$V_1^4 - 4\,XQ_2V_1^2 - 4P_2^2\,X \ge 0 \qquad (24)$$

Since the line is considered lossless, P_1 = -P_2, thus LQP is formulated as

$$LQP = 4\left[\frac{X}{|V1|2}\right]\left[\frac{X}{|V1|2}P_1^2 + Q_2\right] \qquad (25)$$

D. Novel Voltage Stability Index:

Authors in paper [6] solved NVSI by considering the system in Fig. 1 to be in lossless condition initially. The index is mathematically formulated as below:

$$I = V_1\frac{V1}{2a} \qquad (26)$$

$$S_2 = V_2\,I^* \qquad (27)$$

Incorporating (26) into (27) and solving it as from equation (18) until (22) formed

$$(2Q_2\,X + V_1^2)^2 - 4\,X^2\,(Q_2^2 + P_2^2) \ge 0 \qquad (28)$$

This equation is similar as (23). However, in NVSI, it did not encounter further equation simplification. Thus, NVSI can be summarized as

$$NVSI = \frac{2X\sqrt{P_2^2+Q_2^2}}{2Q_2X - |V_1|2} \qquad (29)$$

After implementing the above method if the value of the above index is close to 1 or line that provide the largest index value with respect to load is considered as the most critical line of the bus and it will lead to system instability. The concept of determining the weakest bus is based on the maximum load allowed on a load bus. The weakest bus in the network corresponds to the bus with the smallest maximum tolerable load.

Index	Formulation	Relative Variables	Critical value
Lmn	$Lmn=\frac{4\,X\,|Q_2|}{[|V1|Sin(\Theta-\delta)]^2}$	Q ,X, V, Θ	1.00
FVSI	$FVSI=\frac{4\,|Z|^2Q_2}{|V1|^2X}$	Q, V ,Z, X	1.00
LQP	$LQP= 4\left[\frac{X}{|V1|2}\right]\left[\frac{X}{|V1|2}P_1^2 + Q_2\right]$	P, Q, X, V,Q	1.00
NVSI	$NVSI=\frac{2X\sqrt{P_2^2+Q_2^2}}{2Q_2X - |V_1|2}$	P ,Q, X, V, Q	1.00

Table I:

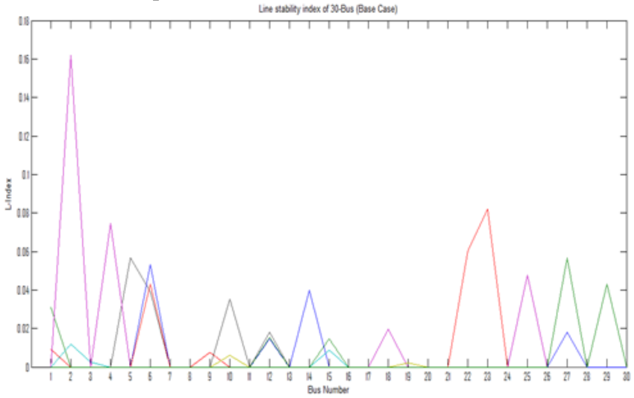
III. Analysis Tools

For the analysis purpose power system analysis toolbox (PSAT) is used. PSAT is one of the most important open source Matlab and GNU/Octave-based software package for the analysis of electric power systems. PSAT includes various operation such as power flow, continuation power flow, optimal power flow, small-signal stability analysis, and time-domain simulation, as well as several static and dynamic models. All the simulation has been implemented on the power system analysis tool.

IV. Test Results and Discussion

The analysis of the voltage stability indices and their comparison were performed on the IEEE 30-Bus test system. The simulation results are investigated in order to document the theory analysis. The system comprises of 6 generator buses, 24 load buses, and 41 interconnected lines. The configuration is shown in the fig 2. The different indices are tested for the different loading conditions of the IEEE 30-Bus system. The increased loading will affect the lines connected to the buses .The loading of the bus can be slowly varied for its base value to its 10% loading, 20%

loading and so on. The small addition of load fails to converge the load flow solution and indicates that the loading is critical and very close to system instability. At a time only one bus is take into consideration and load on the other buses keep constant at base load.



A. Reactive Power Load Changes:

Fig-3 shows the variation of line stability index (Lmn) vs. bus number for the base case of the IEEE 30-Bus system. From the graph, it shows that line $L_{2\text{-}5}$ is having highest index value. Further increment of reactive load on bus number 2 leads line $L_{2\text{-}5}$ to be critical first.

