

# Flow Resistance in Gravel Bed Rivers

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**Abstract**— This paper illustrates that the flow resistance equations require further testing and development for gravel bed rivers. Such rivers are characterized by coarse bed materials, steep slopes, and low depths. Comparison of data with the similar resistance equation relating the Darcy - Weisbach friction factor to the logarithm of relative submergence shows that the equation tends to overestimate the resistance in uniform flow. The main objective of this paper is review the current knowledge about flow resistance in gravel bed rivers.

**Keywords:** Gravel-bed Rivers; Flow resistance

## I. INTRODUCTION

Flow resistance relationships are a classical component of river hydraulics as flood routing, prediction of flow depths and velocities for design floods. However, gravel bed rivers, characterized by coarse bed materials and steep slopes, are poorly served by existing relationships and currently available formulae may typically have errors of ±35 %.

This paper seeks to improve the accuracy and applicability of flow resistance equation for gravel Bed Rivers, by assembling a different range of published data sets and following the at-a-site approach to their analysis.

The type of river considered has a bed of coarse material (gravel, cobbles, and boulders) and slopes in the approximate range 0.1- 5.0 %. New formulae for flow resistance estimation are derived for log type and power type equations for  $d_{50}$ ,  $d_{65}$ ,  $d_{84}$ , and  $d_{90}$  size of bed material data.

Computed values of  $(8/f)^{1/2}$  is compared with observed values of  $(8/f)^{1/2}$ . Based on the obtained results, graphical plots shows variations of  $(8/f)^{1/2}$  with different range of  $d_{50}$ ,  $d_{65}$ ,  $d_{84}$ , and  $d_{90}$  size of bed material data.

## II. FLOW RESISTANCE

Flow resistance is an expression used in hydraulics to describe the dynamic interaction between the water and the contour in which it flows. An important issue is the fact that the river bed retards or opposes the water flow, causing in turn energy losses. The appropriate inclusion of factors determining the flow resistance, as a result of water- river contour interaction, in a functional relationship by means of easily measured hydraulic parameters and a roughness coefficient allows a reliable prediction of velocity flows. In this relationship, the roughness coefficient quantifies the retarding capacity of the contour to the flow. Such relationships are known as flow resistance equations, which are described as bellow:

The resistance relationship can be expressed in dimensionless form as,

Darcy-Weisbach Equation :	$V = \left[ \frac{8gRS}{f} \right]^{1/2}$	(1)
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Manning's Equation:	$V = \frac{1}{n} R^{2/3} S^{1/2}$	(2)
Chezy's Equation :	$V = C(RS)^{1/2}$	(3)

where, V= mean velocity; R=hydraulic radius; S = friction slope; g=acceleration due to gravity; and f, n and C are, respectively, the Darcy-Weisbach's, Manning's, and Chezy's, resistance coefficients, related by:

$$\frac{V}{V_*} = \left( \frac{8}{f} \right)^{1/2} = \frac{C}{g^{1/2}} = \frac{R^{1/6}}{ng^{1/2}} \quad (4)$$

Where,  $V_* = (gRS)^{1/2}$  = the mean shear velocity.

## III. FORMULATION OF FLOW RESISTANCE EQUATION

Using the Prandtl-von Ka'rma'n universal velocity distribution equation for wide channel flow, i.e.

$$\frac{v}{V_*} = 5.75 \log \left( \frac{y}{k_s} \right) + 8.50 \quad (5)$$

in which v = point velocity at a height of y from the bed; and  $k_s$  = roughness height.

