Analysis of Queueing Model Vacation with using Maximum Entropy Principle Approach

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Abstract—This investigation deals with a general queueing model with vacation by using the method of entropy maximization. The probability distribution and other measures of performance have obtained for GE/G/1 and MX/G/1 vacation models of non-exhaustive service. The special cases have been analyzed in such models. The steady state queue size distribution of the number of customers in the system is obtained. In special cases we deduce the results for particular distributions. The numerical examples for illustration purpose are provided for special cases.

Key words: GE/G/1 queue, Non-exhaustive, Vacation, Maximization entropy

I. Introduction

To predict the investigation of computer network performance, the queueing models have played important role in the form of mathematical modeling. The classical analysis of queueing theory fails to give exact solutions for models having general distributions for input and service processes. By using the concept of maximization entropy principle, we are in the position to provide the approximate results for queue size distributions in terms of operational mean values of computer network. The application of maximization entropy models is not limited to computer network. It is the important technique in Statistical mechanics, Bio-Statistics and its modeling which can be successfully applied to analyze the congestion situations of day-to-day as well as industrial organizations.

The origin of entropy and maximum entropy principle as a measure of the amount of uncertainty is due to Shannon (1948). Jaynes (1968) extended this principle in the different situations. Shore and Johnson (1980) explained axiomatic derivation of the principle of maximum entropy in the system modeling. The elementary models of queueing theory such as M/G/1 and G/M/1 with the help of maximum entropy principle were introduced by El–Affendi and Kouvatsos (1983). Guiasu (1986) analyzed a probabilistic model for an M/G/1 queueing system in a maximum entropy condition. He obtained mean queue size distribution by using the expected number of customers given by Pollaczek Khinchine formula. The maximum entropy analysis of queues and queueing networks was established by Wu (1988). Wu (1992) obtained a maximum entropy analysis of open queueing networks with group arrivals. The maximum entropy analysis for Mx/G/1 queueing system at equilibrium state was presented by Jain (1998). Jain and Dhakad (2002) established queue size distribution for G/G/1 model by using principle of maximum entropy. Kouvatsos and Awan (2003) considered entropy maximization for open queueing networks with priorities and blocking.

In vacation queueing model, the server goes on vacation for some random interval of time when the system becomes idle. If at the end of vacation, there are customers in the system, the server provides service immediately; otherwise, a new vacation starts immediately. A new arrival to the system has to wait in the queue for the completion of the current service or vacation. Many authors analyzed the queueing models with vacation. Kehson and Servi (1986) determined oscillating random walk models for GI/G/1 vacation system with Bernoulli schedules. Lee at el. (1992) studied on a batch service queue with single vacation. Takagi (1994) analyzed M/G/1/K/N queue with server vacations and exhaustive service. Machihara (1996) considered a pre-emptive priority queue as a model with server vacation. Li et al. (1997) discussed reliability analysis of M/G/1 queueing systems with server breakdowns and vacations. The transient solution for M/G/1 queue with server vacations was obtained by Tang (1997). Alfa (2003) described vacation models in discrete time. Gray et al. (2000) studied a vacation queueing model with service breakdown. Srinivasan et al. (2003) presented computation of queue length probabilities for bulk service queues with vacation and feed back. Wang et al. (2005) considered maximum entropy analysis to the N policy M/G/1 queueing system with server breakdowns and general startup times. Principle of maximum entropy for G/G/1 queue with vacation under N-policy determined by Jain and Jain (2006). Ke (2007) studied batch arrival queues under vacation policies with server breakdowns and startup/closedown times. Wang et al. (2011) analyzed comparative analysis of a randomized N policy queue. Maurya (2013) derived maximum entropy Analysis of MX/(G1,G2)/1 retrial queueing model with second phase optional service and Bernoulli vacation schedule. Jain et al. (2013) discussed multiple vacation policy for MX/Hk/1 queue with un-reliable server.