Fig-3

The reactive load is slowly increased, only in one bus of the IEEE 30-Bus system at a time, from the base case till its maximum permissible load, keeping the load on the other buses constant at base load. Fig.4 shows the variation of line stability index $L_{(23\text{-}24)}$ for different value of reactive power Q (MVAR) in p.u. at bus number 24 of 30 bus system.

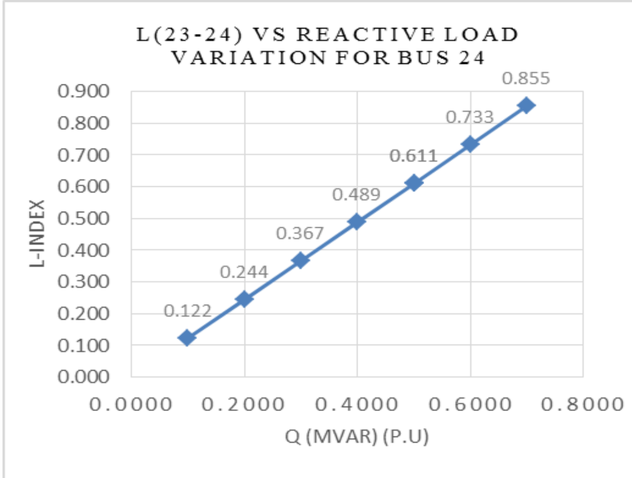


Fig. 4:

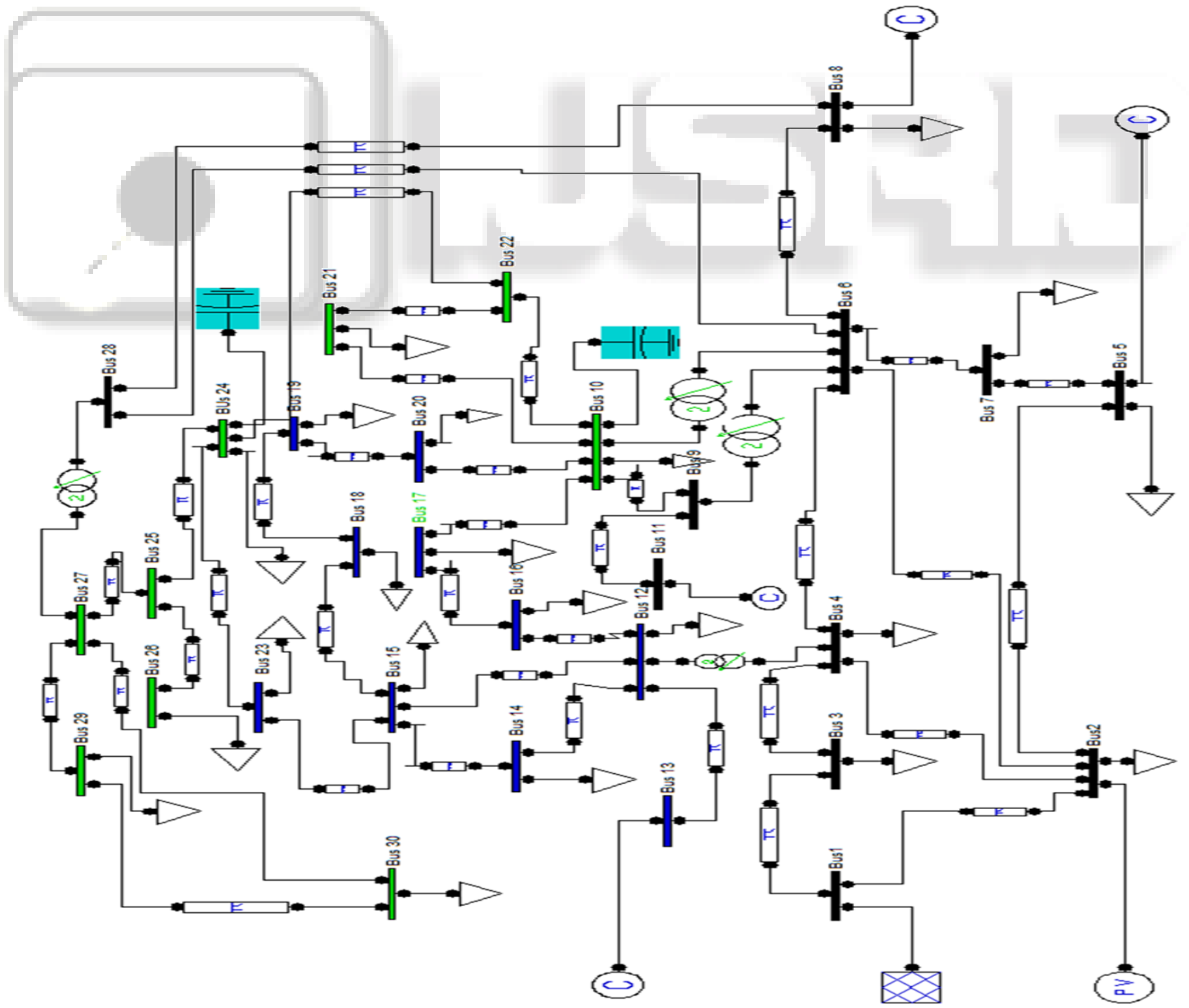


Fig. 2:

