

Rough Set based Natural Image Segmentation under Game Theory Framework

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Abstract- The Since past few decades, image segmentation has been successfully applied to number of applications. When different image segmentation techniques are applied to an image, they produce different results especially if images are obtained under different conditions and have different attributes. Each technique works on a specific concept, such that it is important to decide as to which image segmentation technique should for a given application domain. On combining the strengths of individual segmentation techniques, the resulting integrated method yields better results thus enhancing the synergy of the individual methods alone. This work improves the segmentation technique of combining results of different methods using the concept of game theory. This is achieved through Nash equilibrium along with various similarity distance measures. Using game theory the problem is divided into modules which are considered as players. The number of modules depends on number of techniques to be integrated. The modules work in parallel and interactive manner. The effectiveness of the technique will be demonstrated by simulation results on different sets of test images.

I. INTRODUCTION

A Image segmentation is an important research area of image processing as it has wide range of application like Medical Image Analysis, Satellite Image Analysis, Robotic vision, Surveillance, etc. The process of image segmentation can be defined as follow:

“The goal of image segmentation is domain independent partitioning of an image into a set of disjoint regions that are visually different, homogeneous and meaningful with respect to some characteristics or computed properties to enable easy image analysis.” [4]

Classical Definition of Image segmentation: “Let the image domain be $\Omega \in R$ and P_i be the partition of Ω , such that

$P_i \subset \Omega$, $\Omega = \bigcup_{i=1}^n P_i$ where, $\forall P_i$ and P_j adjacent where P_i

$\cap P_j = \phi$, for $i \neq j$, and each P_i is connected.” [4]

There are many techniques available for image segmentation such as thresholding, clustering, histogram, region findings, edge detection, split and merge, etc., but they all are either based on discontinuity among pixels or similarity among pixels. One another approach is based on integration of the strength of two different approaches to improve the performance of both the methods. Both Region based methods and Edge based methods have their advantages and disadvantages. It has seen that both problems are not identical. The region based method has

better noise tolerance properties, as noise is a high frequency component in image while region growing deals with homogeneity which is mostly low frequency components. The boundary based methods performs good on shape variation and change in gray level. So to achieve strength of both approaches an integration approach must be developed that can facilitate an interactive image segmentation.

A. Game Theory

Game theory [7] has capability to define strategy in conflicting situation and this important property is very useful in area of Image processing. Game theory is used in many areas of image processing, like Image denoising, Image Segmentation, etc. In Image Segmentation, game theory is used to facilitate parallel execution of edge detection module and region based module in iterative and interactive manner, and produce better segmentation results than either of the region based approach or edge detection approach. The main advantage is that it can bring together the region and boundary methods that operate in different probability spaces into a common information sharing framework.

A game is a formal description of a strategic situation and Game theory is the formal study of decision-making where several players must make choices that potentially affect the interests of the other players. A player is an agent who makes decisions in a game. In a game in strategic form, a strategy is one of the given possible actions of a player. In an extensive game, a strategy is a complete plan of choices, one for each decision point of the player. A Nash equilibrium, also called strategic equilibrium, is a list of strategies, one for each player, which has the property that no player can unilaterally change his strategy and get a better payoff. A payoff is a number, also called utility that shows the desirability of an outcome to a player, for whatever reason. When the outcome is random, payoffs are usually weighted with their probabilities. The expected payoff incorporates the player's attitude towards risk.

B. Rough set Theory

Rough set theory [3] [5] is a mathematical model for imperfect data analysis. In Rough set theory it is assumed that with every object of the universe some information or data is associated. Objects characterized by the same information are homogeneous (similar). The similar relation generated in this way is the mathematical basis of rough set theory.

Any set of all homogeneous objects is called an elementary set, and forms a basic granule (atom) of knowledge about the universe. Any union of some elementary sets is referred to as a crisp set – otherwise the

set is rough. Each rough set has boundary-line cases, i.e., objects which cannot be with certainly classified by employing the available knowledge, as members of the set or its complement. With any rough set a pair of precise sets, called the lower and the upper approximation of the rough set, is associated. The lower approximation consists of all objects which surely belong to the set and the upper approximation contains all objects which possibly belong to the set. The difference between the upper and the lower approximation constitutes the boundary region of the rough set. Approximations are fundamental concepts of rough set theory.

