Sensitivity Analysis of GRA Method for Interval Valued Intuitionist Fuzzy MADM: The Results of Change in the Weight of One Attribute on the Final Ranking of Alternatives

Poovarasan.V[1] John Robinson P[2]
[1,2]Assistant Professor
[1,2]Department of Mathematics
[1]Sri Vidya Mandir Arts & Science College, Uthangarai [2]Bishop Heber College, Tiruchirappalli

Abstract— The aim of this paper is to investigate the multiple attribute decision making problems with intuitionistic fuzzy information, in which the information about attribute weights are incompletely known, and the attribute values take the form of intuitionistic fuzzy numbers. In order to get the weight vector of the attribute, we establish an optimization model based on the basic ideal of traditional gray relational analysis (GRA) method, by which the attribute weights can be determined. For the special situations where the information about attribute weights are completely unknown, we establish another optimization model. By solving this model, we get a simple and exact formula, which can be used to determine the attribute weights. Then, based on the traditional GRA method, calculation steps for solving an interval-valued intuitionistic fuzzy environment and developed modified GRA method for interval-valued intuitionistic fuzzy multiple attributes decision-making with incompletely known attribute weight information. This paper provides a new method for sensitivity analysis of MADM problems so that by sing it and changing the weights of attributes, one can determine changes in the final results for a decision making problem. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

***Keywords**:* MADM, Attribute weight, IVIFs, Gray relational analysis (GRA) method, sensitivity analysis, Ranking methods

I. INTRODUCTION

The aim of this paper is to extend the concept of gray relational analysis (GRA) to develop a methodology for solving MADM problems with an interval valued intuitionistic fuzzy environment and developed modified GRA method for interval-valued intuitionistic fuzzy multiple attribute decision-making with incompletely known attribute weight information. In order to do so, the rest of this paper is organized as follows: next section briefly introduce some basic concepts related to intuitionistic fuzzy sets. In Section 3, we have extended the above results to an interval-valued intuitionistic fuzzy environment and developed modified GRA method for interval-valued intuitionistic fuzzy multiple attributes decision-making with incompletely known attribute weight information. . In Section 4, we offer a new method for sensitivity analysis of MADM problems so that by using it and changing the weights of attributes, one can determine changes in the results of a decision making problem. In Section 5 we illustrate our proposed algorithmic method with an example. Thus, The final section concludes.

II. PRELIMINARIES

In the following, we introduce some basic concepts related to intuitionist fuzzy sets.

A. *Definition 1*

Let X be a universe of discourse, then a fuzzy set is defined as

$$A=\{\langle x,\mu_A(x)\rangle \mid x\in X\} \quad (1)$$

Which is characterized by a membership function $\mu_A : X\to[0,1]$, where $\mu_A(x)$ denotes the degree of membership of the element *x* to the set *A* extended the fuzzy set to the IFS, shown as follows

B. *Definition 2*

An IFS *A* in *X* is given by

$$A=\{\langle x,\mu_A(x),\nu_A(x)\rangle \mid x\in X\} \quad (2)$$

where $\mu_A : X\to[0,1]$ and $\nu_A : X\to[0,1]$, with the condition $0\le \mu_A(x)+\nu_A(x)\le 1,\ \forall x\in X$

The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element *x* to the set *A*.

C. *Definition 3*

For each IFS *A* in *X*, if

$$\pi_A(x)=1-\mu_A(x)-\nu_A(x),\ \forall x\in X \quad (3)$$

Then $\pi_A(x)$ is called the degree of indeterminacy of *x* to *A*.

III. GRA METHOD FOR MULTIPLE ATTRIBUTE DECISION MAKING PROBLEMS WITH INTERVAL-VALUED INTUITIONISTIC FUZZY INFORMATION

Further introduced the interval-valued intuitionistic fuzzy set (IVIFS), which is a generalization of the IFS. The fundamental characteristic of the IVIFS is that the values of its membership function and non-membership function are intervals rather than exact numbers.

