Numerical modeling of Degradation due to Sediment diminution

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Abstract— Alluvial channels attain equilibrium state over many years. Changes in the flow rates or in sediment load can disturb the equilibrium. These changes can prove detrimental to the proper functioning of the engineering works located in the river. Various researchers have developed few mathematical and numerical models based on various numerical and analytical schemes. In this paper MacCormack explicit finite difference scheme is used to develop numerical model for simulation of bed level of alluvial channels. The Saint-Venant equations are used for numerical solution of degradation rates due to sediment diminution. The main objective is to develop a numerical model for prediction of degradation rates. Also validation of the developed model will be done by the observed data of experiments.

Key words: Degradation, numerical modeling, MacCormack finite difference scheme, river bed variation.

I. INTRODUCTION

Many rivers are so delicate that, any artificial or natural change, of a permanent nature, in the flow of water and/or sediment may affect the entire system. The rate at which the river will adjust its regime to artificial river works or the natural causes may be very slow or rapid depending on its nature. Such adjustment of the river to the imposed changes may prove detrimental to the proper functioning of the engineering works located in the river.

Hydraulic engineers and geologists since the origin of their professions have been concerned with river dynamics. The engineer's main concern is knowledge of river response to natural and man-induced disturbances.

Mathematical modeling can be used for quantitative prediction of the nature of this adjustment.

An alluvial stream flowing under equilibrium condition is called a graded stream. No natural stream however is truly in equilibrium. The discharge of a natural stream varies continuously with time, with a ratio of high discharge to low discharge ranging from unity to several hundred or more. The temperature and the rate of sediment supply by tributaries and even the character of the sediment also vary. The result of continuous change is that the stream under in consideration be in true equilibrium.

A change in controlling factors, namely, slope, sediment size, water and sediment discharge, will necessitate changes in one or more factors to restore the equilibrium. Thus, if the sediment equilibrium can again be restored only if the water discharge and/or slope are increased sufficiently.

The changes in discharge, slope, sediment load and sediment size can be due to natural factors such as occurrence of landslides, addition of sediment load by tributaries in excess of what the stream can carry, etc. It can also be due to artificial means such as construction of dams, provision of silt exclusion works, withdrawals of clear water, etc.

These changes potentially destroy previously existing equilibrium states leading to river bed AGGRADATION or DEGRADATION.

II. DEGRADATION

A. *Introduction*

The stream bed is constantly getting scoured to reduce excess land slope, that process is known as Degradation.

When the delicate equilibrium of an alluvial river is disturbed and amount of sediment load entering in a reach of the river is less than the amount of sediment load going out of the same river reach, than Degradation occurs.

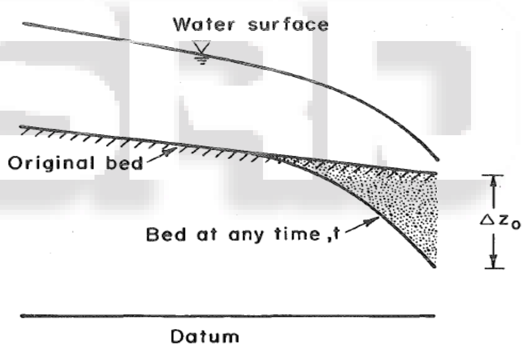

Fig. 1: Degradation Phenomenon

Channel degradation refers to the general lowering of the bed elevation that is due to erosion. The geometry of the cross section, flow depth, the bed slope, of the considered river reach as well as the size and composition of bed material and the dimensions of bed forms which determine the value of friction factor, affect significantly the nature and magnitude of the bed degradation.

Bed degradation occur in natural streams if the delicate balance among water discharge, sediment flow, and the channel shape is disturbed by natural or manmade factors, e.g.,

1) The construction of a dam
2) Change in the sediment supply rate
3) Base-level lowering

B. *Typical examples of Degradation due to sediment diminution*

Cases of degradation below dams, where most or all of the sediment discharge is retained in the reservoir, have been reported frequently in technical publications (Lane, Regression of Levels in River-Beds Below Dams, 1934), (Lane, Sediment Engineering as a Quantative Science, 1947), (Lane, 1955) (Todd, 1940), (Joglekar and Wadekar, 1951), (Hathaway, 1948). Degradation was the cause for the failure of a major dam in India and for the complete reconstruction of Fort Summer Dam on the Pecos River in New Mexico (Lane, 1955). A 2.5 m lowering in the bed elevation of the Missouri River in 10 years was observed downstream from Gavin Point Dam, South Dakota (Sayre and Kennedy, 1958). Bed degradation of up to 10 m over a period of 32 years occurred in the Ratmau Torrent downstream of a level-crossing with the Upper Ganga canal at Dhanauri, India (Vittal and Mittal, 1980). Bed material removal at a striking rate of 12000 cubic meters in one year caused by the increase in flow discharge was observed in Five Mile Creek, Wyoming. The material eroded from the stream bed and banks provoked the rapid silting of the downstream reservoir (Lane, 1955).