Keulegan (1938) has derived equations for mean velocity of turbulent flow in open channels as-

$$Q = \int v \cdot dA = V \cdot A \quad (6)$$

Substituting the pertinent expressions for 'v' in the foregoing equation and by subsequent simplification the equation for mean velocity can be obtained as:

$$\frac{V}{V_*} = 5.75 \log \left( \frac{R}{k_s} \right) + 6.25 \quad (7)$$

The dimensionless Darcy-Weisbach friction factor 'f' is widely used as a measure of resistance to flow in open channels. Thus, from the Chezy's formula i.e.  $V = C(RS)^{0.5}$  and from the definition of friction velocity –

$$\frac{V}{V_*} = \frac{V}{(gRS)^{0.5}} = C^* = \frac{C}{(g)^{0.5}} = (8/f)^{1/2} \quad (8)$$

and thus from Eqns. (7) and (8)-

$$\frac{V}{(gRS)^{0.5}} = \left( \frac{8}{f} \right)^{1/2} = 5.75 \log(R/k_s) + 6.25 \quad (9)$$

or

$$\frac{V}{(gRS)^{0.5}} = \left( \frac{8}{f} \right)^{1/2} = A \log(R/k_s) + B \quad (10)$$

in which A and B are numerical constant and  $k_s$  is the equivalent roughness of the boundary which can be recommended for different size of very rough bed materials

as given by different investigators. Thus, the friction factor defined by equation  $f=8gRS/V^2$  can be also evaluated by a semi-logarithmic type of equation, such as-

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = A \log(R/d_{xx}) + B \quad (11)$$

Equation: (7) can be also written as a power type of equation i.e.

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = a(R/d_{xx})^b \quad (12)$$

Here, again a, and b are some other numeric constants.

#### IV. DATA

Data source	Bed surface slope range, S, (%)	Mean Velocity range V (m/s)	Hydraulic radius R (m)	Bed material Size range (m)			
				d <sub>50</sub>	d <sub>65</sub>	d <sub>84</sub>	d <sub>90</sub>
Bathurst (1978)	0.80-1.74	0.245-0.896	0.165-0.403	0.170-0.277	-	0.280-0.485	-
Bathurst (1981)	0.80-1.74	0.146-1.301	0.008-0.070	0.006-0.073	-	0.008-0.090	-
Bathurst (1985)	0.40-3.14	0.171-3.577	0.102-1.600	0.060-0.343	-	0.113-0.740	-
Bray (1979)	0.02-0.015	0.053-0.238	0.442-6.920	0.019-0.145	0.025-0.200	-	0.036-0.275
Griffith (1981)	0.06-1.10	0.091-3.320	0.120-6.420	0.013-0.301	-	-	-
Thorne (1985)	1.43-1.98	0.518-1.416	0.289-0.615	0.130-0.162	0.200-0.240	0.337-0.393	-

Table 1: Source and range of data for the flow resistance analysis

#### V. RESULTS AND DISCUSSION

In the present study, the field data of different mountainous rivers and of different size of bed materials by different investigators such as Bathurst(1978,1981,1985); Bray(1979); Griffiths(1981); Hey(1979); Thorne(1986) et al. have been used, and following equation has been derived. By the regression analysis of these data with the help of equations (11) and (12) two set of equations for different size of bed materials have been obtained as discuss bellow.

##### A. Logarithmic Type Formulae

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 5.76 \log\left(\frac{R}{d_{50}}\right) + 3.28 \quad 13(a)$$

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 5.95 \log\left(\frac{R}{d_{65}}\right) + 3.40 \quad 13(b)$$

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 6.10 \log\left(\frac{R}{d_{84}}\right) + 3.50 \quad 13(c)$$

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 6.16 \log\left(\frac{R}{d_{90}}\right) + 3.60 \quad 13(d)$$

with a regression coefficient of 0.807, 0.643, 0.68 and 0.73, respectively.

##### B. Power Type Formulae

In the same way by the regression analysis of data as discussed earlier and with the help of equation (12) following power type of resistance equations have been obtained for different size of bed materials:

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 3.76 \left(\frac{R}{d_{50}}\right)^{0.288} \quad 14(a)$$

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 4.15 \left(\frac{R}{d_{65}}\right)^{0.278} \quad 14(b)$$

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 4.98 \left(\frac{R}{d_{84}}\right)^{0.288} \quad 14(c)$$

$$\frac{V}{(gRS)^{0.5}} = \left(\frac{8}{f}\right)^{1/2} = 5.12 \left(\frac{R}{d_{90}}\right)^{0.260} \quad 14(d)$$

with a regression coefficients of 0.81, 0.68, 0.75, and 0.76, respectively.

