

# Effect of Particle Size on Convective Heat Transfer in Flat Plate Through Nano Fluid

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**Abstract**— The significant research has seen with nano fluid flow and heat transfer. A theoretical investigation has attempted in this paper to study the chemical reaction effects. We have studied convective heat transfer of MHD nano fluid flow past inclined, oscillating permeable flat plate with radiation, heat source and suction. It is found that velocity and Diffusion increases for both kind of chemical reactions..

**Key words:** Nano - fluid, MHD, Inclined plate, Radiation, Method of lines.

## I. INTRODUCTION

The study of nano fluid attracted many researches since last few decades because of its vital applications. The exponential growth of the study of nano fluids made the latest technology more convenient and user friendly.

As the nano fluids carries the metallic particle the heat transfer and diffusion together is an interesting phenomenon with wide application in the field of biomedical, drug delivery etc. Sarit Kumar Das et.al (6) studied that many researches could not explain the study of diffusivity. M.I Anwar et.al (1) studied the heat and mass transfer of nano fluids over a non-linear stretching sheet for low constriction. Hyun UK Kang (7) has studied the effect of Brownian motion and inertial migration of particles and construed that the nano particle migration to the center occurs slowly. Hence the thermal conductivity at the walls increases. Many authors Buongiorno J (2), M. A. A. Hamad, I. Pop (3), Khan, W.A., A. Aziz (4), Kuznetsov AV, and Nield DA (5) studied the nano fluid flow and heat transfer effects in various geometries and various effects. All the Researches used the Maxwell or Hamilton & crosser model to describe the thermal conductivity of the nano fluid. In these models there is no description about the particle diameter or layer around the particle.

Many of the researches carried out so far ignored the natural phenomenon of formation of liquid like layer around nano particle because of the chemical reaction with the solvent. The formation of liquid like layer limits the contact of the metallic nano particle with the boundary. This reduces the effective heat transfer.

Keeping in view all the above fact we want to study the heat transfer through copper and water nano fluid with chemical reaction over a permeable, inclined, and oscillating flat plate under suction or injection.

## II. MATHEMATICAL FORMULATIONS

Consider the unsteady three dimensional free convection flow of a nano-fluid past a vertical permeable semi-infinite plate in the presence of an applied magnetic field with constant heat source and radiation. We consider a Cartesian

coordinate system  $(\bar{x}, \bar{y}, \bar{z})$ . The flow is assumed to be in the  $\bar{x}$  direction, which is taken along the plate, and  $\bar{z}$  - axis is normal to the plate. We assume that the plate has an oscillatory movement on time  $\bar{t}$  and frequency  $\bar{n}$  with the velocity  $u(0,t)$ , which is given  $u(0,t) = U_0 (1 + \varepsilon \cos(\bar{n}t))$ , where  $\varepsilon$  is a small constant parameter ( $\varepsilon \ll 1$ ) and  $U_0$  is the characteristic velocity. We consider that initially ( $t < 0$ ) the fluid as well as the plate is at rest. A uniform external magnetic field  $B_0$  is taken to be acting along the  $\bar{z}$ -axis. We consider the case of a short circuit problem in which the applied electric field  $E = 0$ , and also assume that the induced magnetic field is small compared to the external magnetic field  $B_0$ . The surface temperature is assumed to have the constant value  $T_w$  while the ambient temperature has the constant value  $T_\infty$ , where  $T_w > T_\infty$ . The conservation equation of current density  $\nabla \cdot J = 0$  gives  $J_z = \text{constant}$ . Since the plate is electrically non-conducting, this constant is zero. It is assumed that the plate is infinite in extent and hence all physical quantities do not depend on  $\bar{x}$  and  $\bar{y}$  but depend only on  $\bar{z}$  and  $\bar{t}$ ,

$$\text{i.e. } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

It is further assumed that the regular fluid and the suspended nano-particles are in thermal equilibrium and no slip occurs between them. Under Bossinesq and boundary layer approximations, the boundary layer equations governing the flow and temperature are,

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + w \frac{\partial u}{\partial z} = \frac{1}{\rho_{nf}} \left[ \mu_{nf} \frac{\partial^2 u}{\partial z^2} + (\rho \beta_T)_{nf} g (T - T_\infty) \cos \gamma \right] \tag{2}$$

$$\frac{\partial T}{\partial t} + w \frac{\partial T}{\partial z} = \alpha_{nf} \frac{\partial^2 T}{\partial z^2} - \frac{Q}{(\rho c_p)_{nf}} (T - T_\infty) - \frac{1}{(\rho c_p)_{nf}} \frac{\partial q_r}{\partial z} \tag{3}$$

$$\frac{\partial c}{\partial t} + w \frac{\partial c}{\partial z} = D_{nf} \frac{\partial^2 c}{\partial z^2} + k_l (c - c_\infty) \tag{4}$$