Fig. 2: Erosion to downstream bedrock due to sediment diminution due to dam construction

III. METHODOLOGY AND TOOLS

A. *Overview*

Many researchers have done work on predicting rates of aggradation-degradation with the use of various analytical and mathematical methods.

With the advancement in high speed electronic computers and numerical methods for solving partial differential equations, a substantial part of the research effort concerning non-equilibrium flows has been concentrated on numerical models.

(Tinney, 1962); presented an analysis of the process of degradation, which consisted in the combination of the sediment continuity equation, Duboys' sediment-transport equation, and Manning's resistance relation. Both equations were initially solved by the method of characteristics and later by means of a finite difference method that automatically takes care of the shock fitting. The method was initially applied to a bed-aggradation process induced by the withdrawal of clear water.

(Little, Mayer, 1970); performed a series of degradation experiments. They focused on the variation in sediment gradation of the bed surface during the armoring process.

(Newton, 1951): obtained laboratory data for degradation due to sediment diminution.

(Soni et al., Garde, Rangaraju, 1980)Studied aggradation due to sediment overloading.

(Begin et al., 1981)Experimentally studied degradation of alluvial channels in response to lowering of the base level.

(Suryanarayana, Shen, 1969)Obtained laboratory data for degradation in alluvial channels downstream of a dam.

(Bhamidipaty, Shen, 1971) Concluded from analysis of Newton's data and their own experimental data that the bed elevation in a degrading channel decreases exponentially with time.

(Lu and Shen, 1986)Tested several numerical aggradation and degradation models by taking data of (Suryanarayana, Shen, 1969)

(Singh et al., 2004)Developed a fully coupled model using a Pressman implicit finite difference scheme. They tested their model for aggradation due to sediment overloading by using data of Soni (1975) and Mehta (1984).

Various methods used by the investigators are listed below:

1) Linear Model
2) Nonlinear Parabolic Model
3) Diffusion Model
4) Regression Analysis
5) Finite Difference Schemes

In the Explicit finite-difference schemes, the spatial partial derivatives and/or the coefficients are replaced in terms of the values at the known time level. The unknown variables, therefore, appear explicitly in the algebraic equations and the methods are called explicit methods. Some of them are,

1) Unstable scheme
2) Diffusive scheme
3) MacCormack Scheme

In this paper the Saint-Venant's equations for water flow and sediment continuity are solved using MacCormack explicit finite difference scheme.

MacCormack's scheme is second order accurate explicit scheme.

Using this scheme a numerical model is developed which is applicable to: degradation due to sediment diminution.

B. *Governing equations*

The successful application of a model largely depends on the correct path to develop the model for a particular problem. The assumptions used in the model development to simplify a phenomenon are sometimes crucial to the extent of their validity. Considering all these constraints, a model is developed from the basic equations describing the problem.

The basic one-dimensional partial differential equations for unsteady flow in a wide rectangular channel with deformable bed are;

Flow Continuity Equation

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad \text{...} \quad (1)$$

Flow Momentum Equation

$$\frac{\partial q}{\partial t} + \frac{\partial(\frac{q^2}{h} + g\frac{h^2}{2})}{\partial x} + gh\frac{\partial z}{\partial x} + ghs_f = 0 \ldots\ldots (2)$$

Sediment continuity Equation

$$\frac{\partial z}{\partial t} + \frac{1}{\Upsilon_s(1\text{-}\lambda)}\frac{\partial q_b}{\partial x} = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

In which,

q = water discharge per unit width;

h = flow depth;

z = bed elevation;

q_b = sediment transport load;

λ = porosity of bed material;

Υ_s = specific weight of sediment.

The friction slope S_f is determined using the Manning equation (SI unit):

$$S_f = \frac{q^2 n^2}{h^{3.333}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Where, n = The Manning roughness coefficient.

C. Overview on MacCormack scheme

The MacCormack scheme is an explicit, two-step predictor-corrector scheme [(MacCormack, 1969)] that is second-order accurate in space and time and is capable of capturing the shocks without isolating them. This scheme has been applied for analyzing one-dimensional, unsteady, open-channel flows by (Fennena, 1985) (Chaudhary, 1987)and (Bhallamudi and Chaudhary, 1991).

For one-dimensional flow, two alternatives of this scheme are possible. In one alternative, backward finite-differences are used to approximate the spatial partial derivatives in the predictor part and forward finite-differences are utilized in the corrector part. The values of variables determined during the predictor part are used during the corrector part. In the second alternative, forward finite-differences are used in the predictor part and backward finite-differences are used in the corrector part. A general recommended procedure is to alternate the direction of differencing from one time step to the next; i.e., use alternative 1 during one time step, alternative 2 during the next time step, and alternate this sequence thereafter. Recent investigations show that better results are produced if the direction of differencing in the predictor step is the same as that of the movement of the wave front.

