Image segmentation Based on Chan-Vese Active Contours using Finite Difference Scheme

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Abstract— There are a lot of image segmentation techniques that try to differentiate between backgrounds and object pixels but many of them fail to discriminate between different objects that are close to each other, e.g. low contrast between foreground and background regions increase the difficulty for segmenting images. So we introduced the Chan-Vese active contours model for image segmentation to detect the objects in given image, which is built based on techniques of curve evolution and level set method. The Chan-Vese model is a special case of Mumford-Shah functional for segmentation and level sets. It differs from other active contour models in that it is not edge dependent, therefore it is more capable of detecting objects whose boundaries may not be defined by a gradient. Finally, we developed code in Matlab 7.8 for solving resulting Partial differential equation numerically by the finite differences schemes on pixel-by-pixel domain.

Keywords: Image Segmentation, Chan-Vese model, Active contours, Level set method, Finite difference method, Partial Differential Equation.

I. INTRODUCTION:

Image segmentation is a fundamental problem in image processing and computer vision. In most image study operations, example classifiers need individual objects to be divided from the image, so the explanation of those objects can be transformed into a proper structure for computer processing [1,3]. The function of segmentation is to dividing an image into its basic and disjoint sub-regions, which are identical according to their property [2], e.g. quality, intensity and colour. Extensive study has been made and many techniques have been proposed, among which the Active contour method is one of the most successful methods [6]. The basic idea of active contour method is to evolve a curve under some constraints to extract the desired object. According to the nature of constraints, the existing active contour method can be categorized into two types: edge-based models and region-based models. Edge based segmentation partitions an image based on discontinuities with sub-regions, while region-based segmentation does the similar function based on the uniformity of a desired property within a sub-region.

Active contours or snakes, proposed by Kass [8] et al. are connectivity-preserving relaxation methods, valid to the image segmentation problems. The fundamental idea is to start with first boundary shapes represented in a type of closed curves, i.e. contours, and iteratively change them by applying shrink/expansion operations according to the constraints of images. Those shrink/expansion operations, called contour evolution, are done by the minimization of an energy function. The main drawbacks of this method are its sensitivity to initial conditions and the difficulties associated with topological changes like the merging and splitting of the evolving curve. Since the active contour model [7] was proposed, many methods have been proposed to improve it, in which level set method proposed by Osher and Sethian [10] is the most important and successful one.

Level set method [10,16] is based on active contour model and particularly designed to handle the segmentation of deformable structures. Generally, the classical active contour model uses spline curves to model the boundary of an object. However, the level set method is to use a deformable curve front for approximating the boundary of an object. In the level set framework, the curve is represented by the zero level set of a smooth function, usually called the level set function. Moving the curves can be done by evolving the level set functions instead of directly moving the curves. Therefore, level set methods exhibit interesting elastic behaviours and can efficiently handle the topological changes which is also a main advantage compared with classical active contour model. Formally, the evolution of the curve is driven by a time-dependent partial differential equation (PDE) where the so called velocity term reflects the image features characterizing the object to be segmented.

A. *Definitions and Problem Statement*

An image I can be defined as a bounded real positive function on some (rectangular) domain $\Omega \in R^2$. Let its boundary be denoted $\partial\Omega$. Typically, in image processing, $I \in \{0,1,....,255\}$, and referred to as a grey scale image. We denote $C:[0,1] \to \Omega$ curve parameterized by arc length. We assume C is piecewise $C^1([0,1])$. Although the problem of image segmentation [2] has various forms, we will consider the simplest case where we segment the image into two regions. The definition of the problem is as follows: The image segmentation problem:- Let I be an image in Ω. Suppose Ω can be partitioned into Ω_1 and Ω_2, where for some image property P, $I|\Omega_1$ differ from $I|\Omega_2$. Find a curve $C \in \Omega$ such that it partitions $\Omega = \Omega_1 \cup \Omega_2$. Here, P is typically the image intensity i.e. the function value of I, but can also be texture, pattern curvatures, etc. A conventional approach in solving image segmentation is to start with some initial guess $C(0) = C_0$ and evolve $C(t)$ in a time dependent Partial differential equation $C_t = F(C(t))$ such that $\overline{C} = \lim_{t\to\infty} C(t)$ solves the problem. In other words, ideally, the model should reach a steady state when $C(t)$ is at the correct solution see Fig-1.

