Use of PMU and Observability Analysis with Power System State Estimation

Meera .R[1] Dr. R. Jegatheesan[2]
[1, 2]Electrical and Electronics Engineering Department
[1, 2]SRM University Chennai, India

Abstract---Phasor measurement units are one of the most important measuring devices in the future of power system monitoring, protection and control applications. This paper presents, the advantages of using PMU in power system state estimation calculations. By considering optimal placement and observability ,an alternate state estimation is formed using WLS algorithm. Also this paper presents the observability analysis using nodal variable formation.
Keywords: state estimation, PMUs, optimal placement, obsrvability analysis

I. INTRODUCTION

State estimation gives the capabilities of the SCADA system computers, resulting in establishment of the Energy Management Systems (EMS), which would now be equipped with, among other application functions. For finding the current operating state of the system, state estimators facilitate accurate and efficient monitoring of operational constraints on quantities such as the transmission line loadings or bus voltage magnitudes. They provide a reliable real-time data base of the system.Measurement set for state estimation includes power flows, injections, voltage and current magnitudes[2]. All measurements should want to be synchronized and it is difficult ,if not possible to assure that all measurements are synchronized.

For increasing the scope of SEs, in regional electricity markets, in which long-distance transactions are to be perfectly monitored. In that, collection of measurements
and synchronizing it will be a challenging one[3].

One solution of that problem is phasor measurement unit (PMU). PMU will accurately determine the oscillating voltage and current measurements and also it determines the phasor with respect to a GPS clock[1].

Phasor measurement units are devices that provide synchronized measurements of real-time phasor of voltages and currents. Synchronization is achieved by same-time sampling of voltage and current waveforms using timing signals from the Global Positioning System Satellite (GPS). Synchronized phasor measurements elevate the standards of power system monitoring, control, and protection to a new level.

Use of PMU influences state estimation in many different ways. Adding number of PMUs to an system is not sufficient for state estimation and it cause non linear and iterative solutions. State estimation issues like network observability, optimal placement etc will have to be considered and an alternate state estimation using PMU is proposed in this paper.

II. STATE ESTIMATION

The idea of state estimation in power systems was first recognized and subsequently addressed by Fred Schweppe . Inorder to identify the current operating state of the system, state estimators gives accurate and efficient monitoring of operational constraints on quantities such as the transmission line loadings or bus voltage magnitudes. They provide a reliable real-time data base of the system[2].

The main objective of the power systems state estimator is to find a robust estimate for the unknown complex voltage at every bus. Since measurements from a SCADA system are used to calculate the complex voltages, the estimate will also be inexact. This introduces the problem of how to obtain a best estimate for the voltages given the available measurements. Two most common estimators are (i) Maximum Likelihood (ii) Weighted least square(WLS).
Some equations are

$$z = h(x) + e \qquad (2.1)$$

Where
z is the Measurement vecto
x is the State variable
h is the vector of functions,usually non linear
e is the measurement error.
WLS State Estimation involves the iterative solution of the Normal equations given by (2.2)

$$G(x)\Delta(= H^T(x)R^{-1}[z - h(x)] \qquad (2.2)$$

Where
$H(x) = \frac{\partial h}{\partial x}$ is the jacobian matrix
$G(x) = H^T(x)WH(x)$ is the gain matrix
$R^{-1} = W$ is the weight matrix
Δx is the change in state variable
The WLS estimator will minimize the following objective function

$$J = \sum_{i=1}^{m}(z - h(x))^2/R^{ii} = (z - h(x))^T R^{-1}(z - h(x)) \qquad (2.3)$$

iterations ends when Δx comes to an appropriate tolerance value.

III. PHASOR MEASUREMENT UNIT (PMU)

Phasor measurement units are devices which measures frequency, magnitude and phase angle of voltage and current with respect to a time reference. PMU is a dedicated device, its function can be used in protective relays or other devices.

Power system security analysis is based on the output of the state estimator, which traditionally process measurements provided by Supervisory Control and Data

Acquisition (SCADA) systems. These measurements are commonly provided by the Remote Terminal Units (RTU) at the substations and include real/reactive power flows, real/reactive power injections, magnitudes of bus voltages and magnitudes of branch currents.