A. *Definition 4*

Let *X* is a universe of discourse. An IVIFS $\tilde{A}$ over *X* is an object having the form

$$\tilde{A}=\{\langle x,\tilde{\mu}_A(x),\tilde{\nu}_A(x)\rangle \mid x\in X\} \quad (4)$$

Where $\tilde{\mu}_A(x)\subset[0,1]$ and $\tilde{\nu}_A(x)\subset[0,1]$ are interval numbers, and $0\le \sup(\tilde{\mu}_A(x))+\sup(\tilde{\nu}_A(x))\le 1,\ \forall x\in X$. For convenience, let

$\tilde{\mu}_A(x)=[a,b], \tilde{\nu}_A(x)=[c,d]$, so $\tilde{A}=([a,b],[c,d])$, where $[a,b]\subset[0,1],\ [c,d]\subset[0,1],\ b+d\le 1$

B. *Definition 5*

Let $\tilde{a}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{a}_2 = ([a_2, b_2], [c_2, d_2])$ be two interval-valued intuitionistic fuzzy values, then the normalized Hamming distance between $\tilde{a}_1 = ([a_1, b_1], [c_1, d_1])$ and $\tilde{a}_2 = ([a_2, b_2], [c_2, d_2])$ is defined as follows:

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{4}\left(|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|\right) \quad (5)$$

Let *A*, *G*, *w* and *H* be presented as in Section 2. Suppose that $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])_{m\times n}$ is the interval-valued intuitionistic fuzzy decision matrix, where $[a_{ij}, b_{ij}]$ indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, $[c_{ij}, d_{ij}]$ indicates the degree that the alternative A_i does not satisfy the attribute G_j given by the decision maker, $[a_{ij}, b_{ij}] \subset [0,1], [c_{ij}, d_{ij}] \subset [0,1]$, $b_{ij} + d_{ij} \le 1$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.

In the following, we apply GRA method to solve interval-valued intuitionistic fuzzy MADM with incompletely known weight information. The method involves the following steps:

1) *Step 1*

Determine the positive ideal with interval-valued intuitionistic fuzzy information.

$$\tilde{r}^+ = ([a_1^+, b_1^+], [c_1^+, d_1^+], [a_2^+, b_2^+], [c_2^+, d_2^+], \ldots, [a_n^+, b_n^+], [c_n^+, d_n^+]) \quad (6)$$

Where

$$\tilde{r}^+ = ([a_j^+, b_j^+], [c_j^+, d_j^+]) = \left(\left[\max_i a_{ij}, \max_i b_{ij}\right], \left[\max_i c_{ij}, \max_i d_{ij}\right]\right), \quad j = 1, 2, \ldots, n$$

2) *Step 2*

Calculate the gray relational coefficient of each alternative from PIS using the following equation:

$$\zeta_{ij}^+ = \frac{\min_{1\le i\le m} \min_{1\le j\le n} d(\tilde{r}_{ij}, \tilde{r}_j^+) + \rho \max_{1\le i\le m} \max_{1\le j\le n} d(\tilde{r}_{ij}, \tilde{r}_j^+)}{d(\tilde{r}_{ij}, \tilde{r}_j^+) + \rho \max_{1\le i\le m} \max_{1\le j\le n} d(\tilde{r}_{ij}, \tilde{r}_j^+)} \quad (7)$$

$$i = 1, 2, \ldots, m, \; j = 1, 2, \ldots, n$$

where the identification coefficient $\rho = 0.5$.

3) *Step 3*

Calculating the degree of gray relational coefficient of each alternative from PIS using the following equation:

$$\zeta_i^+ = \sum_{j=1}^{n} w_j \zeta_{ij}^+, \quad i = 1, 2, \ldots, m \quad (8)$$

The basic principle of the GRA method is that the chosen alternative should have the ‘‘largest degree of gray relation’’ from the positive ideal solution. Obviously, for the weight vector given, the larger ζ_i^+, the better alternative A_i is.