Fig. 1: Evolving Curve (in green) to the

For the application point of view, Given an image in Fig-2, what is its segmentation and how to find it. E.g. if a following brain image is given, the problem could be described to "what is the image component inside of the skull", i.e. "how to truncate the part of the brain

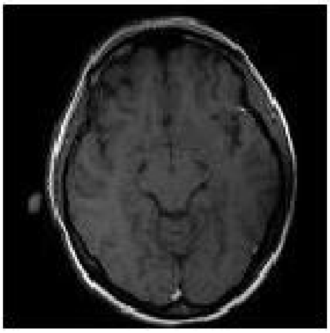

Fig. 2: Brain MRI Image

Image bounded by the 'white' skull contours ". There are many approaches [14] for you to get the segmentation of the skull shape. Even those simple ones, like a simple Sobel mask, will give you good detection on the contour of the skull. In order to get a better edge map, you may also use Canny Edge detector [13]. Of course, when we apply the above edge detector operators of segmentation for this brain image is not that precisely, because we want to know the details of the brain but not everything inside of the skull. However, the segment of brain is not an easy problem. Although you could use 'high edge value' and 'low edge value' to distinguish the contour of the skull and that of the brain, the method used here is called active contours, more precisely Chan-Vese Active Contours without edges.

1.2 Mathematical description of Chen-Vese Model:

Let Ω be a bounded open subset of R^n, $\partial\Omega$ its boundary and let I be a given bounded image function defined by $I:\overline{\Omega}\rightarrow R$. For the purpose of illustration, we consider $n=2$ although the model can be easily extended to the case $n=3$. The Chan-Vese model aims at finding the best approximation of the image as a function taking only two values. It is a particular case of the minimal partition problem and it is based on the piecewise-constant Mumford-Shah [9] functional. Contrary to classical methods which involve a stopping criterion based on the image gradient [13], the Chan-Vese model is region-based and does not require the computation of an edge-detector function. The evolving contour C is embedded in a higher-dimensional lipschitz continuous function ϕ. More precisely, it is implicitly represented as the zero level line of the function ϕ such that:

$$C=\partial w=\{(x,y)\in\Omega \mid \phi(x,y)=0\}$$
$$inside(C)=w=\{(x,y)\in\Omega \mid \phi(x,y)>0\}$$
$$outside(C)=\Omega-\overline{w}=\{(x,y)\in\Omega \mid \phi(x,y)<0\}$$
......(1)

An energy minimization problem is then introduced including both a fidelity term that ensures that the curve will match the edges of the object to be detected and a regularizing term based on the length of the curve. Chan-Vese active contour algorithm comes from segmentation problem formulated by Mumford and Shah.

The outline of the paper is as follows. In Section 1, we briefly review the background of the image segmentation then problem definition and Mathematical description. All our segmentations are carried out using the Mumford–Shah model, the Chan–Vese active contours and level set method, which we review in Section 2. This is followed by numerical experiments in Matlab using the Finite difference method in Section 3. Finally, some conclusive remarks are included in Section 4.

II. DESCRIPTION OF THE IMAGE SEGMENTATION MODEL:

A. Mumford–Shah model

The Chan–Vese model is the curve evolution [7] implementation of a piecewise constant case of the Mumford–Shah model. The Mumford–Shah model [9] is energy based method introduced by Mumford and Shah via energy functional. The basic idea is to find a pair of (u,C) for a given image I where, u is a nearly piecewise smooth approximation of I, and C denotes the smooth and closed segmenting curve.