The appropriate initial and boundary conditions for the problem are given by

$$\begin{aligned} u(z,t) = 0, T = T_\infty, c = c_\infty \text{ for } t < 0 \forall z \\ u(0,t) = U_0 \left[ 1 + \frac{\varepsilon}{2} (e^{i\bar{n}t} + e^{-i\bar{n}t}) \right], T(0,t) = T_w, c(0,t) = c_w, \\ u(\infty,t) \rightarrow 0, T(\infty,t) \rightarrow T_\infty, c(\infty,t) \rightarrow c_\infty, \varepsilon \ll 1 \text{ for } t \geq 0 \end{aligned} \tag{5}$$

Thermo-Physical properties are related as follows:

$$\rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$(\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s$$

$$(\rho\beta)_{nf} = (1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s$$

$$\frac{\mu_{nf}}{\mu_f} = 1 + 2.5\phi + 4.5 \left( \frac{1}{\frac{h}{d_p} \left( 2 + \frac{h}{d_p} \right) \left( 1 + \frac{h}{d_p} \right)^2} \right)$$

$$k_{nf} = k_f(1-\phi) + \beta_1 k_p \phi + c_1 \frac{d_f}{d_p} k_p \text{Re}^2 d_p pr \phi \quad (6)$$

Where  $\beta_1 = 0.01$  is a constant for considering the Kapitza resistance per unit area.

$c_1 = 18 \times 10^6$  is proportionality constant

$$\text{Re } d_p = \frac{d_p}{\gamma_f} \frac{\kappa T}{3\pi \mu d_p l_f} = \frac{1.381 \times 10^{23} T}{\gamma_f 3\pi \mu_f (0.738)}$$

$$d_f = 0.384 \text{ nm for water } \quad p_r = \text{prandtl number} = \frac{\gamma_f}{\alpha_f}$$

$l_f = \text{mean free path} = 0.738$

$k = \text{Boltzmann constant}$

$T = 300 \text{ K}$

The Thermo-physical properties (values) of the materials used are as follows:

| Physical Properties            | Water                 | Copper(Cu) |
|--------------------------------|-----------------------|------------|
| $C_p$ (J/kg K)                 | 4,179                 | 385        |
| $\rho$ (kg/m <sup>3</sup> )    | 997.1                 | 8,933      |
| $\kappa$ (W/m K)               | 0.613                 | 400        |
| $\beta_T \times 10^{-5}$ (1/K) | 21                    | 1.67       |
| $\mu$                          | $8.94 \times 10^{-4}$ | -----      |

Table 1: The Thermo-physical prop

We consider the solution of Eq (1) as  $w = -w_0$  (7)

Where the constant  $w_0$  represents the normal velocity at the plate which is positive for suction ( $w_0 > 0$ ) and negative for blowing or injection ( $w_0 < 0$ ). Thus, we introduce the following dimensionless variables:

$$z = \left( \frac{\psi_f}{U_0} \right) Z, \quad t = \left( \frac{\psi_f}{U_0^2} \right) t^*, \quad n = \left( \frac{U_0^2}{\psi_f} \right) \eta,$$

$$u = UU_0, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$q_r = -\frac{4\sigma_1}{3\delta} \frac{\partial T^4}{\partial y} \quad (8)$$

We assume that the temperature differences within the flow are sufficiently small so that  $T^4$  can be expressed as a linear function after using Taylor series to expand  $T^4$

about the free stream temperature  $T_\infty$  and neglecting higher-order terms. This result is the following approximation:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4$$

By using above, we obtain

$$\frac{\partial q_r}{\partial z} = -\frac{16\sigma_1}{3\delta} \frac{\partial^2 T^4 T_\infty^3}{\partial z^2} \quad (9)$$