If the information about attribute weights are incompletely known, in order to get the ζ_i^+, firstly, we must calculate the weight information. The gray relational coefficient between PIS and itself are (1, 1... 1), so the comprehensive gray relational coefficient deviation sum is

$$d_i(w) = \sum_{j=1}^{n} (1 - \zeta_{ij}^+) w_j \quad (9)$$

So, we can establish the following multiple objective optimization models to calculate the weight information:

$$\begin{cases} \min d_i(w) = \sum_{j=1}^{n} (1 - \zeta_{ij}^+) w_j, \quad i = 1, 2, \ldots, m \\ \text{subject to}: w \in H \end{cases} \quad (10)$$

Since each alternative is non-inferior, so there exists no preference relation on the all the alternatives. Then, we may aggregate the above multiple objective optimization models with equal weights into the following single objective optimization model:

$$\begin{cases} \min d(w) = \sum_{j=1}^{n} d_i(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} (1 - \zeta_{ij}^+) w_j \\ \text{subject to}: w \in H \end{cases} \quad (11)$$

By solving the model (11), we get the optimal solution *w = (w$_1$,w$_2$, . . . ,w$_n$)*, which can be used as the weight vector of attributes. Then, we can get ζ_i^+ *(i=1,2,…,m)* by Eq. (8).

If the information about attribute weights are completely unknown, we can establish another multiple objective programming model as follows:

$$\begin{cases} \min d_i(w) = \sum_{j=1}^{n} \left[(1 - \zeta_{ij}^+) w_j\right]^2, \\ \text{subject to}: \sum_{j=1}^{n} w_j = 1 \end{cases} \quad (12)$$

Similarly, we may aggregate the above multiple objective optimization models with equal weights into the following single objective optimization model:

$$\begin{cases} \min d(w) = \sum_{i=1}^{n} d_i(w) = \sum_{i=1}^{m}\sum_{j=1}^{n} \left[(1 - \zeta_{ij}^+) w_j\right]^2 \\ \text{subject to}: \sum_{j=1}^{n} w_j = 1 \end{cases} \quad (13)$$

To solve this model, we construct the Lagrange function:

$$L(w, \lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n} \left[(1 - \zeta_{ij}^+) w_j\right]^2 + 2\lambda\left(\sum_{j=1}^{n} w_j - 1\right) \quad (14)$$

where λ is the Lagrange multiplier. Differentiating Eq. (14) with respect to w_j *(j = 1,2,. . . ,n)* and λ, and setting these partial derivatives equal to zero, we get a simple and exact formula for determining the attribute weights as follows:

$$w_j = \left[\sum_{j=1}^{n}\left(\sum_{i=1}^{m} (1 - \zeta_{ij}^+)^2\right)^{-1}\right]^{-1} \Big/ \sum_{i=1}^{m} (1 - \zeta_{ij}^+)^2 \quad (15)$$

Then, we can get ζ_i^+ *(i=1,2,…,m)* by Eq. (8).

4) *Step 4*

Rank all the alternatives A_i *(i = 1, 2,. . . ,m)* and select the bet one(s) in accordance with ζ_i^+ *(i=1,2,…,m)*. If any alternative has the highest ζ_i^+ value, then, it is the most important alternative.

5) Step 5
End.

IV. Developing New Method FOR Sensitivity Analysis OF MADM Problems

Earlier researches on the sensitivity analysis of MADM problems often focused on determining the most sensitive attribute. They also focused on finding the least value of the change. However, a new method for sensitivity analysis of MADM problems is considered in this article that calculates the changing in the final score of alternatives when a change occurs in the weight of one attribute.

The effects of change in the weight of one attribute on the weight of other attributes are,

The vector for weights of attributes is $W^t = (w_1, w_2, ..., w_k)$ wherein weights are normalized with a sum of 1, that is:

$$\sum_{j=1}^{k} w_j = 1 \qquad (11)$$

With these assumptions, if the weight of one attribute changes, then the weight of other attributes change accordingly, and the new vector of weights transformed into $W'^t = (w'_1, w'_2, ..., w'_k)$

The next theorem depicts the changes in the weight of attributes.

