

Performance Characteristic of Infinitely Short Bearing

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Abstract— An attempt has been made to analyse to performance of an infinitely rough short bearing. Here in solving the associated Reynold's type equation in new boundary conditions involving the standard deviation associated with roughness has been used. The pressure, load carrying capacity and friction are calculated. It is found that the roughness parameter significantly affects all the above performance characteristic. It is observed that effect due to the roughness compounded by taking the effect of standard deviation on the boundary conditions.

Key words: Journal Bearing, Pressure Distribution, Roughness, Load Carrying Capacity, friction

I. INTRODUCTION

Journal bearings are used to carry radial loads, for example, to support a rotating shaft. A simple journal bearing consists of two rigid cylinders. The outer cylinder (bearing) wraps the inner rotating journal (shaft). Normally, the position of the journal center is eccentric with the bearing center. A lubricant fills the small annular gap or clearance between the journal and the bearing. The amount of eccentricity of the journal is related to the pressure that will be generated in the bearing to balance the radial load. The lubricant is supplied through a hole or a groove and may or may not extend all around the journal. Patel et al. [1] study and analyze the performance of a hydrodynamic short journal bearing under the presence of a magnetic fluid lubricant. With the usual assumptions of hydrodynamic lubrication, the associated Reynolds equation for the fluid pressure is solved with appropriate boundary conditions. It was clearly observed that the load carrying capacity increases nominally while the coefficient of friction decreases significantly. Besides, it was seen that the bearing can support a load even when there is no flow of lubricant. Christensen and Tonder[2] described a theoretical analysis of the effects of surface roughness in a finite width bearing. The analysis was based upon a stochastic theory of hydrodynamic lubrication developed previously. It was shown how the effect of surface roughness on the bearing characteristics was closely tied up with features of nominal geometry as well as with operational factors. Christensen and Tonder [3] explained the application of this theory to the analysis of the full journal bearing of finite width. The analysis demonstrated how the roughness influences the characteristics of the bearing and also shows how roughness interacts with features of nominal geometry and operating factors to determine the bearing response. Andharia et al.[4] analyzed the performance of a rough porous journal bearing in the presence of a magnetic fluid lubricant. The stochastic film thickness was characterized by a random variable with non-zero mean, variance and skewness. Patil[5] in infinitely short journal bearings, the length of bearing in an axial direction was very short. Hence, there was considerable flow of fluid in an axial direction or z-direction. The journal bearings with length to diameter ratio (l/d ratio) less than

0.5 were considered as infinitely short journal bearings. Andharia et al.[6]described effect of transverse roughness on the behaviour of a hydrodynamic squeeze film between a non-rotating spherical surface and a hemispherical bearing under a steady load was discussed. It was assumed that the bearing surface as well as the surface of the approaching sphere has random roughness which was distributed throughout the surfaces. The modified Reynolds equation was solved and then the expressions for pressure, load carrying capacity and the response time were obtained and numerically computed. Lund [7] the effects of lubricant compressibility on the pressure profile and total load capacity in a partial journal bearing; pivot pressure as a measure of shoe load capacity; shoe friction characteristics; and optimum shoe clearance ratios for various speeds. The parameter λ was used as an indicator of the compressibility effect of gas in the bearing clearance. Ocvirk [8] the mathematical analysis presented here was the r e s u l t of attempts to compare existing solutions with the experimental data on pressure distribution in the oil film of a short journal bearing available in NACA TN 2507. It was apparent that the end leakage of oil flow had a predominant effect in narrow bearings, and that a simple approximation including end-leakage effects would be useful. Prakash and Prawal [9] explained the problem considered was that of a steady laminar flow of an incompressible micropolar fluid in a journal bearing. A two dimensional flow field was considered and order of magnitude arguments were made, which reduce the governing balance equations to a system of coupled, ordinary differential equations. Hence it has been proposed to study and analyze the effect of certain type of roughness on the performance characteristics of a infinitely short rough hydrodynamic journal bearing. The geometry of the bearing system and equilibrium of forces on fluid element are presented in figures below.