The general form for the Mumford–Shah energy functional can be written as

$$E^{MS}(u,C)=\int_{\Omega}|I(x,y)-u(x,y)|^2\,dxdy+\mu\int_{\Omega\setminus C}|\nabla u(x,y)|^2\,dxdy+v.Length(C)$$

---- (2)

Where μ and v are positive constants, Ω denotes the image domain, the segmenting curve $C\subset\Omega$. To solve the Mumford–Shah problem is to minimize the energy functional over u and C. Note that the removal of any of the above three terms in (2) will result in trivial solutions for u and C. However, with all three terms, it becomes a difficult problem to solve since u is a function in the *N*-dimensional space (*N* = 2 in 2D image segmentation), while *C* is an (*N*−1)-dimensional data set.

A reduced case of the model is obtained by restricting the segmented image u to piecewise constant function, i.e. u = constant c_i inside each connected component Ω_i. Then this problem is called minimal partition problem and its functional is

$$E^{MS} = \sum_i \int_\Omega (u - c_i)^2 dxdy + v|C| \quad ......(3)$$

It is easy to see that, for a fixed *C*, the energy from above is minimized in the variables c_i by setting $c_i = mean(I)$ in Ω_i.

B. Chen-Vese active contours without edges model:

The Chan–Vese (CV) model [6] is an alternative solution to the Mumford–Shah problem which solves the minimization of (2) by minimizing the following energy functional:

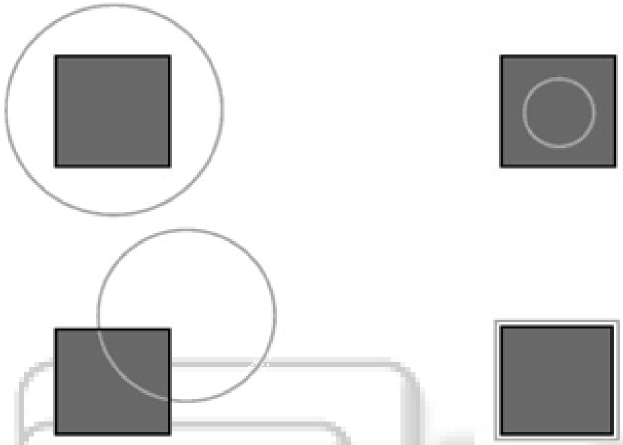

Fig. 3: Minimum "energy" is obtained when the curve C being on the boundary of the object

$$E^{CV}(c_1, c_2, C) = \lambda_1 \int_{imside(c)} (I(x,y)-c_1)^2 dxdy + \lambda_2 [\int_{outside(c)} (I(x,y)-c_2)^2 dxdy + \mu.Length(C)] \quad ---- (4)$$

Where μ, λ_1 and λ_2 are positive constants, usually fixing $\lambda_1 = \lambda_2 = 1$. c_1 and c_2 are the intensity averages of I inside *C* and outside *C*, respectively.

In order to understand what's going on with this idea, let's see Figure 3. As we discussed in section-1, the mathematical description of the Chen-Vese model says that the evolving contour C is embedded in a higher-dimensional [11] lipschitz continuous function ϕ. More precisely, the first two terms in (4) have been interpreted to two forces. The first term is the force to shrink the contour and the second term is the force to expand the contour. These two forces get balanced when the contour reaches the boundary of our interested object.

C. Level set Formulation

To solve this minimization problem, the level set method [15,16] is used which replaces the unknown curve *C* by the level-set function $\phi(x,y)$. Thus, the energy functional $E^{CV}(c_1, c_2, C)$ can be reformulated in terms of the level set function $\phi(x,y)$ as follows:

$$E_\varepsilon^{CV}(c_1, c_2, \phi) = \lambda_1 \int_\Omega |I(x,y) - c_1|^2 H_\varepsilon(\phi(x,y))dxdy + \lambda_2 \int_\Omega |I(x,y) - c_2|^2 (1 - H_\varepsilon(\phi(x,y)))dxdy + \mu \int_\Omega \delta_\varepsilon(\phi(x,y)) |\nabla \phi(x,y)| dxdy \quad ..... (5)$$