A. Theorem 4.1.1

In the MADM model, if the weight of the P^{th} attribute, changes by Δ_p, then the weight of other attributes change by Δ_j, where:

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}; j = 1, 2, ..., k, \quad j \neq p \qquad (16)$$

Proof

If the new weight of the attribute is w'_j and the new weight of the P^{th} attribute changes as:

$$w'_p = w_p + \Delta_p \qquad (17)$$

Then, the new weight of the other attributes would change as

$$w'_j = w_j + \Delta_j; j = 1, 2, ..., k, \quad j \neq p \qquad (18)$$

And because the sum of weights must be 1 then:

$$\sum_{j=1}^{k} w'_j = \sum_{j=1}^{k} w_j + \sum_{j=1}^{k} \Delta_j \Rightarrow \sum_{j=1}^{k} \Delta_j = 0 \qquad (19)$$

Therefore:

$$\Delta_p = -\sum_{\substack{j=1 \\ j \neq p}}^{k} \Delta_j \qquad (20)$$

Where:

$$\Delta_j = \frac{\Delta_p . w_j}{w_p - 1}; j = 1, 2, ..., k \;, j \neq p \qquad (21)$$

Since:

$$-\Delta_p = \sum_{\substack{j=1 \\ j \neq p}}^{k} \Delta_j = \sum_{\substack{j=1 \\ j \neq p}}^{k} \frac{\Delta_p . w_j}{w_p - 1} = \frac{\Delta_p}{w_p - 1} \sum_{\substack{j=1 \\ j \neq p}}^{k} w_j = \frac{\Delta_p}{w_p - 1}(1 - w_p) = -\Delta_p \qquad (22)$$

Main result. In a MADM problem, if the weight of the P^{th} attribute changes from w_p to w'_p as:

$$w'_p = w_p + \Delta_p \qquad (23)$$

Then, the weight of other attributes would change as:

$$w'_j = \frac{1 - w_p - \Delta_p}{1 - w_p} . w_j = \frac{1 - w'_p}{1 - w_p} . w_j \qquad j = 1, 2, ..., k \;, j \neq p \qquad (24)$$

Since, for $j = 1, 2, ..., k \;, j \neq p$ we have:

$$w'_j = w_j + \Delta_j = w_j + \frac{\Delta_p . w_j}{w_p - 1} = \frac{w_j (w_p - 1) + \Delta_p . w_j}{w_p - 1} \qquad (25)$$

$$\Rightarrow w'_j = \frac{(1 - w_p - \Delta_p) . w_j}{1 - w_p} = \frac{1 - w'_p}{1 - w_p} . w_j; \qquad j = 1, 2, ..., k \;, j \neq p$$

(26)

Then, new vector for weights of attributes would be $W'^t = (w'_1, w'_2, ..., w'_k)$, that is:

$$w'_j = \begin{cases} w_j + \Delta_p & j = p \\ \dfrac{1 - w'_p}{1 - w_p} . w_j & j \neq p, \; j = 1, 2, ..., k \end{cases} \qquad (27)$$

$$w'_p = w_p + \Delta_p \quad \Rightarrow \begin{cases} if \; w'_p > w_p & \Rightarrow w'_j < w_j \\ if \; w'_p < w_p & \Rightarrow w'_j > w_j \end{cases}$$
$$j = 1, 2, ..., k \;, j \neq p \qquad (28)$$

The sum of new weights of attributes that are obtained in (23) is 1,because:

$$\sum_{j=1}^{k} w'_j = \sum_{\substack{j=1 \\ j \neq p}}^{k} w'_j + w'_p = \sum_{\substack{j=1 \\ j \neq p}}^{k} \frac{w_j (1 - w_p - \Delta_p)}{1 - w_p} + w_p + \Delta_p$$
$$= \frac{(1 - w_p - \Delta_p)}{1 - w_p} \sum_{\substack{j=1 \\ j \neq p}}^{k} w_j + w_p + \Delta_p$$
$$= \frac{(1 - w_p - \Delta_p)}{1 - w_p} . (1 - w_p) + w_p + \Delta_p$$
$$= 1 - w_p + w_p + \Delta_p = 1 \qquad (29)$$