The lower approximation RX contains sets that are certainly included in X , and the upper approximation $\bar{R}X$ contains sets that are possibly included in X . R-positive, R-negative and R-boundary regions of X are respectively defined as follows.

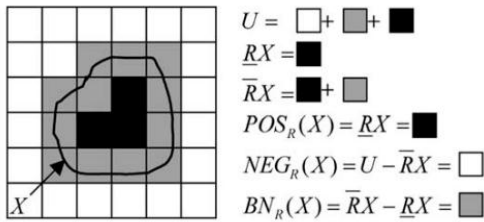


Fig. 1 : An illustrative example of the concept of approximation [3].

C. Expectation Maximization Algorithm

The Expectation Maximization Algorithm [8] provides statistical model of the data and handles the associated measurement and representation of uncertain data. Suppose, In an Image, if each class follows a particular probability density function, then any pixel in the image can be considered as sample drawn from the mixture of the individual class densities.

A Gaussian density function describing a finite mixture model with K component given by :

$$\phi(x^{(i)}, \mu_j, \sigma_j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \exp\left\{-\frac{(x^{(i)} - \mu_j)^2}{2\sigma_j^2}\right\} \quad (1.1)$$

where μ_j and σ_j are mean and standard deviation of j^{th} Gaussian density.

The individual component densities observed in many of the computer vision problems are often approximated by the Gaussian densities, due to which Gaussian Mixture Models (GMM) are the commonly used mixture models.

The parameters can be estimated by using Expectation Maximization Algorithm. The EM algorithm consists of two steps :

– *Expectation step:* In E-step, the hidden variables are estimated using the current estimate of the parameters of the component density.

$$w_h^j(x) = \frac{w_h^j f_h(x/\mu_h^j, \sigma_h^j)}{\sum_i w_i^j f_i(x/\mu_i^j, \sigma_i^j)} \quad (1.2)$$

– *Maximization Step:* Log likelihood function is maximized and parameters are updated.

$$w_h^{j+1} = \sum_{x \in D} w_h^j(x) \quad (1.3)$$

$$\mu_h^{j+1} = \frac{\sum_{x \in D} w_h^j(x)x}{\sum_{x \in D} w_h^j(x)} \quad (1.4)$$

$$\sigma_h^{j+1} = \frac{\sum_{x \in D} w_h^j(x)(x - \mu_h^{j+1})(x - \mu_h^{j+1})^T}{\sum_{x \in D} w_h^j(x)} \quad (1.5)$$

The E-M steps continue until convergence, that is $L(\phi^{t+1}) > L(\phi^t)$.

$$L(\phi) = \sum_{x \in D} \log\left(\sum_{h=1}^k w_h f_h(x/\mu_h, \sigma_h)\right) \quad (1.6)$$

The EM guarantee to converge to some local minimum. And optimal output is found when the above condition becomes false.

II. BASIC ARCHITECTURE

The proposed approach deals with integration of Rough-set based Boundary finding and Rough-set-Initialized-EM based Region growing, under Game theory framework.

Performing integration of region and boundary-based information within a game-theoretic framework shows a variety of advantages: [1]

First, both information sources are equally taken into consideration by the segmentation process, without computing the joint likelihood function.

Second, The benefit of adopting a decoupled scheme for information integration results to computational efficiency, since each module deals with the subset of the entire parameter set, which is related to the corresponding information domain.

Third, Robustness is achieved even when the quality of one of the sources is not high, since each module tends to pull the other away from noise and local maxima on one hand while pushing itself toward the global optimum on the other.