In this paper, we consider the GE/G/1 vacation and MX/G/1 vacation (non – exhaustive) models to present the probability distribution of the possible states. We obtain queue size distribution in steady state, by using maximum entropy principle. The numerical results are also supplemented for special cases.

II. The Principle of Maximum Entropy

We consider the finite discrete case of Shannon's entropy (cf. Guiasu,1986) as follows:

Let $$H(p) = H_n(p_1, p_2,..., p_n) = -\sum_{k=1}^{n} p_k \log p_k \quad \ldots (1)$$

where $H(p)$ is the amount of uncertainty contained by the probability distribution $p = (p_1, p_2,..., p_n)$ and is defined as system's entropy function. Shannon's entropy has the following property

$$H(p) = H_n(p_1, p_2,..., p_n) \le H_n(1/n, 1/n\ ..., 1/n)$$

with equality iff , (k=1,2,…,n).

The above relation shows that the uniform distribution is the most uncertain, when no constraint is imposed on the probability distribution. We consider the vacation (non-

exhaustive service) models and evaluate the queue size distribution with the help of maximum entropy principle.
We have used following notations:

λ	Mean arrival rate of the customers.
μ	Mean service rate of the customers.
λ_g	Average rate of groups arrivals.
n	Number of customers in the system.
P_n	Steady- state probability distribution of number of customers in the system.
P_0	Probability of system being empty.
L	Average queue length.
ρ	Traffic intensity.
$\mathbf{E(V)}$	Average vacation time.
$E(V^2)$	Second moments of vacation time.
C_Y^2	Coefficient of variation of customers inter-arrival time.
C_T^2	Coefficient of variation of service time.
$1/p$	The average group size.

We apply maximum entropy principle, to obtain Pn as follows:

$$\text{Max}\{-\sum_{n=1}^{\infty} P_n \log P_n\} \qquad \ldots(2)$$

subject to constraint

$$\sum_{n=1}^{\infty} P_n = 1 - P_0 \qquad \ldots(3)$$

$$\sum_{n=1}^{\infty} nP_n = L \qquad \ldots(4)$$

By using the Lagrange's multipliers corresponding to above constraints, we obtain Lagrange's function, which can be used to find queue size distribution by using the approach as discussed in Jain [1998]. For analysis purpose, the following theorems are useful.

A. Theorem:

Let L be the expected number of customers in the system. Then by using PME, the probability distribution, for the number of customers in the system is

$$P_n = \frac{(1-P_0)^2 (L-1+P_0)^{n-1}}{L^n}, \quad (n \geq 1) \qquad \ldots(5)$$

where $P_0 = 1 - \rho$

B. Proof:

We maximize the discrete countable entropy given by (1)subject to constraints (2) and (3). We have (cf. Jain, 1998)

$P_n = \exp(-\alpha - \beta n)$ $(\alpha>0, \beta>0)$ $(n = 1,2,\ldots)$ …(6)

Introducing (5) into (2), we get

$$\exp[-\alpha] = \frac{(1-P_0)\{1-\exp(-\beta)\}}{\exp(-\beta)} \qquad \ldots(7)$$

which implies

$$P_n = \frac{(1-P_0).\{1-\exp(-\beta)\}.\exp(-\beta n)}{\exp(-\beta)} \qquad \ldots(8)$$

and $$\exp(-\beta) = \frac{(L-1+P_0)}{L} \qquad \ldots(9)$$

By using equations (7) and (8), the theorem is established.

