ON Fuzzy Linear Programming Technique Application

Dr. R. S. Chanda[1]
[1]University of Calcutta,

Abstract— A solution approach based on fuzzy linear programming is proposed and applied to optimal machine scheduling problem. In this solution approach errors in the demand of various products during the next production period are considered to be fuzzy in nature. In conventional linear programming approach it is assumed that there is no error in the expected demand of various products. A fuzzy linear programming approach is proposed to obtain an optimal solution under fuzzy conditions. In the proposed method expected demand of products & profit are expressed by fuzzy set notations. The proposed fuzzy linear programming formulation is then transformed to an equivalent conventional linear programming problem and solutions obtained by solving this transformed linear programming problem. For illustration purpose the proposed method is applied to a profit maximization related machine scheduling problem.

Keywords: Fuzzy set theory, Crisp linear Programming, Fuzzy linear Programming, Optimization, Membership function.

I. INTRODUCTION

The profit maximization related machine scheduling problems are normally solved by deterministic linear programming (LP) techniques. The main objective of profit maximization related machine scheduling problem is to obtain an optimal schedule of machine hours subject to the system constraints like demand constraints, hour constraints etc. The main drawback of deterministic approach is the assumption of fixed expected demand of various products values in the problem formulation. Actually in real-life situation uncertainties in data are often encountered. The expected demand of products cannot be represented by a probabilistic approach. Information regarding the expected demand of products may be limited to some linguistic declaration about the data e.g. expected demand of product 'i' is approximately 200 units during the next production period. This type of information is neither deterministic nor probabilistic. Such situation is more common in forecasting problems where reflection of data into the future is not stationary and human decisions are involved in an environment. Such type of data is said to be fuzzy and the nature of uncertainty is described as possibilistic. Moreover, researchers tend to search for some sort of flexible solution instead of for some kind of special solution that may be optimal for the expectancy of some variable but prove disastrous if the future does not come as expected which usually happens.

In deterministic LP approaches production balance and machine running hour limits are maintained using forecasted expected demand of different variation of products. So for solving such problem expected demand must be a known parameter because it is an input parameter and it can only be known through forecasting. Errors are associated in the forecasted demands because the demand depends on the customer behavior. So the problem is how to solve such machine scheduling problem when the product demands are not crisp in nature. In this paper a fuzzy linear programming (FLP) based approach is proposed to solve such problems.

Several applications of fuzzy linear programming (FLP) in different fields have been reported[1-5] earlier using the basic concept of fuzzy set theory. Each variable is assigned a degree of membership for each possible value of the variable.

In this paper fuzzy linear programming is applied to profit maximization related machine scheduling problem where forecasted demand of product and profit are expressed by the fuzzy set notations. To obtain the optimal solution of such problem the conventional linear programming (LP) problem is formulated as fuzzy linear programming problem by replacing the crisp constraints and objective function by fuzzy set and then by simply solving a conventional LP problem which is equivalent to the formulated FLP problem. In the present approach a LP package is used to solve this problem.

The proposed method is a more reasonable method of analysis which depends on fuzzy set theory for analyzing different forecasted product demand scenarios. The proposed method is more flexible than the other previously proposed non-fuzzy methods. It is concluded that the arbitrary reduction of fuzzy values to ordinary closed intervals may result in misleading forecasts or unclear risky decisions.

II. MATERIALS AND METHODS

A. *Deterministic Machine Scheduling Problem*

The profit maximization related machine scheduling problem is basically a problem to determine X_{ij} i.e the number of hour machine j should be scheduled to produce product variation i for all i & j so that profit for producing the product variation i for all i on respective machine j for all j is maximum subject to demand constraint for product variation i during the next production period, considering the production rate per hour of the machine j when producing product variation i for all i & j maintained and limit of machine available hours to produce the i variation during the next production period on the machine j for all i & j are also maintained.

B. *Problem Considered*

The following problem is considered for illustration purpose of the proposed approach and is taken from ref.-6.

A plant has 4 machines each capable of producing 3 variations of a single product. The profit per hour when producing the 3 variations on the respective machines are given in Table-I. The production rates per hour of the 4 machines when producing the 3 variations of the products are given in Table-II. The demand for 3 variations during the next production period is expected to be 700, 500 & 400 units of variations 1, 2 & 3 respectively. The maximum available hours to produce the 3 variations during the next production period on the 4 machines are 90, 75, 90 & 85 hours.

