A Compressed Sensing Approach to Image Reconstruction

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Abstract— compressed sensing is a new technique that discards the Shannon Nyquist theorem for reconstructing a signal. It uses very few random measurements that were needed traditionally to recover any signal or image. The need of this technique comes from the fact that most of the information is provided by few of the signal coefficients, then why do we have to acquire all the data if it is thrown away without being used. A number of review articles and research papers have been published in this area. But with the increasing interest of practitioners in this emerging field it is mandatory to take a fresh look at this method and its implementations. The main aim of this paper is to review the compressive sensing theory and its applications.

Keywords: Compressed sensing (CS), Sparsity, sparse matrix, nonlinear image reconstruction

I. INTRODUCTION

With the increase of technology the amount of data being transferred from here to there is also increasing day by day. So there is a need to increase the storage space for this increased data. As storage space can be increased up to some extent, so compression of data is necessary. Data compression can be done by reconstructing the signals or images in a compressed form. [1] Various compression techniques were used till date like JPEG compression, wavelet coding, Predictive coding, bit plane slicing etc. But all these terms are lossy and require a lot of time for the complete process. So a new term compressed sensing or Compressive sampling or sparse sampling comes into picture. Moreover all the information collected is never used to represent the desired signal or image. Only few of Fourier coefficients are used and all other are get wasted. Then the question arises if we have to throw away most of the data collected then why do we have to acquire it at all?[1] And why do we waste so much of time in gathering the data that is simply thrown away? So, CS comes into picture.

A. Introduction to CS

CS is a technique that uses very few samples of a signal to represent it. [4] From the Shannon Nyquist theorem we require double the samples of a signal for its proper reconstruction. But CS theory uses random samples (say N) of that of total M samples of a signal (where N<M). [2][3] Let X be a signal for which we want a sparse representation in the form of its ortho norms Ψ and α, as

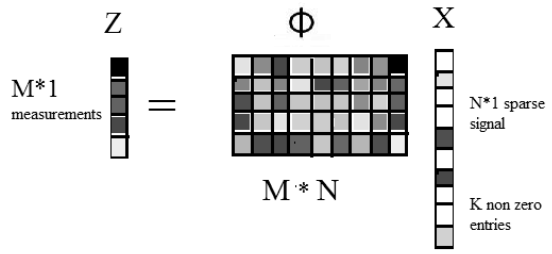
$$X = \Psi * \alpha$$

If we use K samples of α (k<M), we can recover our signal. As we cannot directly measure the recovery of signal, we have to use a representation Z such that

$$Z = \Phi * \Psi * \alpha$$

$$Z = \Phi * X$$

Where Φ is sparse matrix representation of X having few rows that of α. It is necessary for Cs to take place that N<K<M.[3][4]



Now the question arises that if we have to take random samples of a signal, then its location should be known a priori. But CS theory does not use this fact, yet it gives faithful results. For this reason, it uses a number of tools from the theory of probability.

B. Introduction to Sparsity

Sparsity refers to the number of non – zero coefficients of a signal.[2] A matrix represented with very few non – zero terms is a sparse matrix.

C. Image Reconstruction

Generating an image from scattered or under sampled data is the reconstruction of image. For CS, the image must be reconstructed non-linearly, i.e. the apparatus used to collect information must be of nonlinear type.[1][5] Image is reconstructed from projections taken from different directions.

D. CS approach

For the CS theory to get implemented, there are three aspects that must be fulfilled by any signal/image [1][4][10]-

1. Sparsity: The signal must be sparse or compressible in some other domain like wavelet or Fourier… i.e. it must have a number of zero value pixels in case of image or coefficients in case of a signal.
2. In coherence: The different pixels of an image must be loosely connected with the other pixels. They must be independent to each other. There should be no relation between different coefficients of a signal.
3. Non-linear reconstruction: The image/signal must be recovered non-linearly.

E. Working mechanism of CS

A simple look at the working of CS is given below. Steps involved are:

1. Under sampled data:

The very first step is to capture randomly chosen pixels of an image by a camera or scanning system. [7]

2. Fill the missing pixels: If we are concerned with signal a minimization algorithm called l1- minimization is applied that fill the missing places by taking different iterations. For image processing we use a minimization algorithm known as total variation minimization. [6][11]
3. Addition of smaller shapes: The next step is to add the shapes by looking at the pixels which match to the original pixels. For example, if three pixels are of red colour, then the algorithm used adds a triangle there and so on. [6]
4. Adding larger shapes: After giving smaller shapes, the algorithm adds larger shapes taking the same basis.
5. Getting clarity: The last step is to achieve clarity in the recovered image by iterations taken by the algorithm.

II. Reconstruction algorithms

To reconstruct an image or signal a there is a property called RIP (restricted Isometry Property) which must be fulfilled by the matrix chosen for random measurements.

RIP (restricted Isometry property): The sampling matrix Φ for any K-sparse vector X follows a restricted property,[19] that is

$$(1-\delta_K)\,\|X\|^2_2 \le \|\Phi X\|^2_2 \le (1+\delta_K)\,\|X\|^2_2$$

When δ_K is << 1, then Φ can reconstruct K/2 sparse signal X stably with a large probability. So , if we use δ_{2K} , obtain K-sparse signal X. This condition is called Isometry property. Gaussian matrices are used mostly as random measurement matrices that fully satisfy the RIP property.[19]

Recovery algorithms are classified into basis pursuit and greedy pursuit. Basis pursuit works with ℓ-1 norms and greedy pursuit is an iterative algorithm for reconstruction. ℓ-1 norms can exactly recover signal using only M= (K.log (N/K)) i.i.d. (independent and identically distributed) measurements.[9] The reconstruction problem with ℓ-1 norm turns to be

$$X = \underset{X}{\operatorname{argmin}} \|X\|_1 \text{ w.r.t. } Z = \Phi^* X$$

We can represent this equation in linear form as

$$\min \sum_{j=1}^{2N} V_j \text{ w.r.t. } v \ge 0,\ Z = (\Phi, -\Phi)v$$

signal X can be obtained from solution of v* from this equation.[11][12]

In the greedy algorithms [14] signal support is calculated from pseudo inverse Φ` of the random matrices Φ, as

$$X = (\Phi)^{`} Z$$

Where Φ` can be obtained from

$$\Phi^{`} = (\Phi^*\Phi)^{-1}\Phi^*$$

OMP (Orthogonal Matching Pursuit) and CoSaMP (Compressive Sampling Matching Pursuit) are used as greedy algorithms. In OMP largest correlation between Φ and residual of z is calculated and for each iteration one coordinate for support of signal X is generated. Thus the complete support signal X is obtained from total iterations performed by algorithm. CoSaMP works by generating a proxy signal of X and then finding the correlations.[13][15]

ℓ-1 minimization algorithm is slower than the greedy algorithms but it gives stability and uniform guarantees on the result. [12]

III. Conclusion

In this review, our aim was to give a brief theory behind CS and its implementation to reconstruction of images or signals, as it is difficult to cover all of the developments happened in field of recovery via compressive sensing. As the sensing instruments provide some constraints, so we are limited with measurement matrices. We also give the basic difference between the two pursuits used. We hope that this summary will be proved useful to the beginners who are interested in exploring the features of this field.

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