III. QUEUE SIZE DISTRIBUTION FOR GE/G/1 VACATION MODEL

We consider GE/G/1 queueing system with vacation. We assume that the inter-arrival time, service times and vacation times are i.i.d. random variables, which are also independent of each other. At the end of a busy period, the server goes on vacation for time with first and second moments and , respectively. The average number of customers (L) for GE/G/1 vacation (non-exhaustive service) model is given by (cf. Wu, 1992)

$$L = \frac{\lambda E(V^2)}{2E(V)} + \frac{\rho^2 (C_Y^2 - 1) + \rho/(1 + C_T^2)}{2(1-\rho)} \qquad \ldots(10)$$

Equation (5) gives

$$P_n = \frac{\rho^2 (L-\rho)^{n-1}}{L^n}, \quad \text{where } P_0 = 1-\rho \qquad \ldots(11)$$

From equations (10) and (11), we get

$$P_n = \frac{\rho^2 \left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho^2 (C_Y^2 - 1) + \rho/(1 + C_T^2)}{2(1-\rho)} - \rho \right)^{n-1}}{\left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho^2 (C_Y^2 - 1) + \rho/(1 + C_T^2)}{2(1-\rho)} \right)^n} \qquad \ldots(12)$$

A. Some Special Cases:

1) Substituting = 1/ 2 And = 1/2 In Equation (12), We Have:

$$P_n = \frac{\rho^2 \left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho(9\rho - 8)}{12(1-\rho)} \right)^{n-1}}{\left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho(4 - 3\rho)}{12(1-\rho)} \right)^n} \qquad \ldots(13)$$

2) *We Put =0 And = 0 In Equation (12), Yield:*

$$P_n = \frac{\rho^2\left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho}{2}\right)^{n-1}}{\left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho}{2}\right)^{n}} \quad \ldots(14)$$

3) *In This Case = 1 And = 1 Equation (12), Gives:*

$$P_n = \frac{\rho^2\left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho(4\rho-3)}{4(1-\rho)}\right)^{n-1}}{\left(\frac{\lambda E(V^2)}{2E(V)} + \frac{\rho}{4(1-\rho)}\right)^{n}} \quad \ldots(15)$$

IV. QUEUE SIZE DISTRIBUTION FOR MX/G/1 VACATION MODEL

The average number of customers (L) for MX/G/1 vacation (non-exhaustive service) model is given by (cf. Wu, 1992)

$$L = \frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho^2(C_Y^2-1)+2\rho/p}{2(1-\rho)} \quad \ldots(16)$$

Equations (11) and (16) give

$$P_n = \frac{\rho^2\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho^2(C_Y^2-1)+2\rho/p}{2(1-\rho)} - \rho\right)^{n-1}}{\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho^2(C_Y^2-1)+2\rho/p}{2(1-\rho)}\right)^{n}} \quad \ldots(17)$$

A. *Some Special Cases:*

Substituting = 1/ 2 in equation (17), we obtain

$$P_n = \frac{\rho^2\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho(3p\rho-4p+4)}{4p(1-\rho)}\right)^{n-1}}{\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho(4-p\rho)}{4p(1-\rho)}\right)^{n}} \quad \ldots(18)$$

B. *Putting =0 In Equation (17), We Get:*

$$P_n = \frac{\rho^2\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho(p\rho-2p+2)}{2p(1-\rho)}\right)^{n-1}}{\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho(2-p\rho)}{2p(1-\rho)}\right)^{n}} \quad \ldots(19)$$

C. *When = 1 Equation (17), Gives:*

$$P_n = \frac{\rho^2\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho(1-p+\rho p)}{p(1-\rho)}\right)^{n-1}}{\left(\frac{\lambda_g E(V^2)}{2E(V)} + \frac{\rho}{p(1-\rho)}\right)^{n}} \quad \ldots(20)$$

V. NUMERICAL RESULTS

In this section, we present the numerical results to evaluate queue size distribution by using analytical results derived in previous section. To demonstrate the effect of the arrival rate (λ), on probabilities, by varying traffic intensity (ρ) are presented in tables 1-2. For fixed parameters $C_Y^2 = 1$, $C_T^2 = 3.4$, $\frac{\lambda E(V^2)}{2E(V)} = 0.28$ and varying ρ, the numerical results for GE/G/1 model are summarized in table 1. Table 2, display results by varying ρ and fixed values $C_Y^2 = 0.003$, $\frac{\lambda_g E(V^2)}{2E(V)} = 0.1$ and K= 2 for M^X/G/1 model.