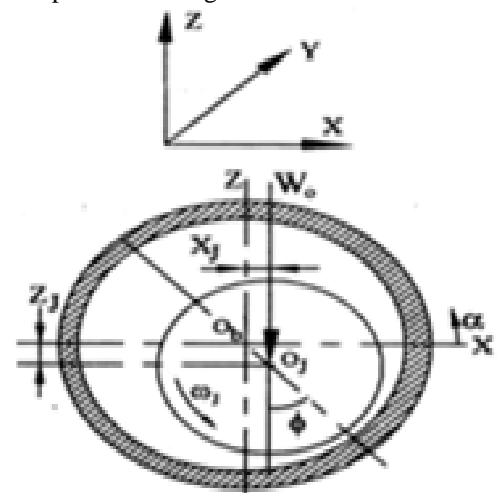


Fig. 1(a): Geometry and Coordinate system

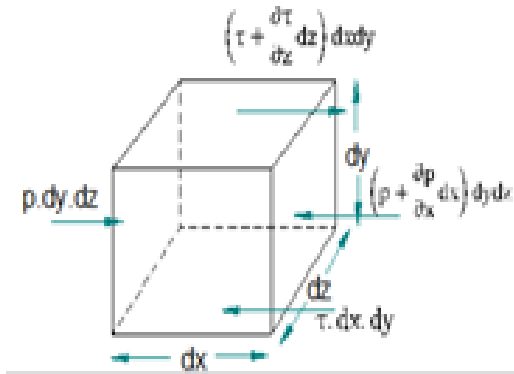


Fig. 1(b): Equilibrium of Forces on Fluid Element

Normal forces due to fluid pressure p act upon the left and right surfaces of the fluid element whereas shear forces due to viscosity act upon the bottom and top surfaces of the fluid element. Considering equilibrium of forces in X-direction, one finds that

$$p dy dz + \left(\tau_x + \frac{\partial \tau_x}{\partial y} dy \right) dx dz - \tau_x dx dz - \left(p + \frac{\partial p}{\partial x} dx \right) dy dz = 0$$

$$\therefore \frac{\partial \tau_x}{\partial y} = \frac{\partial p}{\partial x} \quad (1)$$

According to the Newton's law of viscosity

$$\tau_x = \mu \frac{\partial u}{\partial y}$$

Substituting the value in equation (1), one obtains that

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

which gives pressure gradient in X-direction. Since the pressure of the lubricant is constant in the direction of film thickness that is

$$\frac{\partial p}{\partial y} = 0$$

While the pressure gradient in z direction is

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2}$$

For time being we will restrict to analysis in X-direction only. Integrating equation (2) twice with respect to y, this results in

$$u = \frac{1}{2\mu} \frac{\partial p}{\partial x} y^2 + C_1 y + C_2 \quad (3)$$

The constants of integrations C_1 and C_2 are evaluated from the following boundary conditions:

$$y=0 \quad u=0$$

$$y=h+\sigma \quad u=U$$

Where U is the velocity in X-direction. In view of boundary conditions one arrives at

$$u = \frac{1}{2\mu} \frac{\partial p}{\partial x} (y^2 - (h+\sigma)y) + \frac{U}{h+\sigma} y \quad (4)$$

The equation gives the velocity distribution of lubricant in the film as a function of y and pressure gradient. The flow rate of lubricant in X-direction per unit width of Z-direction is given by

$$q_x = \int_0^{h+\sigma} u dy \quad (5)$$

Similarly, the flow rate of lubricant in the Z-direction per unit width of X-direction is found

$$q_z = -\frac{1}{12\mu} \frac{\partial p}{\partial z} (h+\sigma)^3 + \frac{W}{h} (h+\sigma) \quad (6)$$

Assuming there is no flow of lubricant in the Y-direction, the Reynolds's equation is obtained as

$$\frac{\partial}{\partial x} \left[\frac{(h+\sigma)^3}{\mu} \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[\frac{(h+\sigma)^3}{\mu} \frac{\partial p}{\partial z} \right] = 6 \left[\frac{\partial}{\partial x} U(h+\sigma) + \frac{\partial}{\partial z} W(h+\sigma) \right] \quad (7)$$

As U and W are not a function of x and z respectively.