RTUs do not record voltage phase angles. It is importance to measure voltage phase angles because the difference in phase angle between a sink and source will determine power flows, resulting current and stability limits.

Measurements are collected from different locations in the power system over a few seconds causing a lack of measurement synchronicity with respect to time. Synchronized phasor measurement is a key technology that can enhance the measurement of critical aspects of transmission grid, directly. This will pave way for improved real time monitoring, real time control and real time state estimation. PMUs are devices that provide synchronized measurements of real-time voltage phasors and current phasors. Synchronization is achieved using timing signal obtained from the Global Positioning Satellite (GPS) ensuring synchronicity among PMUs.

PMU was invented by Dr. Arun G. Phadke and Dr. James S. Thorp at Virginia Tech in 1988[4,5]. Steinmetz's technique of phasor calculation evolved into the calculation of real time phasor measurements that are synchronized to an absolute time reference provided by the Global Positioning System. Early types of the PMU were built at Virginia Tech, and Macrodyne built the first PMU in 1992.

PMU gives multi channel input,so that volatage and current in more than one line can be processed by a single unit. it provides phasor information in real time.Advantage of connecting phase angle to a global time reference is that,it gives a wide screen snot of the power system.

A. PMU representation

Consider a sinusoidal signal

$$y(t)=Y_m\cos(wt+\varphi) \qquad (3.1)$$

Phasor representation is

$$y(t)=\frac{Y_m}{\sqrt{2}}e^{j\varphi}=\frac{Y_m}{\sqrt{2}}(\cos\varphi+j\sin\varphi s) \qquad (3.2)$$

Sinusoidal signal and its phasor are represented in fig

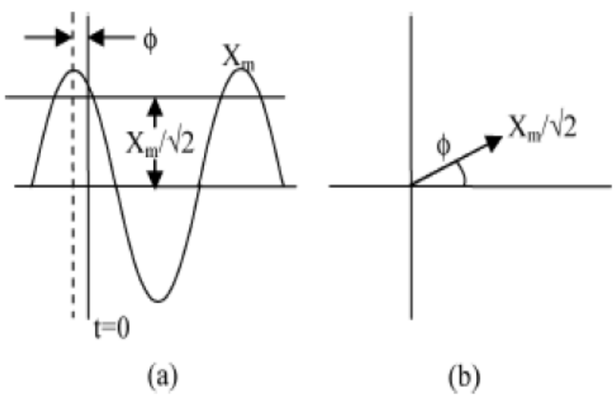

Fig. 1: Sinusoidal signal and its phasor

IV. OPTIMAL PLACEMENT OF PMU

PMUs are very expensive and it is not economical to place a PMU at every bus in the system. PMUs should be minimally placed at the buses that provide maximum observability for the system. An optimization problem is formulated that minimizes the PMU installation cost and maximizes system observability. A full-fledged optimal PMU placement algorithm should minimize total cost of PMUs and maximize observability taking into account contingencies, adaptability and user friendly[6,7].

A. Formulation for optimal PMU placement

Unlike traditional measurement units, the PMU is able to measure the voltage phasor of the installed bus and the current phasors of all the lines connected to that bus. As a consequence of Ohm's law, when a PMU is placed at a bus,all the neighboring buses also become observable as shown in

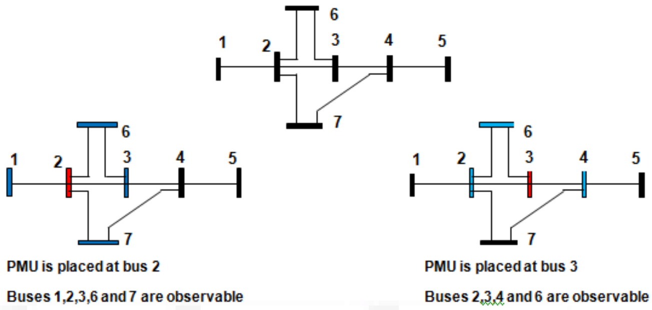

Fig. 2: PMU Placement

That is a PMU can make the installed bus and its neighboring buses observable.