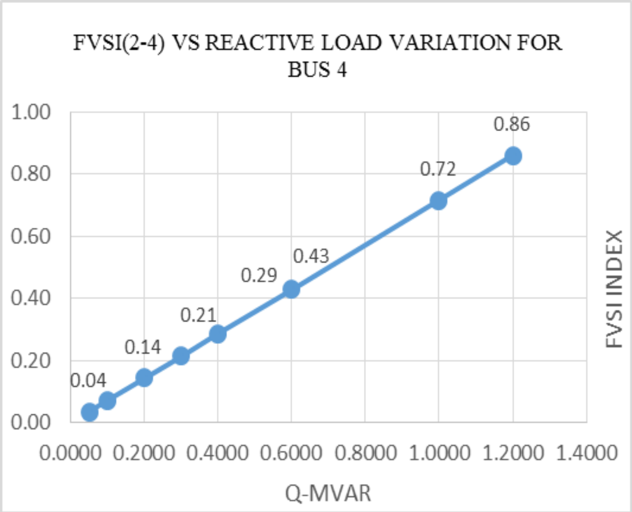


Fig. 5:

Fig.5 Figure shows the variation of fast voltage stability index FVSI(2-4) for different value of reactive power Q (MVAR) in p.u. at bus number 4.Fig.6 Figure shows the variation of line stability index LQP(2-4) for different value of reactive power Q (MVAR) in p.u. at bus number 4.

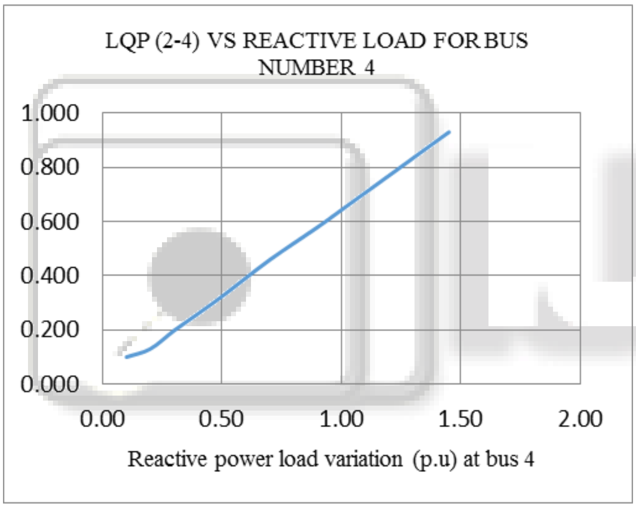


Fig. 6:

From the graph it shows the line is going to be critical when the reactive power load at bus 4 is 1.42 p.u as the stability index shows a value of 0.92 which is close to 1.Fig.7 shows the variation of novel voltage stability index NVSI (2-4) for different value of reactive power Q (MVAR) in p.u. at bus number 4.

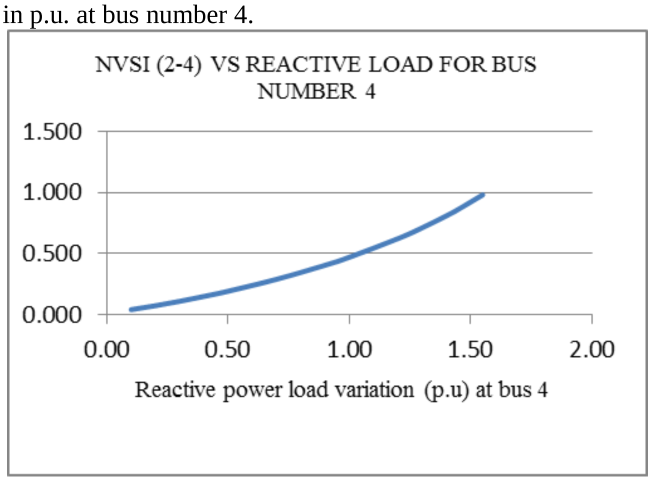


Fig. 7:

From the graph it shows the line 2-4 is going to be critical when the reactive power load at bus 4 is 1.55 p.u as the stability index shows a value of 0.981 which is close to 1. But as compared to other methods for reactive power variation NVSI does not give an appropriate results based on the loading condition because as shown from the graph index corresponds to 0.981 at the reactive load of 1.55 p.u at bus 4 which is far from the actual value at which line is going to be critical.