Using equations 5, 6,7,8,9 the Equations 2, 3 & 4 can be written in the following dimensionless form:

$$\begin{aligned} & \left[ 1 - \phi + \phi \left( \frac{\rho_s}{\rho_f} \right) \right] \left( \frac{\partial U}{\partial \tau} - S \frac{\partial U}{\partial Z} \right) \\ & = 1 + 2.5\phi + 4.5 \left( \frac{1}{\left( \frac{h}{d_p} \right) \left( 2 + \frac{h}{d_p} \right) \left( 1 + \frac{h}{d_p} \right)^2} \right) \frac{\partial^2 U}{\partial Z^2} \\ & \quad + \left[ 1 - \phi + \phi \left( \frac{\rho \beta_T}{\rho \beta_T} \right)_s \right] \theta \text{Cos } \gamma \\ & \left[ 1 - \phi + \phi \left( \frac{\rho c_p}{\rho c_p} \right)_s \right] \left( \frac{\partial \theta}{\partial \tau} - S \frac{\partial \theta}{\partial Z} \right) \\ & = \frac{1}{p_r} \left( 1 - \phi + 0.01 \phi \frac{k_p}{k_f} + \frac{k_p}{k_f^2} \phi \frac{\rho_f^2 c_{pf}}{d_p \mu_f^3} 28632.9991 \times 10^{-52} \right) \frac{\partial^2 \theta}{\partial Z^2} \\ & \quad - \frac{1}{p_r} Q_H \theta + \frac{1}{p_r} \frac{4}{3} \frac{1}{R_a} \frac{\partial^2 \theta}{\partial Z^2} \\ & \left( \frac{\partial C}{\partial \tau} - S \frac{\partial C}{\partial Z} \right) = \frac{1}{Sc} \frac{\partial^2 C}{\partial Z^2} + K C \end{aligned}$$

Where the corresponding boundary conditions (5) can be written in the dimensionless form as:

$$\begin{aligned} & U(z,t) = 0, \theta(z,t) = 0, C(z,t) = 0 \text{ for } t < 0 \quad \forall z \\ & U(0,t) = U_0 \left[ 1 + \frac{\varepsilon}{2} (e^{int} + e^{-int}) \right], \theta(0,t) = 1, C(0,t) = 1 \quad \forall t \geq 0 \\ & U(\infty,t) \rightarrow 0, \theta(\infty,t) \rightarrow 0, C(\infty,t) \rightarrow 0 \end{aligned}$$

Here  $p_r$  is the Prandtl number,  $S$  is the suction ( $S > 0$ ) or injection ( $S < 0$ ) parameter,  $R_a$  is the Radiation parameter and  $Q_H$  is the heat source parameter,  $Sc$  is the Schmidt number,  $K$  is the chemical Reaction parameter, which are defined as:

$$\begin{aligned} p_r &= \frac{\mu_f}{\alpha_f}, \quad S = \frac{w_0}{U_0}, \quad R_a = \frac{4\alpha\sigma_1 T_\infty^3}{\delta k_{nf}}, \\ Q_H &= \frac{Q \psi_f^2}{k_f U_0^2}, \quad Sc = \frac{\psi_f}{D_{nf}}, \quad K = \frac{k_1 \psi_f}{U_0^2} \end{aligned}$$

Where the velocity characteristic  $U_0$  is defined as:

$$U_0 = \left[ g\beta_f (T_w - T_\infty) \psi_f \right]^{1/3}$$

The local Nusselt number  $Nu$  in dimension less form:

$$Nu = -\frac{k_{nf}}{k_f} \theta'(0)$$

III. RESULTS AND DISCUSSIONS

The governing equations are solved by using Method of lines with the help of Mathematica package and Z is limited to 6. The variations of velocity U, temperature  $\theta$  and diffusion C are graphically exhibited. By keeping  $Pr = 6.2$ ,  $nt = \pi/2$  and  $\epsilon = 0.02$ . The effects of various parameters are as follows.

The velocity increases with increase in volume fraction ( $\phi$ ) of the nano-fluid. It indicates the nano particle density dominates the motion of the solute more. It is exhibited in Fig. 1. The variation of U with liquid like layer thickness (h) is exhibited in Fig.2. The flow is resisted with increase in h. As the thickness increases the flow of the nano-fluid is very much decreasing. From Fig. 3. It is observed that the flow increases with particle size ( $d_p$ ). The friction among the liquid and the metal particles enhance the velocity. From Fig. 4. The velocity decreases with increase in the inclination angle ( $\gamma$ ) of the plate. From Fig. 5. The velocity decreases with increase in the heat source ( $Q_H$ ). There is no significant influence on the flow field by the moderate heat source. From Fig. 6. It observed that the radiation parameter ( $R_a$ ) decreases the velocity. From Fig. 7. The increase in kinematic viscosity/decrease in diffusivity ( $Sc$ ) decreases the velocity. From Fig. 8. The velocity increases from generative ( $K < 0$ ) to destructive ( $K > 0$ ) chemical reactions and found moderate for no reaction.

From Fig. 9. It is observed that the increase in solid particles ( $\phi$ ) increases the temperature. Fig. 10 exhibits the variation of temperature with size of the solid particle ( $d_p$ ). The temperature increases with increase in size of the particle. Fig. 11 is temperature profile varying with heat source parameter ( $Q_H$ ). It found that the temperature decreases with increase in heat source. The increase in radiation parameter decreases temperature, it shown in Fig. 12.