Variation			Machine	
	1	2	3	4
1	5	6	4	3
2	5	4	5	4
3	6	7	2	8

Table (1): Profit in money unit/hr. for each variation by machine

Variation	Machine			
	1	2	3	4
1	8	2	4	9
2	7	6	6	3
3	4	8	5	2

Table (2): Production rate/hr. for each machine by variation

If we let X_{ij} be the number of hours machine j should be scheduled to produce variation i for all i & j, then the LP model can be written as

$$\text{MAXIMIZE } F = 5X_{11} + 6X_{12} + 4X_{13} + 3X_{14} + 5X_{21} + 4X_{22} + 5X_{23} + 4X_{24} + 6X_{31} + 7X_{32} + 2X_{33} + 8X_{34}$$

Subject to

2) *the demand constraint*

$$8X_{11} + 2X_{12} + 4X_{13} + 9X_{14} = 700$$
$$7X_{21} + 6X_{22} + 6X_{23} + 3X_{24} = 500$$
$$4X_{31} + 8X_{32} + 5X_{33} + 2X_{34} = 400$$

And

3) *the hour constraint*

$$X_{11} + X_{21} + X_{31} \le 90$$
$$X_{21} + X_{22} + X_{32} \le 75$$
$$X_{13} + X_{23} + X_{33} \le 90$$
$$X_{14} + X_{24} + X_{34} \le 85$$

By applying rule mining algorithms, frequent itemsets are generated from large data sets e.g. Apriori algorithm. It takes so much computer time to compute all frequent itemsets. We can solve this problem much efficiently by using Genetic Algorithm (GA). GA performs global search and the time complexity is less compared to other algorithms. Genetic Algorithms (GAs) are adaptive heuristic search & optimization method for solving both constrained and unconstrained problems based on the evolutionary ideas of natural selection and genetic. The main aim of this work is to find all the frequent itemsets from given data sets using genetic algorithm & compare the results generated by GA with other algorithms. Population size, number of generation, crossover probability, and mutation probability are the parameters of GA which affect the quality of result and time of calculation.

The above deterministic formulation can be simplified as

Maximize $F = CX$

Subject to

$$\left.\begin{aligned} AX &= b \\ \text{And} \quad DX &\le d \\ X &\ge 0 \end{aligned}\right\} \qquad (1)$$

Where,

X = Control variable vector
F = Total profit to be maximized
C = Profit parameter vector
A = Constraints matrix relating to constraint (a)
b = R.H.S column vector relating to constraint (a)
d = R.H.S column vector relating to constraint (b)
D = Constraints matrix relating to constraint (b).

C. *Fuzzy Linear Programming Approach*

Fuzzy set theory is an extension of conventional crisp set theory. Fuzzy set theory is now being widely used to formulate optimization related industrial management problems under uncertain conditions to obtain better solutions to the problems.

In the proposed approach fuzzy linear programming model is introduced to the machine scheduling problem based on fuzzy constraints and fuzzy objective function. Here the demand constraints for each product variation are treated as fuzzy constraints because they are related to uncertain forecasted demand of the product. The fuzzy objective function is denoted by $\tilde{F}$ related to the total profit. As the objective is to maximize the total profit, a membership function for the fuzzy profit $\tilde{F}$ can be defined such that a low profit is given a low membership value and a high profit is given a high membership value.

The original deterministic LP formulation in equation (1) is now reformulated as a partially fuzzy LP formulation [1-5].

Maximize $\tilde{F} = CX$

Subject to

$$\left.\begin{aligned} A_i X &= b_i, \ i = 1,2,\ldots\ldots,N_1 \\ D_j X &\le d_j, \ j = 1,2,\ldots\ldots,N_2 \\ X &\ge 0 \end{aligned}\right\} \qquad (2)$$

Where,

N_1 = Number of product variation of a single product

N_2 = Number of machine

The method of solution of the above equation (2) is as follows [1-5].

Find a solution in such a way that,

1) the value of the objective function CX exceeds at least the predetermined level b_o and
2) the constraints $A_iX=b_i$, $i=1,2,\ldots\ldots,N_1$ are satisfied as well as possible. While
3) the restriction $D_jX \le d_j$, $j=1,2,\ldots\ldots,N_2$ are satisfied strictly.
4) So the problem contains two types of fuzzy constraints, which are stated as follows.

