Fuzzy Soft Aggregation Operator to Identify Best Student among the Students with Different Grading Systems

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Abstract— In this paper, we estimate the set data normalization using soft set theory. These sets are based on the grading system implemented by different universities. Often to select a best candidate among the students with different grades is done by normalizations for decision making. In this paper a fuzzy soft aggregation operator is presented to construct more efficient decision making method. Finally, we give an example which shows that the method can be successfully applied to select best candidate among the universities that contain uncertainties with different grading system.

Key words: Soft set, normalization, fuzzy set, cardinal set, membership function

I. INTRODUCTION

Uncertainty is a common phenomenon of our daily existence because our world is full of uncertainties. In our daily life we face many situations where we do take account of these uncertainties. For example, the different universities have different grading systems. Uncertainty is raised for select the best one among them if the persons applies for a job with different grading systems. Therefore it is natural for man to understand and try to model this uncertainty prevailing in physical world.

Normalization is the process of scaling individual samples to have unit norm. In order to normalize data [1] different methods are used. These methods include Rescaling, Standardization and Scaling to unit length and Gaussian Membership function. However, these methods cause uncertainty in the normalization. To overcome these problems we proposed a new method called as fuzzy soft aggregation which is based on soft sets and fuzzy sets.

In 1965, L. A. Zadeh [Zadeh, 1965] introduced remarkable theory of Fuzzy sets whose elements have degrees of membership. Fuzzy sets were introduced by Lotfi A. Zadeh and Dieter Klaua in 1965 as an extension of the classical notion of set. At the same time, Salii (1965) defined a general structure called L-relations, in an abstract algebraic context. Fuzzy relations, are used in different areas, such as linguistics (De Cock, et al., 2000), decision-making (Kuzmin,1982) and clustering (Bezdek, 1978), are special cases of L-relations when L is the unit interval [0, 1].

Molodtsov proposed soft sets and applied this theory to several directions [2, 3, 4], and then formulated the notions of soft number, soft derivative, soft integral, etc. in [5]. Maji et al. [6] worked on theoretical study of soft sets in detail, and [7] presented an application of soft set in the decision making problem using the reduction of rough sets [8]. Chen et al. [9] proposed parameterization reduction of soft sets, and then Kong et al. [10] presented the normal parameterization reduction of soft sets.

Fuzzy soft aggregation operator is introduced by Maji et al. This definition is more realistic as it involves handling of uncertainty in the selection of a fuzzy set corresponding to each value of the parameter. Relations on generalized fuzzy soft sets are defined and Fuzzy soft aggregation operator was studied and as an application a decision making problem is solved.

The organization of this paper is as follows: In Section 2, some preliminary definitions are given which will be used in the rest of the paper. In Section 3, a definition of normalization is given and some goals are studied. In Section 4, Fuzzy soft aggregation operator is introduced. An application of this similarity measure in grading system has been shown in Section 5. Section 6 concludes the paper.

II. PRELIMINARIES

In this section, the basic definitions of soft set theory [2] and fuzzy set theory [11] that are useful for subsequent discussions are presented. These definitions and more detailed explanations related to the soft sets and fuzzy sets can be found in [12, 13, 2] and [14, 15], respectively. We have defined $fs-sets$ and their operations for normalize the data. In the soft sets, given in Section 2, the parameter sets and the approximate functions are crisp. But in the $fs-sets$, while the parameters sets are crisp, the approximate functions are fuzzy subsets of U. The notion $\rho_A, \rho_B, \rho_C, \ldots$, etc is used for $fs-sets$ and $\lambda_A, \lambda_B, \lambda_C, \ldots$, etc. for their fuzzy approximate functions, respectively.

A. Definition 2.1:[16]

Let U be an initial universal set and let E be a set of parameters. Let P (U) denote the power set of U. A pair (F, E) is called a soft set over U iff F is a mapping given by F: E → P (U).