The diffusivity decreases with increase in kinematic viscosity from Fig. 13. The diffusivity increases with chemical reaction changes from generative to destructive from Fig.14

A. Nusselt Number:

The variation of rate of heat transfer coefficient for various volume fractions, size of the particle and heat source is displayed in Table – 2. The rate of heat transfer increases with increase in heat source for all volume fractions of the solid particles. But the rate of heat transfer is almost constant for low volume fractions for various sizes of the particle where as a significant enhancement of rate of heat transfer is observed for increase in size of the particle for higher volume fractions.

|        |                         |                         |                          |                         |                          |
|--------|-------------------------|-------------------------|--------------------------|-------------------------|--------------------------|
| $\phi$ | $Q_H = 5$<br>$d_p = 20$ | $Q_H = 5$<br>$d_p = 40$ | $Q_H = 5$<br>$d_p = 100$ | $Q_H = 1$<br>$d_p = 20$ | $Q_H = 10$<br>$d_p = 20$ |
|--------|-------------------------|-------------------------|--------------------------|-------------------------|--------------------------|

|          |             |             |             |             |             |
|----------|-------------|-------------|-------------|-------------|-------------|
| 0.0<br>5 | 1.999<br>86 | 1.99986     | 1.99984     | 1.56649     | 2.456<br>31 |
| 0.1<br>5 | 2.643<br>86 | 2.644<br>38 | 2.644<br>39 | 2.051<br>06 | 3.266<br>4  |