D. *Fuzzy Constraints Related to The Expected Demand of Product Variation*

There are always errors in the forecasted demand of product variation during the next production period. Therefore the actual demand is the forecasted demand plus error ‘t’. Here forecasted demand is crisp in nature but the error ‘t’ is associated with fuzziness and is represented by a membership function as shown in Fig.1. Larger error indicates smaller degree of satisfaction. The degree of satisfaction can be defined by by $\mu_i(X)$, as follows.

$$\mu_i(X) = \begin{cases} 1-(t_i/P_i), & A_i X = b_i + t_i, \quad 0 \le t_i < P_i \\ 1+(t_i/P_i), & A_i X = b_i - t_i, \quad -P_i < t_i \le 0 \\ 1, & A_i X = b_i, \quad t_i = 0 \\ 0, & \text{otherwise} \end{cases}$$

Where, $\mu_i(X)$ is the membership function with respect to the i^{th} constraint and P_i is the maximum tolerable error in the forecasted demand. In this approach P_i is taken as 10% of the forecasted demand.

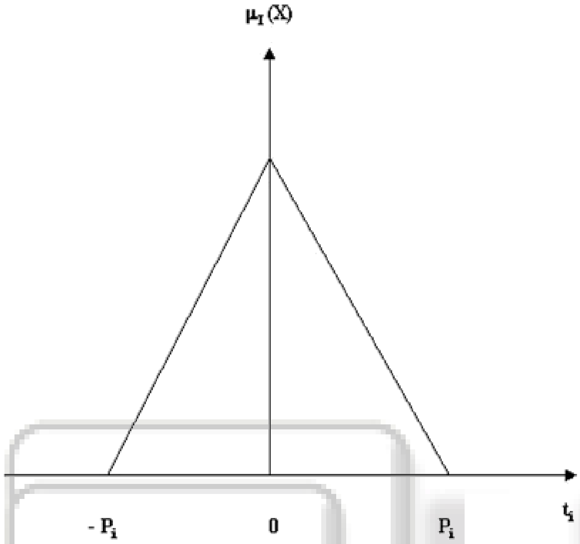

Fig. (1): Membership functions for product variation demand.

E. Fuzzy Constraint Regarding Objective Function

In formulating this type of fuzzy constraints the total profit is kept above a minimum level of profit, which is determined by solving the conventional original LP problem with full error condition of demand i.e. with actual forecasted demand minus 10% of forecasted demand for each variation of product. A high profit must be given a high membership value and a low profit, low membership value. Fig.2 shows the membership function of the profit where P_o is the maximum acceptable tolerance of the level b_o. The profit level b_o is found by solving the conventional original LP problem with zero error condition of demand. A membership value of 1 is given for any profit greater than the level b_o. With the decrease in profit level from b_o , the membership value or degree of satisfaction will decrease linearly to zero value. Here profit becomes equal to the minimum level b_o - P_o. Below this value of profit the degree of satisfaction is zero. The membership function of the objective function is thus expressed as follows.

$$\mu_{\tilde{F}} = \begin{cases} 1+t_0/P_0, & CX = b_0 - t_0, \quad -P_0 < t_0 \le 0 \\ 0, & CX \le b_0 - t_0, \quad t_0 = P_0 \\ 1, & CX \ge b_0, \quad t_0 = 0 \end{cases}$$

F. Final Fuzzy Linear Programming Formulation

The max–min. formulation is used in the present approach because it is widely used for optimization problem formulations. A solution X to the FLP problem is a feasible solution if it satisfies the crisp constraints, the fuzzy constraints and fuzzy Objective function. As it is required to be satisfied the objective function as well as the constraints which are characterized by its membership functions, a fuzzy decision is viewed as the intersection of fuzzy constraints and fuzzy objective function. So the min-operator [1] can be used to find the intersection of fuzzy sets. Therefore the overall membership function for a fuzzy decision X is expressed as

$$\mu_d(X') = \max \mu_d(X) = \max \min\left[\mu_{\tilde{F}}(X), \mu_i(X)\right]$$

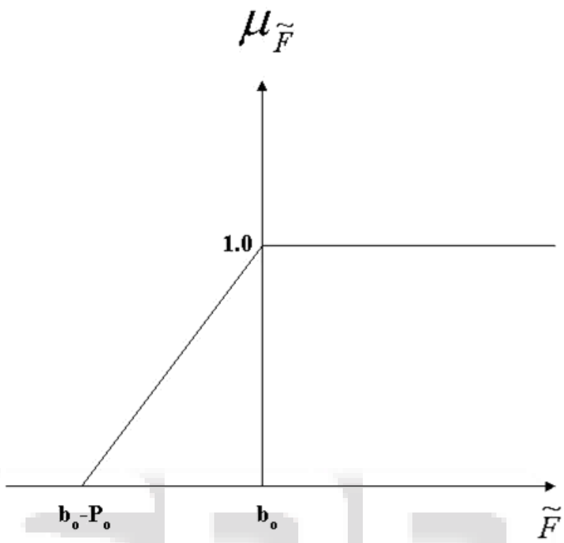

Fig. (2): Membership function for objective function.