It is, in fact very hard to think of any moving system where wedge h vary in two dimensions i.e. X and Y. Hence consideration of

$$\frac{d(h+\sigma)}{dz} = 0$$

$$\frac{\partial}{\partial x} \left[(h+\sigma)^3 \frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z} \left[(h+\sigma)^3 \frac{\partial p}{\partial z} \right] = 6\mu U \frac{d(h+\sigma)}{dx} \quad (8)$$

The pressure distribution along circumference is shown in figure

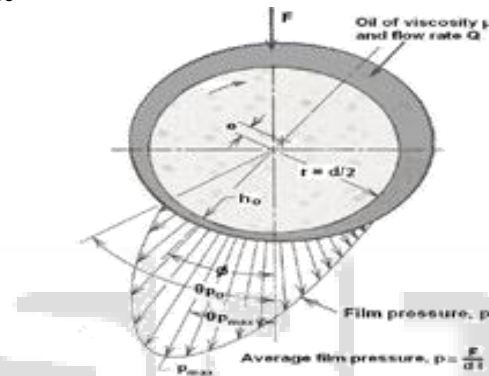


Fig. 2(a): Pressure distribution along circumference

II. PRESSURE DISTRIBUTION

For infinitely short journal bearing

$$\frac{\partial p}{\partial x} = 0 \quad (9)$$

Substituting equation (9) in equation (8)

$$\frac{\partial}{\partial z} \left[(h+\sigma)^3 \frac{\partial p}{\partial z} \right] = 6\mu U \frac{d(h+\sigma)}{dx} \quad (10)$$

Fluid film thickness h is not a function of z , hence equation (10) becomes

$$\frac{d^2 p}{dz^2} = \frac{6\mu U}{(h+\sigma)^2} \frac{d}{dx} (h+\sigma) \quad (11)$$

Integrating above equation

$$p = \frac{3\mu U}{(h+\sigma)^2} \frac{d}{dx} (h+\sigma) z^2 + c_1 z + c_2 \quad (12)$$

Using the boundary conditions

$$z = \pm \frac{l}{2} \quad p = 0$$

$$z = 0 \quad \frac{dp}{dz} = 0$$

and simplifying, one gets the expression for pressure distribution in the bearing system as

$$p = \frac{3\mu U l^2}{c^2 \left(1 + \varepsilon \cos \theta + \frac{\sigma}{c} \right)^3} \left(\frac{\varepsilon \sin \theta}{r} \right) \left[\frac{1}{4} - \frac{z^2}{l^2} \right] \quad (13)$$

Introduction of dimensionless quantities

$$\sigma^* = \frac{\sigma}{c} \quad z^* = \frac{z}{l} \quad P = -\frac{c^2 r}{\mu U l^2} P$$

Leads to the expression for the pressure distribution in the form

$$P = \frac{3\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta + \sigma^*)^3} \left(z^{*2} - \frac{1}{4} \right) \quad (14)$$

III. LOAD CARRYING CAPACITY

By using pressure distribution the load carrying capacity of the bearing can be obtained. The load carrying capacity is determined by using half Sommerfeld condition. Cameron [10] the half Sommerfeld condition states that when the film becomes divergent at the pressure becomes zero. Hence only the positive pressure region that is convergent film region supports the total load in bearing.

The component of total force in the direction of line of centres (N-direction) is described by,

$$W_N = \int_0^\pi \int_{-\frac{l}{2}}^{\frac{l}{2}} pr \, d\theta \, dz \cos \theta \quad (15)$$

Thus, the dimensionless load-carrying capacity in N direction is obtained from

$$W_N = \frac{c^2}{\mu U l^3} W_N = -\frac{\varepsilon^2}{((1 + \sigma^*)^2 - \varepsilon^2)^2} \quad (16)$$

The component of total force in the direction of line of centres (T-direction) is given by,

$$W_T = \int_0^\pi \int_{-\frac{l}{2}}^{\frac{l}{2}} pr \, d\theta \, dz \sin \theta \quad (17)$$

Thus, the dimensionless load-carrying capacity in T direction is obtained from

$$W_T = \frac{\sigma^2}{\mu U l^3} W_T = \frac{\pi \varepsilon}{4((1 + \sigma^*) - \varepsilon^2)^{3/2}} \quad (18)$$