V. NUMERICAL ILLUSTRATION

Let us suppose there is an investment company, which wants to invest a sum of money in the best option. There is a panel with five possible alternatives to invest the money: A_1 is a car company; A_2 is a food company; A_3 is a computer company; A_4 is an arms company; A_5 is a TV company. The investment company must take a decision according to the following four attributes: G_1 is the risk analysis; G_2 is the growth analysis; G_3 is the social-political impact analysis; G_4 is the environmental impact analysis. The five possible alternatives A_i *(i = 1,2,3,4,5)* are to be evaluated using the interval-valued intuitionistic fuzzy information by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{R}=\begin{bmatrix} ([0.4,0.5],[0.3,0.4]) & ([0.4,0.6],[0.2,0.4]) & ([0.3,0.4],[0.4,0.5]) & ([0.5,0.6],[0.1,0.3]) \\ ([0.5,0.6],[0.2,0.3]) & ([0.6,0.7],[0.2,0.3]) & ([0.5,0.6],[0.3,0.4]) & ([0.4,0.7],[0.1,0.2]) \\ ([0.3,0.5],[0.3,0.4]) & ([0.1,0.3],[0.5,0.6]) & ([0.2,0.5],[0.4,0.5]) & ([0.2,0.3],[0.4,0.6]) \\ ([0.2,0.5],[0.3,0.4]) & ([0.4,0.7],[0.1,0.2]) & ([0.4,0.5],[0.3,0.5]) & ([0.5,0.8],[0.1,0.2]) \\ ([0.3,0.4],[0.1,0.3]) & ([0.7,0.8],[0.1,0.2]) & ([0.5,0.6],[0.2,0.4]) & ([0.6,0.7],[0.1,0.2]) \end{bmatrix}$$

In such case, we can utilize the proposed procedure II to get the most desirable alternative (s).

A. *Case 1*

The information about the attribute weights are partly known and the known weight information is given as follows:

$$H=\{0.23\le w_1\le 0.26, 0.15\le w_2\le 0.18, 0.30 \le w_3\le 0.35, 0.25\le w_4\le 0.28, w_j\ge 0, j=1,2,3,4, \sum_{j=1}^{4} w_j=1\}$$

Step 1. Determine the positive ideal

$$\tilde{r}^{+}=[([0.5,0.6],[0.1,0.3]),([0.7,0.8],[0.1,0.2]), ([0.5,0.6],[0.2,0.4]),([0.6,0.8],[0.1,0.2])]$$

Step 2. Calculate the gray relational coefficient of each alternative from PIS

$$d(\tilde{r}_{ij},\tilde{r}_j^{+})=\begin{bmatrix} 0.125 & 0.2 & 0.175 & 0.1 \\ 0.025 & 0.1 & 0.025 & 0.075 \\ 0.15 & 0.475 & 0.175 & 0.4 \\ 0.175 & 0.1 & 0.1 & 0.025 \\ 0.1 & 0 & 0 & 0.025 \end{bmatrix}$$

$$\zeta^{+}=\left(\zeta_{ij}^{+}\right)_{5\times 4}=\begin{bmatrix} 0.556 & 0.5429 & 0.3636 & 0.75 \\ 1.0000 & 0.7037 & 0.7778 & 0.8182 \\ 0.6774 & 0.3333 & 0.5758 & 0.4118 \\ 0.4286 & 0.7037 & 0.4667 & 1.0000 \\ 0.4667 & 1.0000 & 1.0000 & 0.8889 \end{bmatrix}$$

Step 3. Utilize the model (11) to establish the following single objective programming model:

$$\begin{cases} \min \xi(w)=1.8717w_1+1.7164w_2+1.8161w_3+1.1311w_4 \\ \text{Subject to}: w\in H \end{cases}$$