Table 2: Nusselt Number

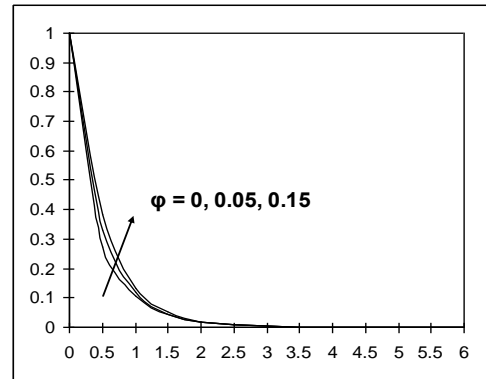


Fig.1: Variation of U with  $\phi$

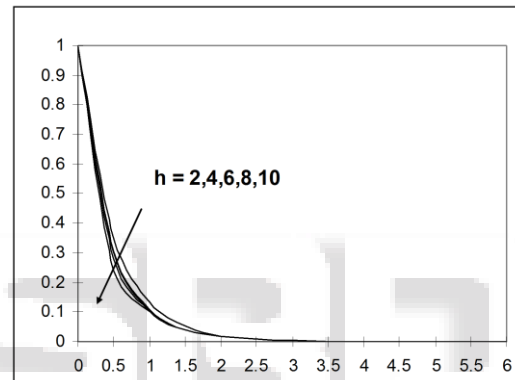


Fig.2: Variation of U with h

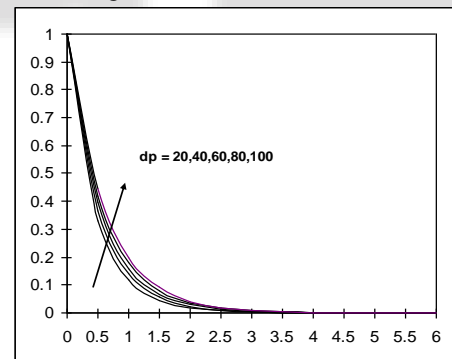


Fig.3: Variation of U with  $d_p$

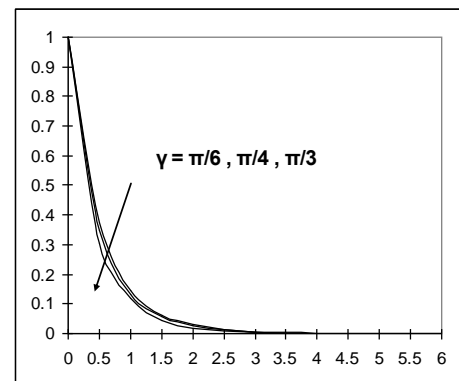


Fig.4: Variation of U with  $\gamma$

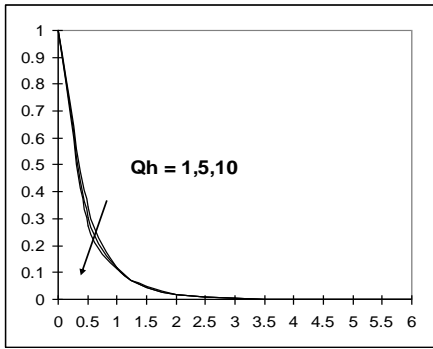


Fig. 5: Variation of U with  $Q_H$

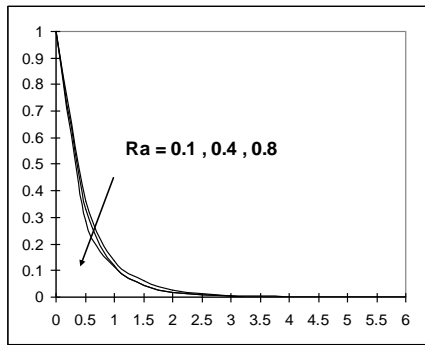


Fig. 6: Variation of U with Ra

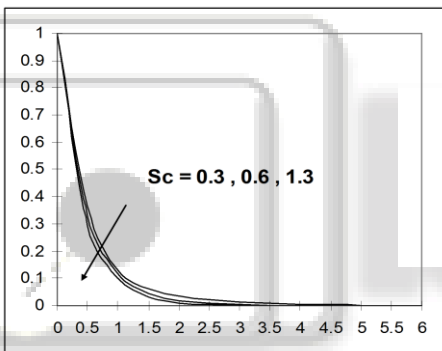


Fig. 7: Variation of U with Sc

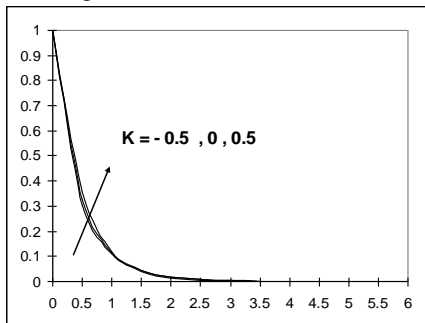


Fig. 8: Variation of U with K

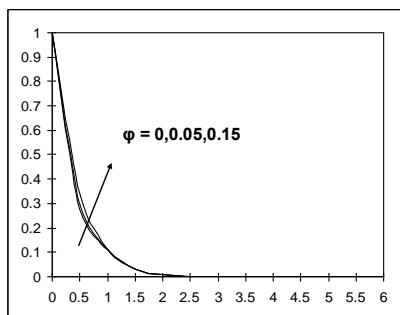


Fig. 9: Variation of  $\theta$  with  $\phi$

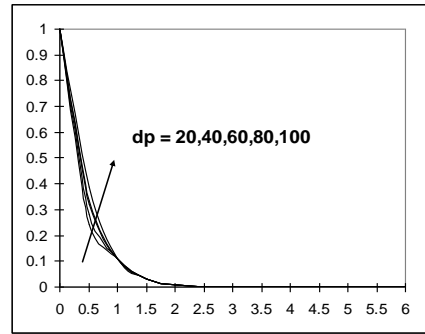


Fig. 10: Variation of  $\theta$  with  $d_p$

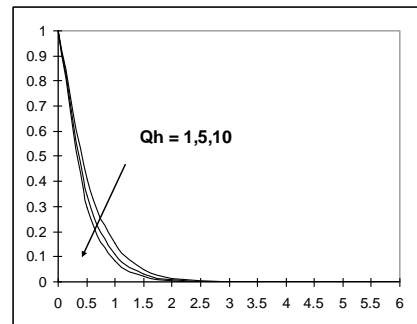


Fig. 11: Variation of  $\theta$  with  $Q_H$

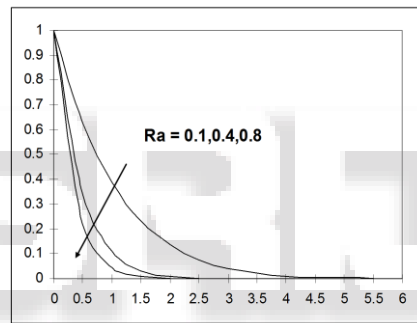


Fig. 12: Variation of  $\theta$  with  $R_a$

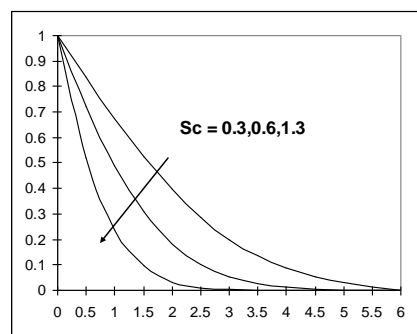


Fig. 13: Variation of C with Sc

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