Therefore, optimal placement of PMUs becomes a problem that finds a minimal set of PMUs such that a bus must be reached at least once by the set of PMUs. With this idea, a matrix T_{PMU} is defined[9]. The elements of T_{PMU} are defined as follows:

$$t_{ij}=\begin{cases}1, & \text{if } i=j\\ 1, & \text{if buses } i \text{ and } j \text{ are connected}\\ 0, & \text{otherwise}\end{cases} \qquad (4.1)$$

For a network with N buses, matrix T_{PMU} will a N x N matrix.

Let X be a binary decision variable vector whose entries are defined as:

$$x_i=\begin{cases}1 & \text{if a PMU is installed at bus i}\\ 0 & \text{otherwise}\end{cases} \qquad (4.2)$$

We are looking for the minimum number of PMUs. Knowing that $\sum_{K=1}^{N} x_k$ is the total number of PMUs, the objective function is Min $\sum_{K=1}^{N} x_k$ where N is the number of buses. With these preliminaries optimal PMU placement problem can be defined for different cases as discussed below.

B. Without Conventional Measurements

The problem of optimal placement of PMUs is formulated as follows:

$$\text{Min} \sum_{K=1}^{N} x_k$$

$$\text{subject to} \quad T_{PMU}\; X \;\geq\; b_{PMU} \qquad (4.3)$$

where $X = [x_1\ x_2\ \ldots\ldots\ x_N]^T$

$x_i \in \{0, 1\}$

and b_{PMU} is a N x 1 column vector given by

$b_{PMU} = [1\ \ 1\ \ \ldots\ldots\ 1]^T$ and N is the number of buses in the network.

C. With Conventional Measurements

As PMUs are very expensive, installation of PMUs shall be carried out in a phased manner. This means that along with certain existing conventional measurements lesser number of PMUs can be added in the existing power system network to ensure observability. Necessary mathematical model can be developed as discussed below.

Let us define a column vector Y as

$$Y = T_{PMU}\, X \qquad (4.4)$$

Element $y_i = T_{PMU\,i}\ X$ where $T_{PMU\,i}$ is the i^{th} row of matrix T_{PMU}. It is to be noted that y_i is the observability constraint of bus *i*. For detail discussion, the following three cases need to be analyzed.

Case 1

If a power flow measurement is on line i - j, one bus voltage can be computed if the other bus is covered by PMU. Knowing that y_i and y_j are the observability constraints for bus *i* and *j* the following inequality needs to be held:

$$y_i + y_j \;\geq\; 1 \qquad (4.5)$$

Case 2

Suppose that a power injection measurement is at bus *k*. Let us assume that bus *k* connects buses *l*, *p* and *q*. Then by knowing the phasor voltages at any three out of the four buses *k*, *l*, *p* and *q*, the voltage at the fourth bus can be calculated using Kirchhoff's Current Law applied at bus *k*. Thus the following inequality needs to be held:

$$y_k + y_l + y_p + y_q \;\geq\; 3 \qquad (4.6)$$

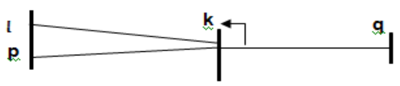

Fig. 3: Power injection at bus k

Case 3

Power flow measurement and power injection measurement may be available simultaneously. If they are NOT associated

inequalities for this case are:

$$y_p + y_k \;\geq\; 1 \qquad (4.7)$$

$$y_l + y_q \;\geq\; 1 \qquad (4.8)$$

If bus i is NOT ASSOCIATED with any conventional measurements, then the corresponding constraint of the minimization problem is still kept as $y_i \geq 1$.