These equations have been derived from all size of bed materials such as gravel, cobbles and boulders. So these equations can be used for mountainous rivers comprising of bed materials of mixture of gravels, cobbles and boulders. Quantification of the resistance effect of bed material size distribution is not possible with the few available data. However, the foregoing equations suggests that much of the effect is accounted for by the inclusion of d<sub>84</sub> (or similarly large percentile of the size distribution) in the resistance relationship and that any remaining size distribution is small. Apart from the inclusion of d<sub>84</sub> or d<sub>90</sub> in the relative submergence, size distribution is therefore, neglected in the approach developed here and also it has been observed that with the increase in percentage size distribution, percentage error in computed value reduced.

- The present formulation provides equations that allow accurate prediction of flow resistance with error of ±20% to ±35% for d<sub>50</sub>, d<sub>65</sub>, d<sub>84</sub>, and d<sub>90</sub>, size of bed materials data.

- Other factors, apart from the measurement errors, are thought to be responsible for the residual variance in the statistical models.

VI. STATISTICAL ANALYSIS

A. Computed Vs Observed  $(8/f)^{1/2}$  for Log type equations

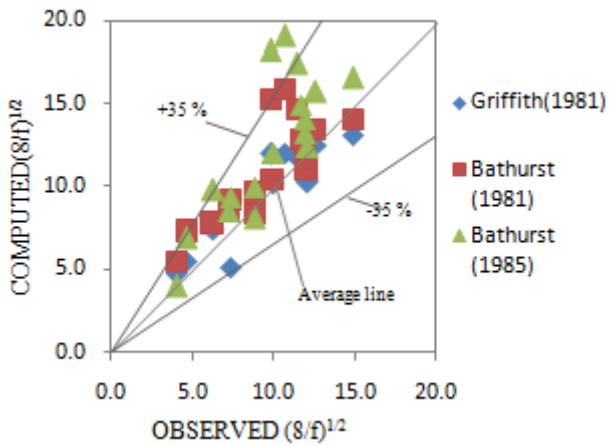


Fig. 1(a): Computed Vs Observed  $(8/f)^{1/2}$  For  $D_{50}$  Size

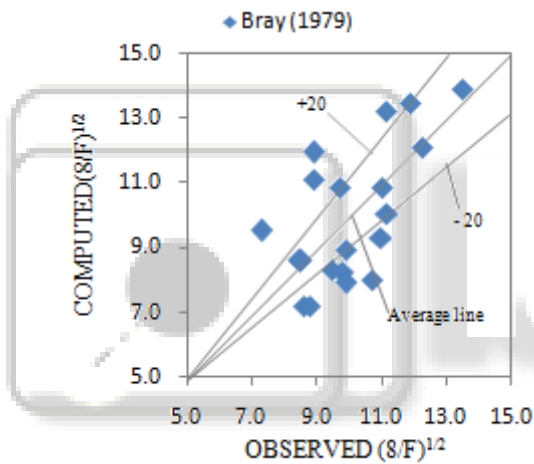


Fig. 1(b): Computed Vs Observed  $(8/f)^{1/2}$  For  $D_{65}$  Size

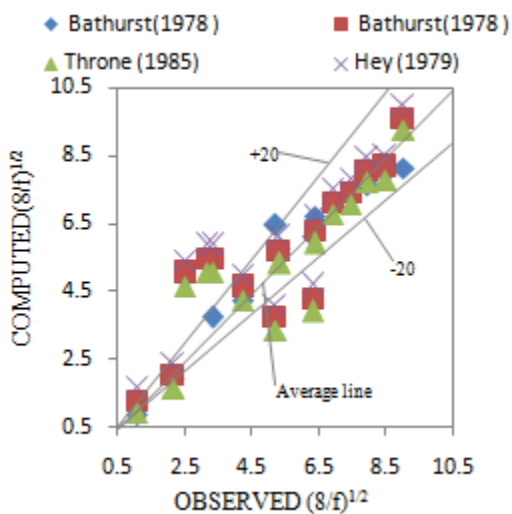


Fig. 1(c): Computed Vs Observed  $(8/f)^{1/2}$  For  $D_{84}$  Size

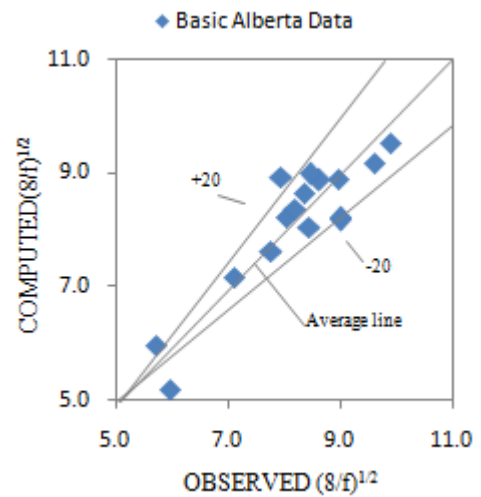


Fig. 1(d): Computed Vs Observed  $(8/f)^{1/2}$  For  $D_{90}$  Size

The observed and corresponding computed values of mean velocity from Eqs. 13(a, b, c & d), have been plotted as shown in Figs. 1(a, b, c, & d), respectively. All are prone to have a maximum percentage error of the order of nearly  $\pm 20$  to  $\pm 35$  %.

B. Computed Vs Observed  $(8/f)^{1/2}$  for Power type equations

- ◆ Griffith(1981)    ■ J.C.Bathurst (1981)    ▲ J.C.Bathurst (1985)