The proposed approach decomposes the entire architecture in three modules:

1. Rough-set based Boundary finding Module
2. Rough-set-Initialized-EM based Region growing Module
3. Game theoretic integration Module

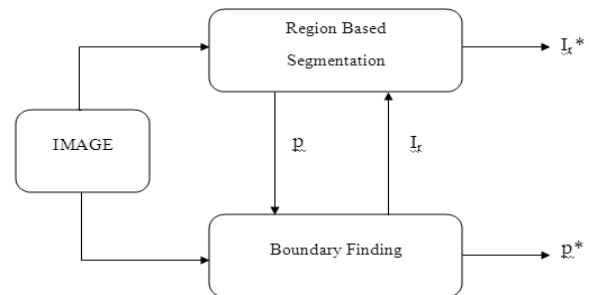


Fig. 2: Game-theoretic integration Framework for region-based segmentation and boundary finding [1].

III. ROUGH-SET-INITIALIZED-EM BASED REGION GROWING MODULE.

In Region growing process, Expectation Maximization algorithm is taken in to account to avoid over-segmented or under-segmented results and Rough-set theory is used to avoid initialization problem of EM algorithm and also helps to determine number of clusters automatically.

A. Discretization of Object Space

Discretization of object space is performed on gray-scale image using any suitable technique.

B. Generation of Rough-Set Reducts

From the discretized image, Rough-Set Reduct Table is generated. Suppose there are 'n' number of objects in a discretized image, so the Reduct table would have 'n' entries. The entire image pixels are categorized to a class or an object.

Now, in every class there are some pixels that are shared by other classes. Based on these common pixels cardinality of shared pixels is calculated for each class. According to the cardinality of the pixels, the classes are placed in descending order. eg. :

$$n_{k_1} > n_{k_2} > \dots > n_{k_m}$$

where, n is a class or object and k' is a cardinality of shared pixels for that class.

A heuristic threshold function is defined so that all entries having frequency less than a certain threshold Th (Here, we have taken Th = 0.5) are eliminated from table.

$$Tr = \left[\frac{\sum_{i=1}^m \frac{1}{n_{k_i} - n_{k_{i+1}}}}{Th} \right] \quad (3.1)$$

The value of Tr is high when small numbers of large clusters are present in image. On the other hand, if Tr attains a lower value, number of clusters with smaller size increases.

C. Mapping Reducts to Parameters for Rough-set based Initialization of EM method

From the Reduct table four parameters are calculated which will be used as Initial estimate for EM method. These four components are: Gaussian density Function, mean, weight and variance

1) *Number of Gaussians (k):* Number of classes or objects in discretized image.

2) *Component weight (w_h):* Weight of each Gaussian component.

$$w_h = \frac{\text{number_of_pixels_in_class}}{\text{total_number_of_pixels_in_entire_image}}$$

3) *Mean (μ):* Here, the pick value is considered as mean for each object or class.

4) *Variance (σ):* Variance of a Gaussian according to object space.

$$\sigma = \frac{\text{difference_between_two_consecutive_boundaries}}{2}$$

D. Optimization of class information using EM algorithm

In E-step, the shared or common pixels are estimated using the current estimate of parameters. Compute the membership probability of x in each cluster 1, 2, k.

$$w_h^j(x) = \frac{w_h^j f_h(x/\mu_h^j, \sigma_h^j)}{\sum_i w_i^j f_i(x/\mu_i^j, \sigma_i^j)} \quad (3.2)$$

In M-step, update the mixture model parameter for next iteration.

$$w_h^{j+1} = \sum_{x \in D} w_h^j(x) \quad (3.3)$$

$$\mu_h^{j+1} = \frac{\sum_{x \in D} w_h^j(x)x}{\sum_{x \in D} w_h^j(x)} \quad (3.4)$$

$$\sigma_h^{j+1} = \frac{\sum_{x \in D} w_h^j(x)(x - \mu_h^{j+1})(x - \mu_h^{j+1})^T}{\sum_{x \in D} w_h^j(x)} \quad (3.5)$$

The Log likelihood for each iteration is calculated, and when $L(\phi^{j+1}) > L(\phi^j)$. The condition is true to some local minimum. And optimal output is found when the above condition becomes false.