Where $H_\varepsilon(z)$ and $\delta_\varepsilon(z)$ are, respectively, the regularized approximation of Heaviside function $H(z)$ and Dirac delta function $\delta(z)$ as follows:

$$H_\varepsilon(z) = \begin{cases} 1, if z > \varepsilon \\ 0, if z < -\varepsilon \\ \frac{1}{2}\left[1 + \frac{z}{\varepsilon} + \frac{1}{\pi}\sin\left(\frac{\pi z}{\varepsilon}\right)\right], if |z| \le \varepsilon \end{cases}$$

$$\delta_\varepsilon(x) = \frac{d}{dz}H(z) = \begin{cases} 0, if |z| > \varepsilon \\ \frac{1}{2\varepsilon}\left[1 + \cos\left(\frac{\pi z}{\varepsilon}\right)\right], if |z| < \varepsilon \end{cases} \quad .. (6)$$

There are many different ways to get a continuous version of Heaviside function and its corresponding delta function. This minimization problem is solved by taking the Euler–Lagrange equations and updating the level set function $\phi(x,y)$ by the gradient descent method:

$$\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)\left[\mu div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - \lambda_1 (I - c_1)^2 + \lambda_2 (I - c_2)^2\right] \quad ...... (7)$$

Where c_1 and c_2 can be, respectively, updated at each iteration by

$$c_1(\phi) = \frac{\int_\Omega I(x,y) H_\varepsilon(\phi(x,y))dxdy}{\int_\Omega H_\varepsilon(\phi(x,y))dxdy},$$

$$c_2(\phi) = \frac{\int_\Omega I(x,y)(1 - H_\varepsilon(\phi(x,y)))dxdy}{\int_\Omega (1 - H_\varepsilon(\phi(x,y)))dxdy} \quad .... (8)$$

Please note that c_1 and c_2 are the global mean intensity of the region outside and inside the zero level set respectively. So the solution image can be expressed as

$$I = c_1 H(\phi) + c_2 (1 - H(\phi)) \quad ...... (9)$$

III. NUMERICAL EXPERIMENTS:

A. Numerical Method-Finite Difference Scheme

The partial differential equation [12] in the continuous domain defined in (7) can be solved by a finite difference method in numerical scheme. All the spatial partial derivatives are approximated by the central difference and the temporal partial derivatives are approximated by the forward difference.

Then, (7) can be discretized using the forward difference [5] as follows:

$$\frac{\phi_{i,j}^{n+1}-\phi_{i,j}^{n}}{\Delta t} = L\left(\phi_{i,j}^{n}\right),$$

Where Δt the time is step and $L\left(\phi_{i,j}^{n}\right)$ is the numerical approximation of the right-hand side in (7).

The corresponding curvature $k = div\left(\frac{\nabla\phi}{|\nabla\phi|}\right)$ in the $L\left(\phi_{i,j}^{n}\right)$ can is discretized using the second order central differencing scheme [4]:

$$k = div\left(\frac{\nabla\phi}{|\nabla\phi|}\right) = \frac{\phi_{xx}\phi_y^2 - 2\phi_{xy}\phi_x\phi_y + \phi_{yy}\phi_x^2}{\left(\phi_x^2+\phi_y^2\right)^{3/2}}$$

……………………… (10)

Where $\phi_x, \phi_y, \phi_{xx}, \phi_{yy}$ & ϕ_{xy} are computed as follows:

$$\phi_x = \frac{1}{2h}\left(\phi_{i+1,j}-\phi_{i-1,j}\right), \quad \phi_y = \frac{1}{2h}\left(\phi_{i,j+1}-\phi_{i,j-1}\right),$$

$$\phi_{xx} = \frac{1}{h^2}\left(\phi_{i+1,j}-2\phi_{i,j}+\phi_{i-1,j}\right),$$

$$\phi_{yy} = \frac{1}{h^2}\left(\phi_{i,j+1}-2\phi_{i,j}+\phi_{i,j-1}\right),$$

$$\phi_{xy} = \frac{1}{h^2}\left(\phi_{i+1,j+1}-\phi_{i-1,j+1}-\phi_{i+1,j-1}+\phi_{i-1,j-1}\right).$$