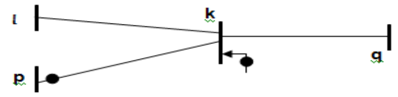

Fig. 4: Power Flow and Power Injection simultaneously

When considering the conventional measurements, optimal placement of PMU can be formulated as a problem of Integer Linear Programming as follows:

$$\text{Min} \sum_{K=1}^{N} x_k$$

$$\text{subject to} \quad T_{con}\, P\, T_{PMU}\; X \;\geq\; b_{con} \qquad (4.9)$$

Where the matrix

$$T_{con} = \begin{bmatrix} I_{M\times M} & 0 \\ 0 & T_{meas} \end{bmatrix} \qquad (4.10)$$

V. STATE ESTIMATION USING PMU

If PMU is added to a certain bus,that bus and all other bus connected to that bus becomes observable. For example if we connect a PMU to bus number 9 of a IEEE 14 bus system,then 4^{th} ,7^{th},10^{th} and 14^{th} buses which are connected to 9^{th} bus become observable. So In state estimation process, number of state variable and number of measurements get reduces. Because of these reasons state estimation calculation becomes easy and jacobian matrix also get reduces.

New jacobian matrix is

$$H_{new} = H_{old}P \qquad (5.1)$$

Where P is the Permutation matrix.

A. Permutation Matrix

It has only one nonzero element of value 1 in each row and in each column. When the matrix P is pre-multiplied with Hnew, the rows of Hnew are arranged in the order as required.

$$\text{Consider a matrix } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad (5.2)$$

If row 2 is to be deleted from A and other two rows are be arranged in the order 3 and 1,

then

$$\text{then } P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad (5.3)$$

$$\text{Thus } PA = \begin{bmatrix} a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \end{bmatrix} \qquad (5.4)$$

VI. NETWORK OBSERVABILITY

If the given set of measurements is sufficient to make state estimation possible, we say the network is observable. Observability depends on the available measurements and their geographical distribution.

If the system is not observable, it will have two or more isolated observable islands, each one having its own phase angle reference that is independent of the rest. Network observability analysis allows detection of such cases and identifies all the observable islands prior to the execution of the state estimator. Observability analysis can be carried out by the help of either topological or numerical approaches.Here we are taking numerical method[2].

The numerical approach is based on the measurement Jacobian and the associated gain matrix. If the gain matrix is non-singular, the system is observable. On the other hand, if the gain matrix is found singular, then all observable islands need to be identified and injection of pseudo-measurements will have to be introduced in order to merge these islands into a single observable island for the entire system[8].

A. *Numerical method based on the nodal variable formulation*

Numerical Observability analysis can be carried out by using the nodal variables. Nodal variable vector is denoted by x and represents the vector of magnitudes and phase angle of all bus voltages in the system. In this method We call a network observable whenever all measurements are equal to zero implies that all flows are zero

For Observable

$$Z_A = H_{AA} * \partial = 0 \tag{6.1}$$

$$P_b = A * \partial = 0 \tag{6.2}$$

Where

$$H_{AA} = \frac{\partial h(x)}{\partial x} \tag{6.3}$$

p_b is the vector of branch flows
A is the branch-bus incidence matrix
δ is the vector of bus voltage phase angles
and for unobservable system

$$Z_A = H_{AA} * \partial = 0 \tag{6.4}$$

$$P_b = A * \partial \neq 0 \tag{6.5}$$

B. *Measurement Placement to Restore Observability*

Once the observable islands are identified, measurements can be added to merge these islands so that eventually a single observable island can be formed. The candidate measurements that can merge islands are the line flows along branches that connect observable islands, and the injection at the boundary buses of observable islands.

Consider the gain matrix G_{AA} ,

$$(H_{AA}^T \; H_{AA}) \; \delta^{\wedge} = H_{AA}^T \; z_A = t_A = 0 \tag{6.6}$$

Calculate the cholesky factors of gain matrix and

$$D= L^{-1} \, G \, (L^{-1})^T \tag{6.7}$$

if it has only one zero pivot then the system is observable. otherwise we want to calculate W from the rows of L^{-1} from the rows of matrix corresponding to zero pivot positions.

Candidate measurements are the measurements such as flows and injections connected to the observable island. Build jacobian matrix from the candidate measurements, Hc and compute B matrix

$$B = H_c \, W^T \tag{6.8}$$

Obtain the row echelon form of B, E. The linearly independent rows of E will correspond to all the measurements required to be placed.