$$\mu_d(X) = \min\left[\mu_F(X), \mu_i(X)\right]$$

If it is defined that the solution with highest degree of membership to the fuzzy decision set is the best solution then best solution X′ is

If $\lambda = \mu_d(X')$ & $\mu_r(X) = [\mu_{\tilde{F}}(X)$, $\mu_i(X)$] then the above equation can be expressed as a typical LP problem.

max λ = max min [$\mu_r(X)$]

Subject to

$\lambda \le [\mu_r(X)]$

$X \ge 0.$

Therefore the optimal solution X′ and λ the membership function associated with it can be determined by solving a crisp LP Problem.

The above formulation can be represented in a simplified form with context to the already formulated problem as follows [1-5]:

Max $\mu(X)$

Subject to

$AX + t = b$, $A=A_i$, $b= b_i$, $i = 0,1,2,\ldots\ldots,N_1$

$t \le P$, $t = t_i$, $P= P_i$, $i = 0,1,2,\ldots\ldots,N_1$

$DX \le d$, $D = D_j$, $d = d_j$, $j= 1,2,\ldots\ldots,N_2$

$X , t \ge 0$

where A_o corresponds to profit parameter vector. The crisp representation of the above formulation is as follows [1-5]:

Max λ

Subject to

$\lambda P + t \le P$

$AX + t = b$
$t \leq P$
$DX \leq d$
$X, t \geq 0$

III. RESULT AND DISCUSSION

The Table-III shows the detail results of crisp LP & FLP approach. From the crisp LP result with zero error, the objective function value is found to be 2062.02 & from the result with full error the objective function value is found to be 2056.79. The difference of this two is 5.23. Based on this result & the developed model the following equations can be written for the considered sample system.

MAX λ

Subject to

$\lambda \times 5.23 + Y_1 \leq 5.23$

$$5X_{11} + 6X_{12} + 4X_{13} + 3X_{14} + 5X_{21} + 4X_{22} + 5X_{23} + 4X_{24} + 6X_{31} + 7X_{32} + 2X_{33} + 8X_{34} + Y_1 = 2062.02$$

$0 \leq Y_1 \leq 5.23$

$\lambda \times 70 + Y_2 \leq 70$

$$8X_{11} + 2X_{12} + 4X_{13} + 9X_{14} + Y_2 = 700$$

$0 \leq Y_2 \leq 70$

$\lambda \times 50 + Y_3 \leq 50$

$$7X_{21} + 6X_{22} + 6X_{23} + 3X_{24} + Y_3 = 500$$

$0 \leq Y_3 \leq 50$

$\lambda \times 40 + Y_4 \leq 40$

$$4X_{31} + 8X_{32} + 5X_{33} + 2X_{34} + Y_4 = 400$$

$0 \leq Y_4 \leq 40$

$$X_{11} + X_{21} + X_{31} \leq 90$$
$$X_{21} + X_{22} + X_{32} \leq 75$$
(a) $$X_{13} + X_{23} + X_{33} \leq 90$$
$$X_{14} + X_{24} + X_{34} \leq 85$$

The result of this model is shown in Table-III for different error settings i.e. say for 60% error setting means 60% of 10% of forecasted demand. Comparing the result with crisp LP, it shows that some changes in the obtained values, this is due to the uncertainty of demand condition. The obtained value of λ and t_o for say minimum 50% error setting, indicates that the solution satisfied with 0.5 degree of satisfaction at $t_o = 2.62$. The total profit is calculated using The variable values obtained to judge the effectiveness of the proposed approach and it is checked that the membership function value for this calculated profit is 0.5 and this profit occurs when $t_o= 2.62$. If the minimum error setting is 80% then the membership function value is found to be 0.2 and $t_o= 4.19$ which indicates a lower profit occurs at $t_o= 4.19$. If the minimum error setting is 30% then the membership function value is found to be 0.7 and $t_o=1.57$. The problem is solved for different minimum error settings varying from 0% to 100% and checked the effectiveness of the proposed approach. It is found satisfactory at every stage.