B. Definition 2.2:[16]

Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U. Let $A \subset E$

A pair (F, E) is called a fuzzy soft set over U, where F is a mapping given by F: $A \rightarrow I^U$.

C. Example 2.3: [16]

As an illustration, consider the following example. Suppose a fuzzy soft set (F, E) describes attractiveness of the shirt with respect to the given parameters, which the authors are going to wear. U ={x1, x2, x3, x4, x5} which is the set of all shirts under consideration. Let I^U be the collection of all fuzzy subsets of U and let E = {e1 = “colorful”; e2 = “bright”; e3 = “cheap”; e4 = “warm”}

Let

$$F(e_1)=\left\{\frac{x_1}{0.5},\frac{x_2}{0.9},\frac{x_3}{0},\frac{x_4}{0},\frac{x_5}{0}\right\}, F(e_2)=\left\{\frac{x_1}{1.0},\frac{x_2}{0.8},\frac{x_3}{0.7},\frac{x_4}{0},\frac{x_5}{0}\right\},$$

$$F(e_3)=\left\{\frac{x_1}{0},\frac{x_2}{0},\frac{x_3}{0},\frac{x_4}{0.6},\frac{x_5}{0}\right\}, F(e_4)=\left\{\frac{x_1}{0},\frac{x_2}{1.0},\frac{x_3}{0},\frac{x_4}{0},\frac{x_5}{0.3}\right\},$$

Then the family {F (ei), i = 1, 2, 3, 4} of I^U is a fuzzy soft set (F , E).

D. Definition 2.4:

An $fs-set$ ρ_A over U is a set defined by a function $\lambda_A(x)$ representing a mapping

$$F_A = \{(x_1,\{0.9/u_2, 0.5/u_4\}),(x_2,U),(x_4,\{0.2/u_1, 0.4/u_3, 0.8/u_5\})\}$$

$\lambda_A : E \rightarrow F(U)$ Such that $\lambda_A(x) = \theta$ if $x \notin A$.

Here, λ_A is called fuzzy approximate function of the fs-set ρ_A, and the value $\lambda_A(x)$ is a set called x-element of the fs-set for all $x \in E$. Thus, an fs-set ρ_A over U can be represented by the set of ordered pairs

$$\rho_A = \{(x, \lambda_A(x)) : x \in E, \lambda_A(x) \in F(U)\}.$$

Note that the set of all fs-sets over U will be denoted by $FS(U)$.

E. Example 2.5:

Let $U = \{u_1, u_2, u_3, u_4, u_5\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If

$$A = \{x_1, x_2, x_4\} \subseteq E, \lambda_A(x_1) = \{{}^{0.9}/_{u_2}, {}^{0.5}/_{u_4}\}, \lambda_A(x_2) = U,$$

And $\lambda_A(x_4) = \{0.2/u_1\ 0.4/u_3, 0.8/u_5\}$, then the soft set F_A is written by

$$F_A = \{(x_1,\{0.9/u_2, 0.5/u_4\}),(x_2,U),(x_4,\{0.2/u_1, 0.4/u_3, 0.8/u_5\})\}.$$

F. Definition 2.6:

An $fs-set$ over U is a set defined by a function λ_A representing a mapping $\lambda_A : E \rightarrow F(U)$ such that $\lambda_A(x) = \theta$ if $x \notin A$

Here, λ_A is called fuzzy approximate function of the $fs-set$ ρ_A, and the value $\lambda_A(x)$ is a set called x-element of the $fs-set$ for all $x \in E$. Thus, an fs-set ρ_A over U can be represented by the set of ordered pairs.