Therefore, resultant Load Carrying capacity of the bearing is given by

$$W = \sqrt{W_N^2 + W_T^2} \quad (19)$$

$$W = \frac{\varepsilon}{4[(1 + \sigma^*) - \varepsilon^2]^2} \sqrt{\frac{(16 - \pi^2)\varepsilon^2}{\pi^2} + (1 + \sigma^*)} \quad (20)$$

A. Friction

The friction force is determined by

$$f = \int_0^{2\pi} \frac{\mu U}{h} LR \, d\theta \quad (21)$$

Non dimensional frictional force is given by

$$F = \frac{fc}{\mu U L r} = \frac{2\pi \varepsilon}{\sqrt{(1 + \sigma^*)^2 - \varepsilon^2}} \quad (22)$$

The coefficient of friction is given by

$$\mu = \frac{F}{W}$$

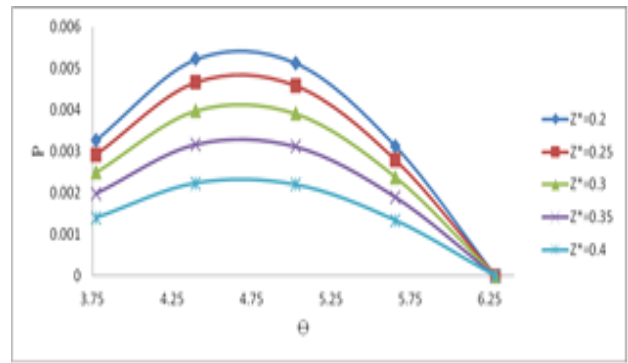


Fig. 3: Non dimensional pressure distribution P versus θ for different value of z^*

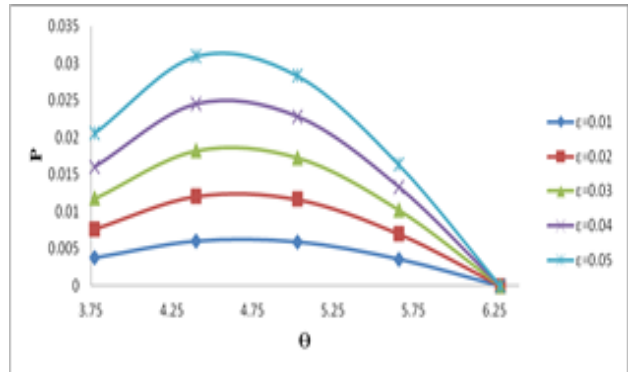


Fig. 4: Non dimensional pressure distribution P versus θ for different value of ε

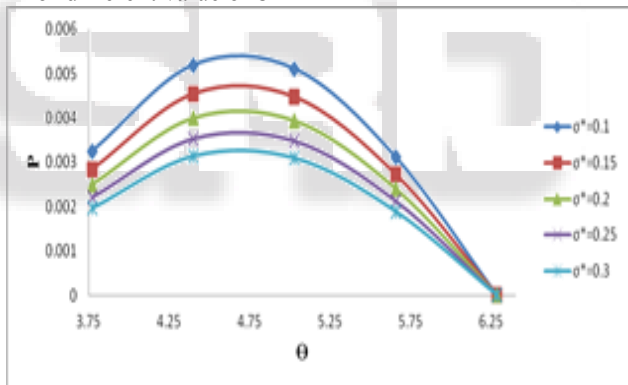


Fig. 5: Non dimensional pressure distribution P versus θ for different value of σ^*

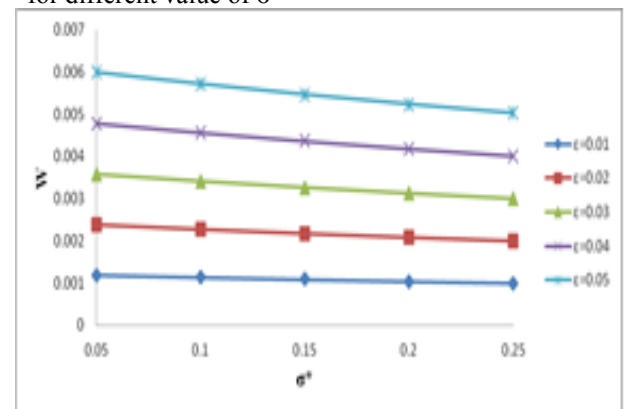


Fig. 6: Non dimensional load carrying capacity W versus σ^* for different values of ε