$$L(\phi) = \sum_{x \in D} \log \left(\sum_{h=1}^k w_h f_h(x/\mu_h, \sigma_h) \right) \quad (3.6)$$

IV. ROUGH-SET BASED BOUNDARY FINDING

The derivation of rough edge map [3] deals with finding thin boundary in image. All the objects in image consist of thin precise boundary. To derive these boundary information, granule based approach is used. Under granule based approach, granules of 2×2, 1×2 or 2×1 pixels are taken in to account. First of all, object boundary for each object is derived from processing histogram. After deriving object boundaries, the entire image is converted into binary image, this thresholding process converts the image in two parts, object of interest that is white, and rest of the entire image as background as black. After achieving threshold image, 2×2 size window is taken and moved on entire image. The logic is, if the window consist of all same valued pixels than it is considered as region and set black, and if all the pixels in window are not same than the set of pixels is considered as boundary and assigned white. After rotating window on entire image, resulting image consists of boundary information for particular object. This process is repeated for all objects and at last union of all the results is taken to get the boundary results. The results obtained using this process consists of 2×2 pixels thick boundary lines.

V. GAME THEORETIC INTEGRATION OF MODULES

This Parallel Game theory Decision Making framework is based on work done by Chakraborty and Duncan. The Game is between two segmentation that are required to be integrated. The game described here is two person game, P¹ is a set of strategies of Player 1, and P² is a set of strategies

of Player 2. Each player is trying to minimize their payoff function.

Here, the goal is to find Nash Equilibrium of the system (\bar{p}^1, \bar{p}^2) such that,

$$F^1(\bar{p}^1, \bar{p}^2) \leq F^1(p^1, \bar{p}^2) \quad (5.1)$$

$$F^2(\bar{p}^1, \bar{p}^2) \leq F^2(\bar{p}^1, p^2) \quad (5.2)$$

There always exists a Nash equilibrium if F1 and F2 are of the following form:

$$F^1(p^1, p^2) = f_1(p^1) + \alpha f_{21}(p^1, p^2) \quad (5.3)$$

$$F^2(p^1, p^2) = f_2(p^2) + \beta f_{12}(p^1, p^2) \quad (5.4)$$

where alpha and beta are some scaling constant.

A. Region Based Segmentation Influenced by Boundary finding

The image is partitioned into connected regions by growing neighboring pixels of similar intensity pixels. Adjacent regions are merged under some criterion involving homogeneity or sharpness of the boundaries.

$$F^1(p^1, p^2) = \min_x \left[\sum_{i,j} [y_{i,j} - x_{i,j}]^2 + \lambda \left(\sum_{i,j} (x_{i,j} - x_{i-1,j})^2 + \sum_{i,j} (x_{i,j} - x_{i,j-1})^2 \right) + \alpha \left[\sum_{(i,j) \in A_{\bar{p}}} (x_{i,j} - u)^2 + \sum_{(i,j) \in A_{\bar{p}}} (x_{i,j} - v)^2 \right] \right] \quad (5.5)$$

y = Intensity of original image

x = Segmented image given by P¹

u = Intensity of image inside contour given by P²

v = Intensity of image outside contour given by P²

In above objective function, in first term, the first summation is trying to minimize the difference between the classification and pixel intensity. In second part, the equation seeks to minimize the difference between the classifications of neighboring pixels to minimize the region boundary. This enforcing of smoothness constraint would force a single black pixel surrounded by white pixels to become white.

The second term is trying to match the region and contour. This contour information comes from Boundary finding module. The α is a coupling coefficient.