It is a two level predictor-corrector scheme. In the predictor part, forward finite differences are used for approximating the spatial partial differential terms. In corrector part, they are approximated by backward differences, but using the predicted variables. The MacCormack scheme is an explicit scheme and the unknowns in each algebraic representing the finite differences of the governing equations are those resulting from the temporal derivative in these equations.

1) Predictor Part

The predictor step is

$$\frac{\partial f}{\partial t} = \frac{f^* - f_i^*}{\Delta t} \ldots\ldots\ldots\ldots (5)$$

$$\frac{\partial f}{\partial x} = \frac{f_{i+1}^* - f_i^*}{\Delta x} \ldots\ldots\ldots (6)$$

In which, i = the node in space

k = the node in time; an asterisk refers to the predicted values of the variable

Δx = the distance step

Δt = the time step.

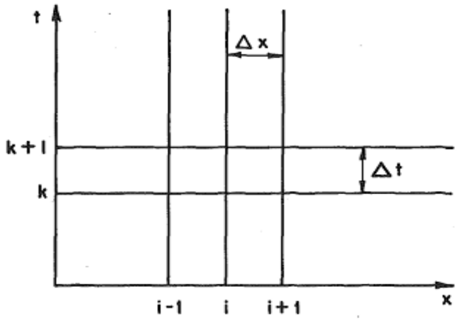

Fig. 3: Finite difference grid

2) Corrector Step

The corrector step is

$$\frac{\partial f}{\partial t} = \frac{f_i^{**} - f_i^*}{\Delta t} \ldots. (7)$$

$$\frac{\partial f}{\partial x} = \frac{f_i^* - f_{i-1}^*}{\Delta x} \ldots. (8)$$

In which two asterisks denote values of the variables.

3) Boundary Conditions

The preceding equations are for the interior points. The boundary grid points may be included in the analysis. The simulation of the boundary points is, therefore, first-order accurate. Whether this first order simulation of boundaries in an otherwise second-order scheme affects the overall accuracy of computed results is controversial. MacCormack heuristically showed that if the order of accuracy of the end conditions is one less than that of the interior points, then the overall accuracy of the computed results is not impaired. However, others question the validity of this statement.

4) Stability

The MacCormack scheme is stable if $C_n \leq 1$. For the computations to be stable, this condition must be satisfied at each grid point during every computational interval. For the stability of the scheme, it is necessary that the Courant number, C_n is less than or equal to 1, where

$$C_n = \frac{Actual\ wave\ velocity}{Numerical\ Wave\ Velocity} = \frac{|V| \pm c}{\frac{\Delta x}{\Delta t}}$$

Thus, the computational time interval depends upon the spatial grid spacing, flow velocity, and celerity, which are functions of the flow depth. Since the flow depth and the flow velocity may change significantly during the computations, it may be necessary to reduce the size of the computational time interval for stability. The time interval should be such that c_n is as close to 1 as possible. If it is substantially less than unity, then the interval size should be increased to improve accuracy and to prevent the smearing of bores and steep waves.

D. Tools

FORTRAN language is used for programming part. Programming algorithm is shown in fig.

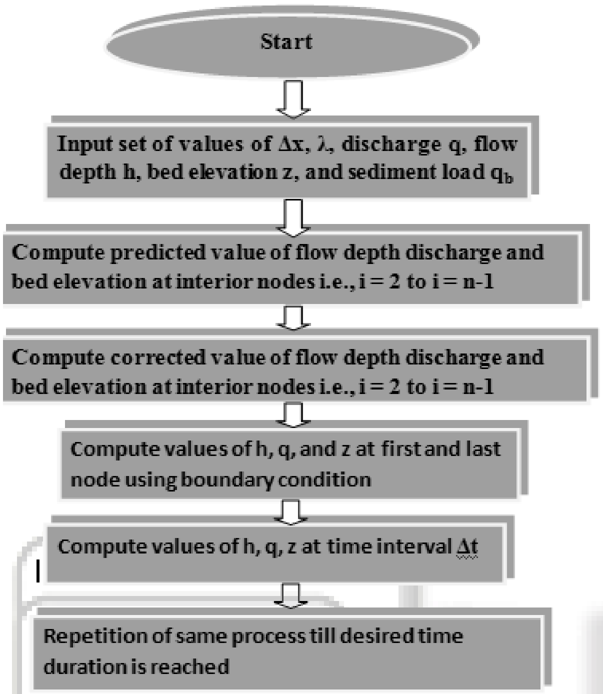

Fig. 4: Programming algorithm

IV. CONCLUSION

MacCormack scheme for simulating bed level variation in alluvial channels has provided significant results.

The methodology used in this study allows us to calibrate and validate the model developed.

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