Solve this model; we get the weight vector of attributes:

w = (0.23, 0.15, 0.35, 0.27)

Then, we can get the degree of gray relational coefficient of each alternative from PIS

$\zeta_1^{+}=0.5389,\ \zeta_2^{+}=0.8287,\ \zeta_3^{+}=0.5185,$

$\zeta_4^{+}=0.6375, \zeta_5^{+}=0.8473,$

Step 4. According to the relative relational degree, the ranking order of the five alternatives are: $A_5\succ A_2\succ A_4\succ A_1\succ A_3$, and thus the most desirable alternative is A_5.

Case 2: If the information about the attribute weights are completely changed, we utilize the new method for Sensitivity Analysis developed to get the most desirable alternative(s).

Now we assume that the weight of the 2nd attribute increased by $\Delta_p=0.1500$ and be $w_2'=w_2+\Delta_2=0.1500+0.1500=0.3000$. Then by equation (24), the weight of other attributes change as (30):

$$w_j'=\frac{1-w_4'}{1-w_4}.w_j;\quad j=1,2,3$$

$$\Rightarrow w'^{T}=(0.1894, 0.3000, 0.2882, 0.2224). \qquad (30)$$

Then, we can get the degree of gray relational coefficient of each alternative from PIS

$\zeta_1^{+}=0.5397,\ \zeta_2^{+}=0.8066,\ \zeta_3^{+}=0.4858,$

$\zeta_4^{+}=0.6492, \zeta_5^{+}=0.8743,$

According to the relative relational degree, the ranking order of the five alternatives are: $A_5\succ A_2\succ A_4\succ A_1\succ A_3$, and thus the most desirable alternative is also A_5.

From the proceeding figures, it can be observed that even though there are much variation in the attribute weights computed from the two different models, there are less variation observed in the coefficients calculated from the two models, hence resulting in same ranking of the alternatives from the two models. Hence the proposed MADM model with coefficient of IVIFs is an effective model because of its uncompromising ranking of the alternatives even when the attribute weights are partially completely unknown and completely changed.

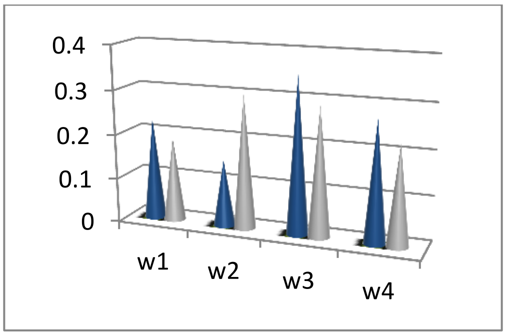

Fig. 1: Comparison of Weights

	W1	W2	W3	W4
Model-1	0.2300	0.1500	0.3500	0.2700
Model-2	0.1894	0.3000	0.2882	0.2224

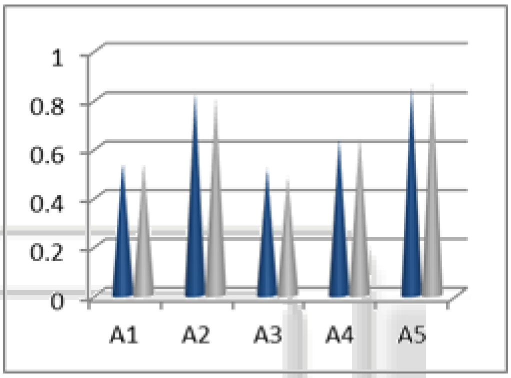

Fig. 2: Comparison of Coefficient

	A1	A2	A3	A4	A5
Model-1	0.5389	0.8287	0.5185	0.6375	0.8473
Model-2	0.5397	0.8066	0.4858	0.6492	0.8743

VI. CONCLUSION

Decision making is the integral part of human life. Regardless of the variety of decision making problems, we can categorize them into two categories, multi objective decision making problems that decision maker must design an approach that has the most utility by considering limited resources and multi-attribute decision making problems that decision maker must select one alternative from among available alternatives so that has the most utility. Naturally, for selecting an alternative we must consider several and often conflicting attributes.