VII. RESULTS AND DISCUSSION

The method of weighted least square is tested on IEEE 14 bus system without PMU. Secondly a PMU is placed in the power system at 9th bus , 4th bus and both in 9th and 4th bus.

Table below shows the results of state estimation without and with PMU at 9th bus. Here PMU is connected at bus 9 ,so 4th,7th,9th,10th,14th buses become observable.

	Without PMU		With pmu at 9th bus	
Bus No	Voltage Magnitude	Voltage Angle	Voltage Magnitude	Voltage Phase Angle
1	1.0606	0	1.0613	0
2	1.0453	-4.8637	1.0473	-4.922
3	1.0102	-12.4135	1.0113	-12.622
4	1.0145	-9.9901	1.0155	-9.990
5	1.0179	-8.5149	1.0267	-8.686
6	1.0706	-14.0943	1.0756	-14.919
7	1.0467	-12.9054	1.0505	-12.905
8	1.0806	-12.9053	1.0916	-12.905
9	1.0316	-14.4485	1.0559	-14.499
10	1.0310	-14.6647	1.0510	-14.664
11	1.0470	-14.9937	1.0557	-15.198
12	1.0540	-14.9260	1.0579	-15.655
13	1.0474	-14.9589	1.0526	-15.719
14	1.0204	-15.6729	1.0355	-15.672

Table. 1: Results of IEEE 14 Bus System

Again PMU is connected to bus number 4 and both to 9 and 4 ,and result is obtained.

Observability analysis is carried out in 6 bus and 14 bus systems

VIII. CONCLUSION

Development of phasor measurement unit has paid the way to monitor the system accurately with help of GPS technology. Since phasor measurement units are expensive optimal placement algorithms have been developed to install PMUs in power system. Conventional state estimation algorithm estimates the state of power system which is no longer if the power system is equipped with PMU.

In this project a new state estimator is developed based on the conventional power system state estimation model,the model of state estimation containing PMU is developed. +The state estimation is performed in IEEE 14 bus system with PMU is placed at bus 9 and 4.Observability analysis using nodal variable formation is also performed in 6 bus amd 14 bus systems

REFERENCES

[1] Ali abur, Antonio G'omez - Exp'osito, Patrica Rousseax, Antonio de la Villa Jaen and Catalina G'omez-Quiles,"On the use of PMU in power system state estimation"17th Power Systems Computation Conference, Stockholm Sweden - August 22-26, 2011

[2] A. Abur, A. G´omez-Exp´osito, Power System State Estimation: Theory and Implementation, Marcel Dekker, April 2004
[3] A.G´omez-Exp´osito,A. de la Villa Ja´en, C. G´omez-Quiles, P. Rousseaux, T. Van Cutsem, “A Taxonomy of Multi-Area State Estimation Methods,”, Electric Power Systems Research, in press.
[4] A.G. Phadke, J. S. Thorp and K. J. Karimi, “State Estimation with Phasor Measurements”, IEEE Transactions on Power Systems, Vol. 1, No. 1, pp. 233- 241,February 1986.
[5] A.G. Phadke, “Synchronized phasor measurements in power systems”, IEEE Computer Applications in Power, Vol. 6, Issue 2, pp. 10-15, April 1993.
[6] Bei Xu and A. Abur, “Observability Analysis and Measurement Placement for System with PMUs”, IEEE PES Power Systems Conference & Exposition, New York, Oct.10-13, 2004.
[7] Bei Xu, Y. J. Yoon, and A. Abur, “Optimal Placement and Utilization of Phasor Measurements for State Estimation”, Power System Computation Conference, Liege, Belgium, Aug. 2005.
[8] S. Chakrabarti, E. Kyriakides, and D. G. Eliades, “Placement of synchronized measurements for power system observability”, IEEE Transactions on Power Delivery, vol. 24, no. 1, pp. 1219, Jan. 2009.
[9] B. Xu and A. Abur,”Observability analysis and measurement placement for the system with PMUs, ”in Proc.2004 IEEE Power Eng. Soc. Conf. Exposition, Oct.10-13,2004,vol.2,pp.943-946.