ρ / P	0.4	0.5	0.6	0.7	0.8
P_0	0.580220	0.506749	0.387024	0.271485	0.169568
P_1	0.236771	0.251676	0.239267	0.195618	0.139912
P_2	0.096620	0.124994	0.143016	0.140952	0.115443
P_3	0.039428	0.062078	0.086938	0.101562	0.095253
P_4	0.016089	0.030831	0.052848	0.073180	0.078594
P_5	0.006566	0.015312	0.032126	0.052730	0.064849
P_6	0.002679	0.007605	0.019529	0.037994	0.053507
P_7	0.001093	0.003777	0.011871	0.027377	0.044149
P_8	0.000446	0.001876	0.007216	0.019726	0.036428
P_9	0.000182	0.000932	0.004387	0.014214	0.030057
P_{10}	0.000074	0.000463	0.002667	0.010242	0.024801
P_{11}	0.000030	0.000230	0.001621	0.007379	0.020463
P_{12}	0.000012	0.000114	0.000985	0.005317	0.016884
P_{13}	0.000005	0.000057	0.000599	0.003831	0.013931
P_{14}	0.000002	0.000028	0.000364	0.002761	0.011495
P_{15}	0.000001	0.000014	0.000221	0.001989	0.009485

Table 1: The effect of traffic intensity (ρ) on probabilities for GE/G/1 vacation model ($C_Y^2 = 1$, $C_T^2 = 3.4$, $\frac{\lambda E(V^2)}{2E(V)} = 0.28$).

ρ / P	0.4	0.5	0.6	0.7	0.8
P_0	0.129688	0.185120	0.232098	0.255363	0.236641
P_1	0.097935	0.035101	0.167358	0.187106	0.182621
P_2	0.073957	0.098596	0.120677	0.137094	0.140933
P_3	0.055849	0.071955	0.087017	0.100449	0.108761
P_4	0.042175	0.052513	0.062745	0.007360	0.083934
P_5	0.031849	0.038324	0.045243	0.053927	0.064774
P_6	0.024051	0.027969	0.023664	0.039513	0.049987
P_7	0.013716	0.020412	0.023524	0.028951	0.038576
P_8	0.010135	0.014996	0.016962	0.021213	0.029770
P_9	0.007822	0.010871	0.012231	0.015543	0.022974
P_{10}	0.005907	0.007934	0.008819	0.011388	0.017730
P_{11}	0.004460	0.005790	0.006359	0.008349	0.013683
P_{12}	0.003368	0.004226	0.004586	0.006114	0.010559
P_{13}	0.002544	0.003084	0.003307	0.004980	0.008149
P_{14}	0.001921	0.002251	0.001719	0.003282	0.006289
P_{15}	0.001451	0.001199	0.001240	0.001762	0.003745

Table 2: The effect of traffic intensity (ρ) on probabilities for $M^X/G/1$ vacation model ($C_Y^2 = 0.003$, $\frac{\lambda_g E(V^2)}{2E(V)} = 0.1$, K=2).

VI. CONCLUSION

In this paper, we have analyzed GE/G/1 and MX/G/1 vacation queueing models with non-exhaustive service by employing the principle of entropy maximization technique. Explicit expressions are derived for the queue size distribution of the system subject to constraints expressed in terms of mean arrival rate, mean service rate, mean number of customers in the system, square coefficient of variation of arrival time, service time and first and second moments of vacation time. Some particular cases are deduced by setting appropriate parameters. Numerical results are obtained for illustration purpose, which demonstrate the validity of analytical results. The performance indices of GE/G/1 and MX/G/1 vacation queue discussed may be applicable for networking mechanism in modern telecommunication system, computer and manufacturing systems, etc.

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