$$\rho_A = \{(x, \lambda_A(x)) : x \in E, \lambda_A(x) \in F(U)\}.$$

G. Example 2.7:

Let $U = \{s_1, s_1, s_1, s_1, s_1\}$ be a universal set and $E = \{x_1, x_2, x_3, x_4\}$ be a set of parameters. If $A = \{x_1, x_2, x_4\} \subseteq E$, $\lambda_A(x_1) = \{0.9/s_2, 0.5/s_4\}, \lambda_A(x_2) = U$, and $\lambda_A(x_4) = \{0.2/s_1, 0.4/s_3, 0.8/s_5\}$, then the soft set FA is written by

$$F_A = \{(x_1,\{0.9/s_2, 0.5/s_4\}),(x_2,U),(x_4,\{0.2/s_1, 0.4/s_3, 0.8/s_5\})\}.$$

III. NORMALIZATION

Normalization is the process of scaling individual samples to have unit norm. The function normalize provides a quick and easy way to perform this operation on a single array-like dataset. The goal is to independently normalize each feature component to the range [0, 1]. Feature scaling or data normalization is a method used to standardize the range of independent variables or features of data. In data processing, it is known as data normalization and performed during the data pre-processing step. The following sections describe four normalization procedures namely Rescaling, Standardization scaling to unit length and Gaussian Membership function method to calculate the data normalization.

A. Rescaling:

The simplest method is rescaling the range of features to scale the range in [0, 1] or [−1, 1]. Selecting the target range depends on the nature of the data or the features selected. The general formula is given as:

$$\text{Normalized } (e_i) = \frac{e_i - E_{\min}}{E_{\max} - E_{\min}}$$

where

E_{min} = the minimum value for variable E

E_{max} = the maximum value for variable E

If E_{max} is equal to E_{min} then Normalized (e_i) is set to 0.5.

By using this formula we can calculate the normalized values of the database.

B. Standardization:

Feature standardization normalizes the value of each feature in the data to have zero-mean and unit-variance. This method is widely used for normalization in handling neural networks and machine learning. The basic idea behind this technique is the computing standard scores which is performed by calculating the distribution mean and standard deviation for each feature. Next subtract the mean from each feature and then divide the values of each feature by its standard deviation.

C. Scaling to Unit Length;

This technique deals with scaling the components of a feature vector to unit vector. Here, each feature vector is divided by the Euclidean length of the vector. In some applications (e.g. Histogram features) it can be more practical to use the L1 norm (i.e. Manhattan Distance, City-Block Length) of the feature vector:

$$x' = \frac{x}{||x||}$$

This is especially important in if Scalar Metric is used as a distance measure.

D. Gaussian Membership Function Method:

The membership function of a fuzzy set is a generalization of the indicator function in classical sets. In fuzzy logic, it represents the degree of belongingness. A degree of belongingness is not similar to probability, because fuzzy sets represent membership in vague. Membership functions were introduced by Zadeh in the first paper on fuzzy sets (1965).For example, to represent the climate of a day whether it is hot, then how much hot the day is represented using fuzzy set.

A fuzzy membership function that is often used to represent vague, linguistic terms is the Gaussian which is given by:

$$\mu_{A^i}(x) = \exp\left(-\frac{(c_i - x)^2}{2\sigma^2{}_i}\right),$$

IV. FUZZY SOFT AGGREGATION OPERATOR

In this section, we define an fs-aggregation operator that produces an aggregate fuzzy set from an fs-set and its cardinal set. The approximate functions of an $fs-set$ is fuzzy. An fs-aggregation operator on the fuzzy sets is an operation by which several approximate functions of an $fs-set$ are combined to produce a single fuzzy set which is the aggregate fuzzy set of them $fs-set$. Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best single crisp alternative from this set.

Cardinality of a set is a measure of the number of elements of the set". For example, the set A = {1, 4, 5 , 6} contains 4 elements, and the cardinality of A is 4. The cardinality of a fuzzy set is given by the sum of all the elements present in the set by the size of the fuzzy set.