B. Comparison of Different Methods:

For the IEEE 30-Bus system, the comparison of different methods based on the variation of reactive power loading at particular bus were presented. For the comparison bus number 4 is taken into consideration and condition of the lines which are connected to bus 4 are checked with the help of all the mentioned methods. The methods are line stability index, fast voltage stability index, line stability factor, novel voltage stability index. The condition of the line 2-4 which is connected to bus number 4 is going to be affected by the variation of the reactive load at bus number 4.During initial state NVSI start at 0.054 index value while other indices start almost at the same index value which is at 0.142.

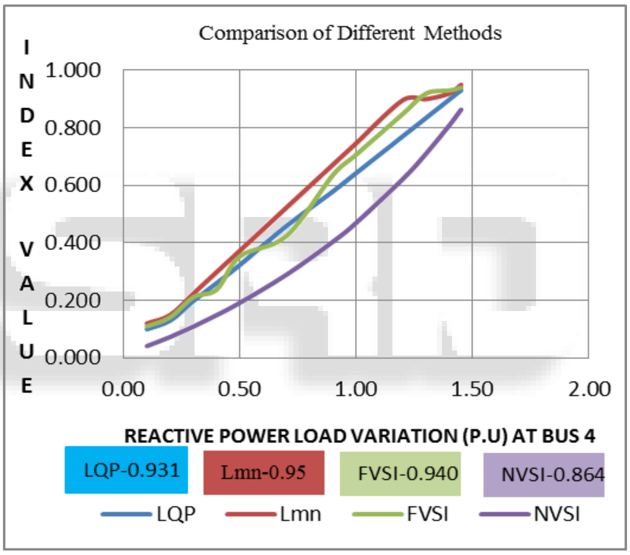


Fig. 8:

V. CONCLUSION

This paper presents the methods to identify the critical lines and buses. Four indices were formulated to identify critical lines based on different loading conditions upto maximum loading condition. The comparison of all the four methods were presented based on the different reactive loading condition. The results are tested on the standard IEEE 30-Bus test system. Based on the analyses, it can be concluded that FVSI and Lmn while LQP and NVSI are equivalent to each other for this two bus system, other than that FVSI, Lmn, LQP are more prone to reactive power load changes as compared to NVSI.

Bus	Voltage (p.u)	Phase (deg)	P load (p.u)	Q load
1	1.0600	0.0000	0.0000	0.0000
2	1.0430	-5.5473	0.2170	0.1270
3	1.0250	-8.1025	0.0240	0.0120

4	1.0171	-9.7787	0.0760	0.0160
5	1.0100	-14.4605	0.9420	0.1900
6	1.0166	-11.6206	0.0000	0.0000
7	1.0061	-13.3117	0.2280	0.1090
8	1.0100	-12.2520	0.3000	0.3000
9	1.0813	-15.5572	0.0000	0.0000
10	1.0813	-15.5591	0.0580	0.0200
11	1.0820	-15.5572	0.0000	0.0000
12	1.0676	-14.8491	0.1120	0.0750
13	1.0710	-14.8491	0.0000	0.0000
14	1.0556	-15.7015	0.0620	0.0160
15	1.0540	-15.8240	0.0820	0.0250
16	1.0661	-15.4415	0.0350	0.0180
17	1.0713	-15.7339	0.0900	0.0580
18	1.0517	-16.4001	0.0320	0.0090
19	1.0534	-16.5566	0.0950	0.0340
20	1.0596	-16.3647	0.0220	0.0070
21	1.0672	-16.0054	0.1750	0.1120
22	1.0672	-15.9970	0.0000	0.0000
23	1.0480	-16.2075	0.0320	0.0160
24	1.0482	-16.3834	0.0870	0.0670
25	1.0369	-16.0981	0.0000	0.0000
26	1.0195	-16.5018	0.0350	0.0230
27	1.0383	-15.6779	0.0000	0.0000
28	1.0154	-12.2315	0.0000	0.0000
29	1.0187	-16.8718	0.0240	0.0090
30	1.0075	-17.7279	0.1060	0.0190

Table II: Load flow results and load bus data

Bus	Q-max	Q-min	Tap setting	
2	50	-40	Bus 4-12	1.0600
5	40	-40	Bus 6--9	1.0430
8	40	-10	Bus 6-10	1.0250
11	24	-6	Bus 28-27	1.0171
13	24	-6		

Table III: Tap Setting and Reactive power limits

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