……………………… (11)

Where h is the grid spacing, then (7) is implemented as follows:

$$\frac{\phi^{n+1}-\phi^{n}}{\Delta t} = \delta_\varepsilon\left(\phi^n\right)\left[\mu k - \lambda_1\left(u_{i,j}-c_1\left(\phi^n\right)\right)^2 + \lambda_2\left(u_{i,j}-c_2\left(\phi^n\right)\right)^2\right]$$

Where δ_ε and k are computed according to (6) and (10) respectively.

B. *Numerical Results:*

Using the Matlab 7.8 software, we used some standard Matlab function and developed the code for image segmentation to solving the partial differential equation using the finite difference scheme [4,5]. Here we consider image as a gray image that has different approaches compare to colour image. So, for the gray image we only need to care about two image forces as we mentioned in section 2. However, if you'd like to treat the image to a colour one, then you have to apply the idea for vector images, that portion we are not consider in this paper. Generally speaking, considering the image to a gray one will simplify the segmentation question and lead to simple calumniation but lose some image information. Therefore, if you have a clean image, which means no noise, then probably you can treat it as a gray image.

We consider Fig.2 – Brain Image, applying different contours following are the result.

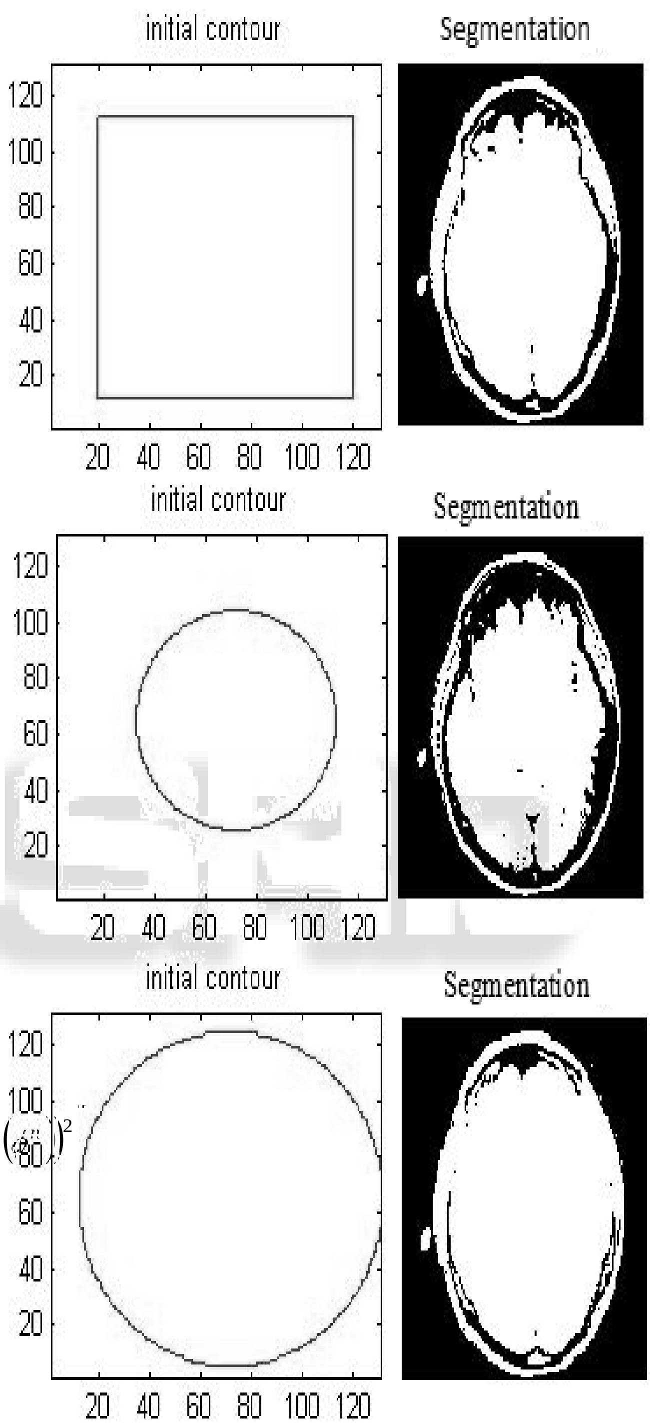

Fig. 4 Result of Segmentation using the Single Phase

Here in this experiment, as shown in Fig. 4, we consider the image as a single phase means you could only divide the original image into two parts, say background and foreground region of the image or say two colors of the image, etc. Also we consider the image as a multiphase as shown in Fig. 5, means you could divide the original image into more than two parts, more precisely parts, and where is the number of phases. Here different contours have different ability for the segmentation of image. So