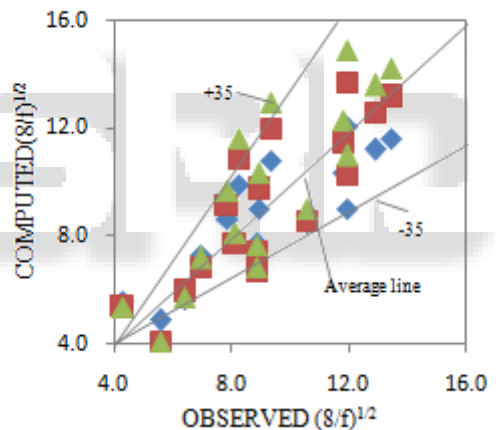


Fig. 2(a): Computed Vs Observed  $(8/f)^{1/2}$  For  $D_{50}$  Size

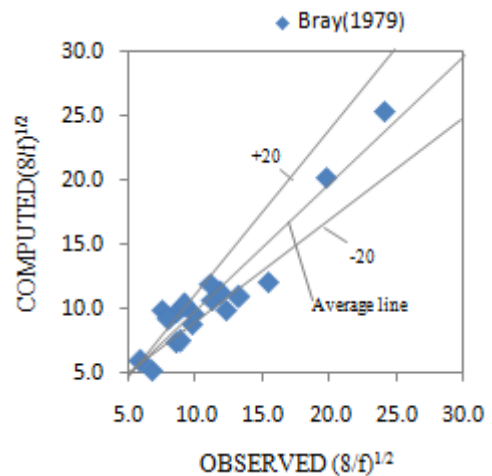


Fig. 2(b): Computed Vs Observed  $(8/f)^{1/2}$  For  $D_{65}$  Size

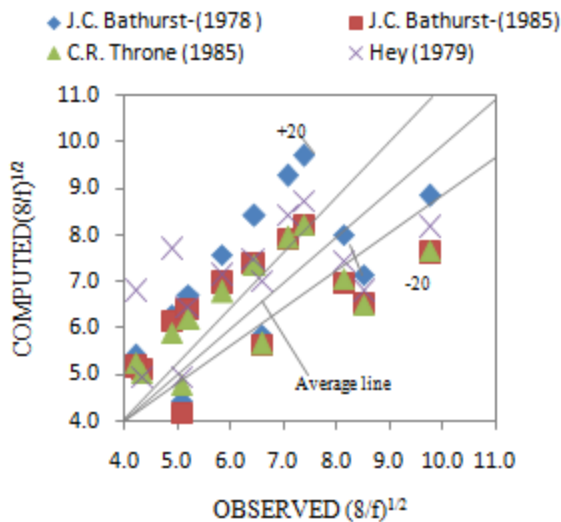
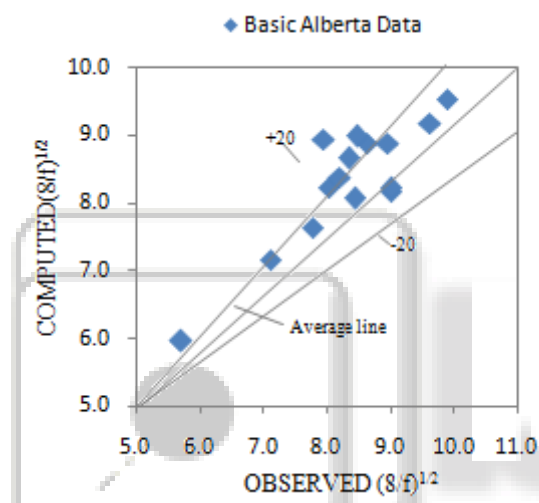


Fig. 2(c): Computed Vs Observed  $(8/f)^{1/2}$  For  $D_{84}$  Size



The observed and corresponding computed values of mean velocity from Eqs. 14(a, b, c & d), have been plotted as shown in Figs. 2(a, b, c, & d), respectively. All are prone to have a maximum percentage error of the order of nearly  $\pm 20$  to  $\pm 35$  %.

## VII. CONCLUSIONS

Gravel bed rivers differ significantly from lowland rivers in several important aspects. As compared to lowland rivers, The slope of gravel bed rivers are steeper(1.0 - 5.0%), the bed is formed in gravels, cobbles and boulders rather than sandy bed and the relative roughness is larger with boulders , cobbles and mixture of boulders ,cobbles and gravels often protruding through the free surface. Because of the differences, the form drag of the gravels, cobbles and boulders, free surface distortion and hydraulic jump associated with locally accelerated flow all contribute significantly to flow