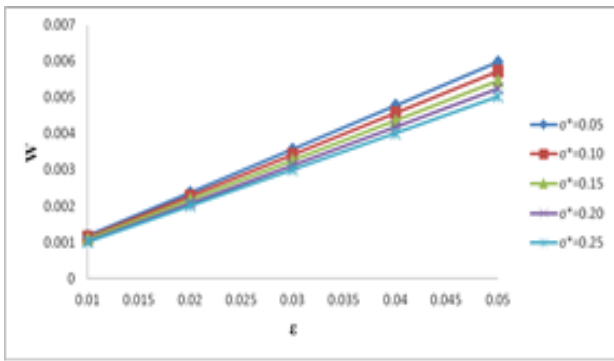


Fig. 7: Non dimensional load carrying capacity W versus ϵ for different values of σ^*

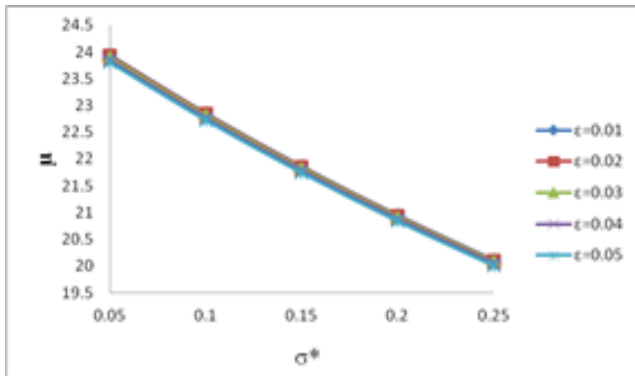


Fig. 8: Coefficient of friction μ versus σ^* for different values of ϵ

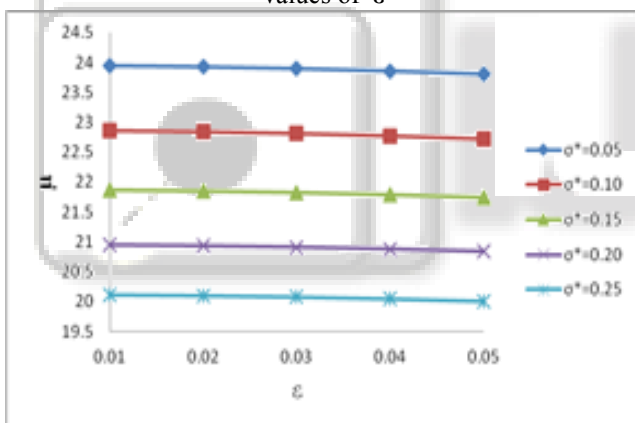


Fig. 9: Coefficient of friction μ versus ϵ for different values of σ^*

IV. RESULT AND DISCUSSION

The nearly parabolic profile of the pressure distribution is manifest in figures 3, 4 and 5. The fact that the influence of ϵ is more sharp as compare to other two parameters. It is noticed that the load carrying capacity decreases as the σ^* increases. Further the load carrying capacity increases as the eccentricity ratio increases, therefore the effect of standard deviation comes out to be adverse and eccentricity ratio appears to be quite significant. Fig 8-9 present the variation of coefficient of friction. It is seen that coefficient of friction increases due to ϵ while it decreases owing to σ^* . Further the effect of skewness in the variation of friction of friction with respect to σ^* is almost negligible.

V. CONCLUSION

This investigation underlines that the roughness aspects must be evaluated while designed this type of bearing system which is essential from bearing life period point of view. Thus load carrying capacity and friction get significantly affected by the roughness.

VI. NOMENCLATURE

- p : Lubricant pressure (N/mm²)
- P : Dimensionless pressure
- w : Load-carrying capacity (N)
- F : Dimensionless friction force
- μ : Coefficient of friction
- W : Dimensionless load carrying capacity
- η : Lubricant viscosity (N.S/mm²)
- e : Eccentricity (mm).
- σ : Standard deviation

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