for some contours give the good result compare to other one but more precisely result is depending on the characteristic of the given image. In our case we have a little bit good result in multiphase compare to the single phase using the finite difference scheme particularly central difference scheme.

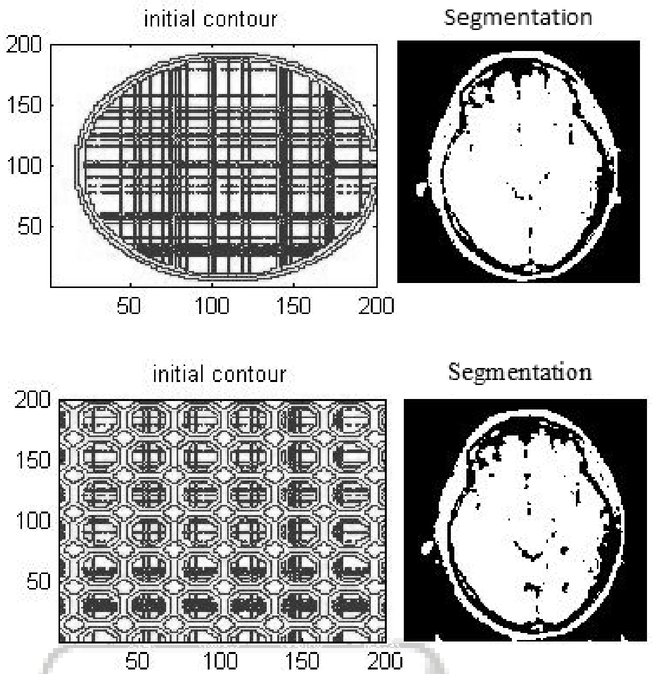

Fig. 5: Result of Segmentation using the Multi Phase

IV. CONCLUSION

Chan-Vese algorithm was implemented in this paper, using the finite difference method. From the results above, it can be seen that this algorithm deals quite well even with images which are quite difficult to segment in the regular methods, such as gradient-based methods or thresholding. The main advantages of this model are: First, it can deal with the detection of objects whose boundaries are either smooth or not necessarily defined by gradient. In such cases, the edge-based level set methods commonly fail and result in boundary leakage. Second, it does not require image smoothing and thus can efficiently process the images with noise. Therefore, the true boundaries are preserved and could be accurately detected. Third, it can automatically detect interior contours with the choice of Dirac delta function $\delta(z)$ that has non-compact support. However, CV model also has some drawbacks i.e., the unsuccessful segmentation of images with intensity inhomogeneity, the sensitivity to the placement of initial contour and the extraordinary time-consumption if re-initialization step is adopted for maintaining stable curve evolution and ensuring more precise results.

Finally, we conclude that using the single phase we have a still good result compare to multiphase. But precise result is shown in multiphase contour, also it is not necessary that you always have to apply multiphase contour for the gray scale image.

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