In this paper, we have investigated the problem of calculating IVIF method for acquiring best alternative of attribute weight. When the attribute weights of alternatives are changed through Sensitive analysis method and when it has been calculated the result which we acquired in the above method that is IVIF method and sensitive analysis both represented the same desirable alternative. Finally, an illustrative example is given to verify the developed approach and to demonstrate its practicality and effectiveness.

REFERENCES

[1] Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 20,* (1986), 87–96.
[2] Atanassov, K. More on intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 33,* (1989), 37–46.
[3] Atanassov, K. Operators over interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 64(2),* (1994), 159–174.
[4] Atanassov, K. Two theorems for intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 110,* (2000), 267–269.
[5] Atanassov, K., & Gargov, G. Interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 31,* (1989), 343–349.
[6] Boran, F. E., Genç, S., Kurt, M., & Akay, D. A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method. *Expert Systems with Applications, 36(8),* (2009), 11363–11368.
[7] Bustince, H. Construction of intuitionistic fuzzy relations with predetermined properties. *Fuzzy Sets and Systems, 109,* (2000), 379–403.
[8] Bustince, H., & Burillo, P. Correlation of interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 74,* (1995), 237–244.
[9] Chen, S. M., & Tan, J. M. Handling multi criteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets Systems, 67,* (1994), 163–172.
[10] Deng, J. L. Introduction to grey system. *The Journal of Grey System (UK), 1(1),* (1989), 1–24.
[11] Hong, D. H. A note on correlation of interval-valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems, 95,* (1998), 113–117.
[12] Hong, D. H., & Choi, C. H. Multi criteria fuzzy decision-making problems based on vague set theory. *Fuzzy Sets Systems, 114,* (2000), 103–113.
[13] Kim, S. H., & Ahn, B. S. Interactive group decision making procedure under incomplete information. *European Journal of Operational Research, 116,* (1999), 498–507.
[14] Kim, S. H., Choi, S. H., & Kim, J. K. An interactive procedure for multiple attribute group decision making with incomplete information: Range based approach. *European Journal of Operational Research, 118,* (1999), 139–152.
[15] Lin, L., Yuan, X. H., & Xia, Z. Q. Multi criteria fuzzy decision-making methods based on intuitionistic fuzzy sets. *Journal of Computer and System Sciences, 73,* (2007), 84–88.
[16] Liu, H. W. Multi-criteria decision-making methods based on intuitionistic fuzzy sets. *European Journal of Operational Research, 179(1),* (2007), 220–233.
[17] Li, D. F., Wang, Y. C., Liu, S., & Shan, F. Fractional programming methodology for multi-attribute group decision making using IFS. *Applied Soft Computing Journal, 9,* (2009), 219–225.
[18] Li, G. D., Yamaguchi, D., & Nagai, M. A grey-based decision-making approach to the supplier selection

problem. *Mathematical and Computer Modeling, 46(3–4),* (2007), 573–581.

[19] G.W. Wei, A method for multiple attribute group decision making based on the ET-WG and ET- OWG operators with 2-tuple linguistic information, *Expert Systems with Applications, vol. 37, no. 12, pp*. (2010), 7895-7900.

[20] Gui-Wu-Wei, Gray relational analysis method for intuitionistic fuzzy multiple attribute decision making. *Expert Systems with Applications 38* (2011) 11671-11677.

[21] Barron.H, Schmidt .C. P, Sensitivity Analysis of additive multi attributes values models. *Operations Research, 46,* (2002), 122-127.

[22] Eshlaghy.A, Paydar. N. R, Joda. K, Paydar. N.R, Sensitivity analysis for criteria values in decision making matrix of SAW method. *International Journal of Industrial Mathematics, 1,* (2009), 69-75.x