From the obtained result it is clear that the FLP approach is more reasonable due to its flexibility and capability of analysis for different forecasted demand scenario. Moreover, FLP approach has the merits to discriminate Between different values of variables inside a given range

Variable	Result of crisp LP		Result of FLP										
			Error setting										
	With zero error	With full error	0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
X_{11}	70.12	53.93	70.12	68.50	66.88	65.26	63.64	62.02	60.40	58.78	57.16	55.54	53.93
X_{12}	56.19	69.29	56.19	57.50	58.81	60.17	61.43	62.74	64.05	65.36	66.67	67.98	69.29
X_{13}	6.67	15	6.67	7.5	8.33	9.17	10.00	10.83	11.67	12.50	18.33	14.17	15
X_{14}	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{21}	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{22}	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{23}	83.33	75	83.33	82.50	81.67	80.83	80	74.17	78.33	77.50	76.67	75.83	75
X_{24}	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{31}	19.88	36.07	19.88	21.50	23.12	24.74	26.36	27.98	29.60	31.22	32.84	34.46	36.07
X_{32}	18.81	5.71	18.81	17.50	16.19	14.85	13.57	12.26	10.95	9.64	8.33	7.02	5.71
X_{33}	0	0	0	0	0	0	0	0	0	0	0	0	0
X_{34}	85	85	85	85	85	85	85	85	85	85	85	85	85
Y_1	-	-	0	0.52	1.05	1.57	2.09	2.62	3.14	3.66	4.19	4.71	5.23
Y_2	-	-	0	7	14	21	28	35	42.01	49.01	56.02	63.02	70
Y_3	-	-	0	5	10	15	20	25	30	35	40	45	50
Y_4	-	-	0	4	8	12	16	20	24	28	32	36	40
λ	-	-	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0

Table (3): Result of crisp LP and FLP formulation

Via the variable membership is an important feature of the FLP approach that cannot be obtained in crisp LP.

IV. CONCLUSION

The method proposed in this paper using fuzzy linear programming to solve optimal machine scheduling problem indicated that fuzzy modeling of demand of product variation would help the industrial engineers to plan the machine schedule in an uncertain environment. In real life situation uncertainty in data is often encountered. The available data are not sufficient for the solution of the problem in most of the cases. The linguistic declaration may be used to describe the validity of the data. Such type of uncertainty is suitably modeled using fuzzy sets. Forecasted demand of product in case of optimal machine scheduling, is one area where this kind of uncertainty can be encountered. The present FLP based approach helps the engineers to incorporate such type of uncertainty in forecasted demand of product for better solution.

After incorporating the fuzziness of product demand and total profit a fuzzy linear programming model is developed which can easily be transferred to an equivalent crisp LP model and the problem is solved by solving a standard crisp LP problem. This makes the proposed FLP approach very efficient. The proposed approach is applied to a sample problem and the result shows that the method is very effective in obtaining the optimal solution of the problem under imprecise product demand condition. The study of the results for various minimum error conditions of demand reveal that the solutions obtained by FLP approach are correct. Finally, fuzzy set theory has the merits of discriminate between different values of variable inside a given range via the variable membership is an important feature of the proposed approach that cannot be obtained in the crisp LP. Moreover, the proposed method is flexible enough for analyzing machine scheduling scenario under uncertainty of product demand.

REFERENCES

[1] Zimmermann H.J., Description and optimization of fuzzy systems, Int. J. General System, 2(1976), pp 209-215.
[2] Lu C.N., Leou R.C., Liu K.C. and Unum M., A load flow and fuzzy linear programming based external network modelling approach, IEEE Trans. On Power Systems, Vol.9, No.3(1994), pp 1293-1301.
[3] Zimmermann H.J., Fuzzy programming and linear programming with several objective functions, Fuzzy sets and Systems, 1(1978), pp 45-55.
[4] Hamacher H., Leberling H. and Zimmermann H.J., Sensitivity analysis in fuzzy linear programming, Fuzzy sets and Systems, 1(1978), pp 269-281.
[5] Tanaka H., Guo P. and Zimmermann H.J., Possibility distribution of fuzzy decision variables obtained from possibilistic linear programming problems, Fuzzy sets and Systems, 113(2000), pp 323-332.
[6] B.E. Gillet , Introduction to Operation Research – A Computer Oriented Algorithmic Approach, Tata McGraw Hill Publishing Co. Ltd., 1976.
[7] A Charnes and W.W. Cooper, Management Models and Industrial Application of Linear programming, Vol.2, Wiley, New York, 1961.
[8] G.J. Klir and T.A. Folger, Fuzzy sets, uncertainty and information, Prentice Hall, 1988.