resistance in Gravel Bed Rivers, but are not explicitly accounted for in resistance equations for low land rivers. This has led to the development of flow resistance equation specifically intended for steep streams with very rough surfaces and further research is required to produce a reliable and process based flow resistance.

### A. Notations:

The different symbols used in this paper are as

C	:	Chezy's coefficient, $(m^{1/2}/s)$ ;
N	:	Manning's roughness coefficient, $(m^{-1/3}s)$ ;
$K_s$	:	Nikuradse roughness size;
$\kappa$	:	Von Karman's constant;
$d_{xx}$	:	Size of the bed particles for which "XX" % are finer, (m);
$d_{50}$	:	grain size in which 50 % of the particles are finer by counting, (m);
$d_{65}$	:	grain size in which 65 % of the particles are finer by counting, (m);
$d_{84}$	:	grain size in which 84 % of the particles are finer by counting, (m);
$d_{90}$	:	Grain size in which 90 % of the particles are finer by counting, (m);
f	:	Darcy-Weisbach's bed friction factor;
g	:	gravitational acceleration, $(m/s^2)$ ;
V	:	mean velocity, $(m/s)$ ;
$V_*$	:	shear velocity, $(m/s)$ ;
R	:	hydraulic radius, (m);
$F_r$	:	Froude number;
Q	:	discharge, $(m^3/sec)$ ; and
S	:	Water surface slope;

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