A. Definition 4.1:

Let $\rho_A \in FS(U)$.Assume that $U=\{u_1,u_2,...u_m\}$, $E=\{x_1,x_2,....x_n\}$ and $A\subseteq E$, then the ρ_A can be presented by the following table,

ρ_A	x_1	x_2.......	x_n
u_1	$\mu_{\lambda A(x_1)}(u_1)$	$\mu_{\lambda A(x_2)}(u_1)$........	$\mu_{\lambda A(x_n)}(u_1)$
u_2	$\mu_{\lambda A(x_1)}(u_2)$	$\mu_{\lambda A(x_2)}(u_2)$.........	$\mu_{\lambda A(x_n)}(u_2)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
u_m	$\mu_{\lambda A(x_1)}(u_m)$	$\mu_{\lambda A(x_2)}(u_m)$.........	$\mu_{\lambda A(x_n)}(u_m)$

Table. 1: Definition 4.1

Where $\mu_{\lambda A(x)}$ is the membership function of λ_A.

If $a_{ij}=\mu_{\lambda A(x_j)}(u_i)$, for $i=1,2,....,m$ and $j=1,2,....,n$, then the $fs-set$ ρ_A is uniquely characterized by a matrix,

$$\left[a_{ij}\right]_{m\times n}=\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

is called an $m\times n$ $fs-$matrix of the $fs-set$ ρ_A over U.

B. Definition 4.2:

Let $\rho_A \in FS(U)$. then, the cardinal set of ρ_A, denoted by c ρ_A and defined by $c\rho_A=\{\mu_{c\rho_A}(x)/x:x\in E\}$, is a fuzzy set over E. The membership function $\mu_{c\rho_A}$ is defined by

$$\mu_{c\rho_A}:E\to[0,1],\ \mu_{c\rho_A}(x)=\frac{|\lambda_A(x)|}{|U|}$$

Where $|U|$ is the cardinality of universe U, and $|\lambda_A(x)|$ is the scalar cardinality of fuzzy set $\lambda_A(x)$. Note that the set of all cardinal sets of the fs-sets over U will be denoted by cFS(U). It is clear that $cFS(U)\subseteq F(E)$.

C. Definition 4.3:

Let $\rho_A \in FS(U)$.and $c\rho_A \in cFS(U)$. assume that $E=\{x_1,x_2....x_n\}$ and $A\subseteq E$, then $c\rho_A$ can be presented as follows.

E	x_1	x_2	$\cdots$	x_n
$\mu_{c\rho_A}$	$\mu_{c\rho_A}(x_1)$	$\mu_{c\rho_A}(x_2)$	$\cdots$	$\mu_{c\rho_A}(x_n)$

If $a_{ij}=\mu_{c\rho_A}(x_j)$ for j=1,2,…,n, then the cardinal set $c\rho_A$ is uniquely characterized by a matrix, $\left[a_{ij}\right]_{1\times n}=[a_{11}\ a_{12}\ \cdots a_{1n}]$ Which is called the cardinal matrix of the cardinal set $c\rho_A$ over E.

D. Definition 4.4:

Let $\rho_A \in FS(U)$ and $c\rho_A \in cFS(U)$. Then fs-aggregation operator, denoted by FSagg, is defined by FSagg $:cFS(U)\times FS(U)\to F(U)$,

$$\text{FSagg}(c\rho_A,\rho_A)=\rho^*_A$$

$$\mu_{A^i}(x)=\exp\left(-\frac{(c_i-x)^2}{2\sigma^2_i}\right),$$

Where

$\rho^*_A=\{\mu_{\rho^*_A}(u)/u:u\in U\}$ is a fuzzy set over U. ρ^*_A is called the aggregate fuzzy set of the fs-set ρ_A. The membership function $\mu_{\rho^*_A}$ of ρ^*_A is defined as follows:

$$\mu_{\rho^*_A}:U\to[0,1],\ \mu_{\rho^*_A}(u)=\frac{1}{|E|}\sum_{x\in E}\mu_{c\rho_A}(x)\mu_{\lambda_A(x)}(u),$$

Where $|E|$ is the cardinality of E.