B. Boundary finding Influenced by Region Based segmentation:

$$F^2(p^1, p^2) = \arg \max_{\bar{p}} \left[M_{grd}(I_g, \bar{p}) + \beta M_{reg}(I_r, \bar{p}) \right] \quad (5.6)$$

\bar{p} = Parameterization of contour given by P²

I_g = gradient of image

I_r = Region segmented image obtained from x and P¹

M_{grd} = Measure of match between the gradient image I_g and contour given by \bar{p}

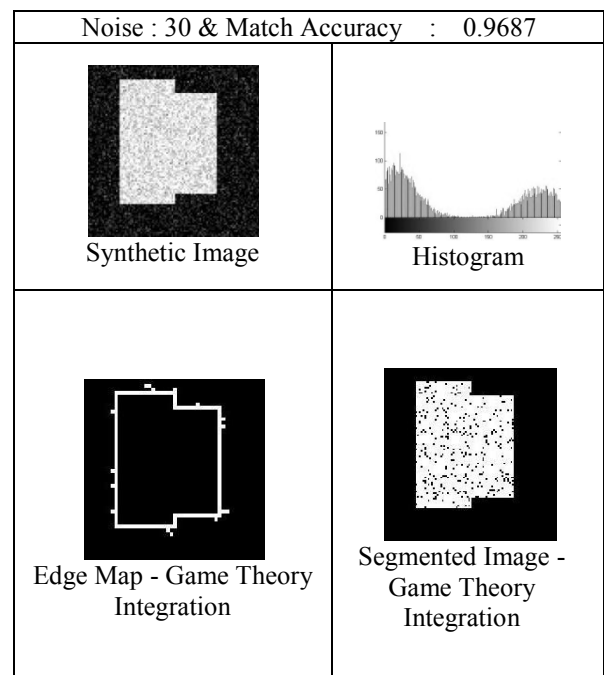
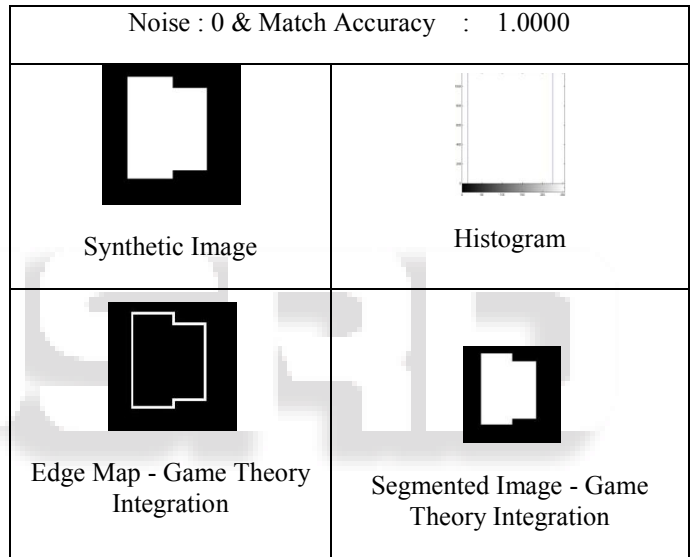
M_{reg} = Measure of match between region segmented image I_r and contour given by \bar{p} .

Here, the Objective function tries to maximize the gradient information. Gradient is calculated iteratively and where the sum of gradient is maximum the contour is defined as actual boundary. The prior information of shape, or boundary parameters are calculated from Region finding module.

VI. SYNTHETIC IMAGE - RESULTS

Image Parameters:

Lambda: 0.7; Alpha: 0.2; Beta: 0.2; W: 50;



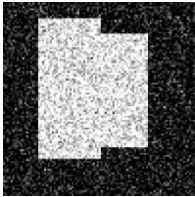
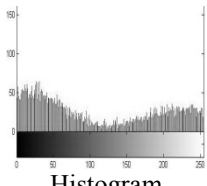
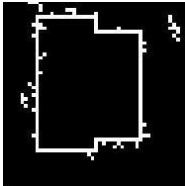
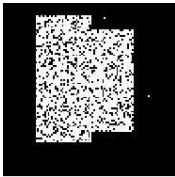
Noise : 50 & Match Accuracy : 0.9194	
 <p>Synthetic Image</p>	 <p>Histogram</p>
 <p>Edge Map - Game Theory Integration</p>	 <p>Segmented Image - Game Theory Integration</p>

Table. 1: Image Results

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