E. Theorem 4.5:

Let $\rho_A \in FS(U)$ and $A\subseteq E$. If $M_{\rho_A}, M_{c\rho_A}$ and $M_{\rho^*_A}$ are representation matrices of $\rho_A, c\rho_A$ and ρ^*_A respectively, then $|E|\times M_{\rho^*_A}=M_{\rho_A}\times M^T_{c\rho_A}$

Where $M^T_{c\rho_A}$ is the transposition of $M_{c\rho_A}$ and $|E|$ is the cardinality of E.

Proof. It is sufficient to consider $\left[a_{i1}\right]_{m\times1}=\left[a_{ij}\right]_{m\times n}\times\left[a_{1j}\right]^T_{1\times n}$.

Theorem 4.5 is applicable to computing the aggregate fuzzy set of normalize fuzzy set.

V. APPLICATION

Once an aggregate fuzzy set has been arrived at, it may be necessary to choose the best alternative from this set. Therefore, we can make a decision by the following algorithm.

- Step 1: Construct an $fs-set$ ρ_A over U ,
- Step 2: Find the cardinal set $c\rho_A$ of ρ_A ,
- Step 3: Find the aggregate fuzzy set ρ^*_A of ρ_A,
- Step 4: Find the best alternative from this set that has the person got the first place by $\max \mu\rho^*_A(u)$.

A. Example 5.1:

Consider the universe as students with different grading system={x1,x2,x3,x4,x5,x6,x7,x8}.The parameter set E be the different grading system={ei} where i=1 for grading system with credits 8, i=2 grading system with credit 7 and

i=3 for grading system 6.From the universe the best student need to be identified.
Consider the marks
E1= {7.2/x1, 5.5/x2, 6.9/x3}
E2= {6.5/x4, 7/x5, 6.3/x6}
E3= {5.6/x7, 4.9/x8}

1) *Step1: Construct fuzzy soft set.*

ΓA = (E1, {0.76/x1, 0.52/x2, 0.92/x3}),
(E2, {0.96/x4, 0.54/x5, 0.7/x6}),
(E3, {0.77/x7, 0.77/x8})}

2) *Step2: Compute the cardinality* $c\rho_A =$ *{0.2760/E1, 0.2763/E2, 0.1947/E3}*

3) *Step3: Find aggregate fuzzy set*

$$\rho^*_A = 1/5 \begin{pmatrix} 0.7600 & 0 & 0 \\ 0.5200 & 0 & 0 \\ 0.9200 & 0 & 0 \\ 0 & 0.9600 & 0 \\ 0 & 0.5400 & 0 \\ 0 & 0.7000 & 0 \\ 0 & 0 & 0.7700 \\ 0 & 0 & 0.7700 \end{pmatrix} \begin{pmatrix} 0.2760 \\ 0.2763 \\ 0.1947 \end{pmatrix} = \begin{pmatrix} 0.0699 \\ 0.0478 \\ 0.0846 \\ 0.0884 \\ 0.0497 \\ 0.0645 \\ 0.0500 \\ 0.0500 \end{pmatrix}$$

$\mu\rho^*_A$ ={0.0699/x1, 0.0478/x2, 0.0846/x3, 0.884/x4, 0.0497/x5, 0.0645/x6, 0.0500/x7,0.0500/x8}

4) *step4: Finally, the largest membership grade is chosen by* $\max \mu\rho^*_A(u)$ *=0.0884.*

Hence the candidate x4 is chosen among the students with different grades.

VI. Conclusion

Soft set is effective tools to handle uncertainty. In the application where the decision making is needed to choose the best student among the students with different grading system soft sets proved to handle the uncertainty by making use of fuzzy soft aggregation operator. The same operator can be used in situations where students have examination with different set of question papers and need to identify the topper. The operator is also applicable to image processing as well, which helps in choosing a best algorithm with multiple qualitative measures.

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