

Review Paper on Numerical Analysis of Natural Frequency of Functionally Graded Materials by ANSYS

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Abstract — This work provides a thorough investigation of the vibration analysis of a rectangular plate made of functionally graded material (FGM). Advanced composite materials known as Functionally Graded Materials (FGMs) are capable of withstanding high temperatures while minimizing thermal stresses. The design of FGM plates uses a variety of methods and theories. The primary goal of this work is to further the goals of scientists and engineers already engaged in the investigation and development of FGM plates.

Keywords: Free Vibration, Functionally Graded Plates

I. INTRODUCTION

Composite materials, due to their high strength to weight and stiffness to weight ratios, have been effectively employed in aviation and other technical applications for many years. Due to their benefits of being able to endure significant temperature gradients while preserving structural integrity, advanced composite materials, which are also known as functionally graded materials, have recently gained great interest in various technical applications. The mechanical characteristics of the functionally graded materials (FGMs), which are microscopically inhomogeneous, vary gradually and continuously from one surface to the next. In order to meet the important criterion of material qualities, they are often manufactured from a combination of ceramics and metals.

It is necessary to have a thorough grasp of FGMs' behaviours under various situations, including their deformable characteristics, stress distribution, natural frequency, and critical buckling load. As a result, research into the characteristics of FGM structures has been ongoing for quite some time. According to the literatures, these activities are done to create FGM plate structures without cracks or other faults. However, there may be certain faults or errors in the FGM or general plate structures during manufacture. Under cyclic pressure in service, fractures can develop and spread from the flaws. It is well known that plate constructions' fractures have an impact on their stability and dynamic responsiveness. They lower the plate structures' ability to support more weight.

II. LITERATURE REVIEW

According to Deepak S.A., Functionally graded materials are the non-homogeneous composite materials whose material properties vary along thickness from one surface to another surface. Within FGM, there are different layers of two or more materials having different functionality which will enhance the overall properties. When compared to individual metals or ceramics, FGM proves to be a better class of material. In this work, finite element analysis is carried out, in order to assess the static behaviour of functionally graded plates and free vibration analysis to ascertain the natural

frequencies. Simply supported (SSSS) and clamped (CCCC) plates are considered for the analysis. A rectangular plate is made up of a ceramic material at the top and a metallic surface at the bottom. Static and free vibration analysis shows that for a length to thickness ratio of above 20, the deflection of plates were lower compared to the lower thickness ratios [1].

According to Trung Thanh Tran, some numerical results from the free vibration analysis of functionally graded material (FGM) plates resting on the elastic foundation (EF) in the thermal environment are presented. An ES-FEM combining with the MITC3 is used to improve the accuracy as well as eliminate the shear locking phenomena for the classical triangular element. The EF is modeled from the Winkler-Pasternak foundation with stiffness two-parameter. Numerical examples are compared with published works to prove the reliability and accuracy of the proposed method. The effects of volume fraction index (n), temperature (T), and two-parameter of the EF on the free vibration of FGM plates are also investigated. Furthermore, an artificial neural network (ANN) is trained to predict exactly the fundamental frequency of FGM plates [2].

According to Pham Van Vinh, a modified single variable shear deformation plate theory is developed for free vibration analysis of rectangular functionally graded plates. By using the equilibrium equations of forces, the relationship between bending and shear parts is established, so that the displacement fields and the governing equation of the modified single variable plate theory contain only one unknown variable. As a consequence, the number of unknown variables is reduced, so it is effective for computation of solid mechanics. The advantage of proposed theory is that it consists of only one unknown variable but it is capable for analysis of heterogeneous plate such as functionally graded plates. The numerical results of the free vibration of functionally graded plates are carried out and compared with other published data to ensure the accurateness and effectiveness of this modification. The paper also explores the impact of some parameters on the free vibration of the functionally graded plates [3].

Mengzhen Li introduces a simple quasi-3D theory with Reddy shear function and a new trigonometric shear function to conduct free vibration analysis of the functionally graded plates resting on the Winkler/Pasternak/Kerr elastic foundation. The proposed transverse shear strain functions satisfy the stress-free boundary conditions on the surfaces of the functionally graded plate without using any shear correction factors. The governing differential equation and boundary conditions are derived based on Hamilton's principle and Winkler/Pasternak/Kerr type mathematical model. The material properties of FGM plates discussed in this paper are assumed to vary through the thickness according to the power-law distribution. The obtained results are compared with other 2D, quasi-3D, and exact 3D

solutions. The results show that the proposed trigonometric shear functions are more accurate than Reddy ones. Some interesting physical discussions of the recorded results are also given in this study [4].

According to Irwan Katili, a shear-locking free quadrilateral element with 5 DOFs per node, called Q4ys, is developed using a discrete shear projection method (DSPM). Independent transverse shear strains are formulated using a tangential coordinate system, and discrete shear strains are applied along each of the element sides to get the constant shear strains. These constant independent transverse shear strains are projected onto each corner node, and the cartesian shear strain components are interpolated over the element domain with standard bilinear shape functions. The applications of the element in static and free vibration analysis of functionally graded material plates show good convergence behavior, high reliability, and accuracy [5].

Vuong Nguyen VanDo intends to analyze free vibration response of functionally graded material (FGM) plates with complex cutouts. The isogeometric analysis (IGA) method combined with a new quasi-3D higher-order shear deformation theory (HSDT) for the vibration analysis is presented to predict the dynamic behaviour. The quasi-3D HSDT is able to account for transverse shear and normal deformations with only four unknown variables using the dissimilar shear and normal shape functions. In the refined quasi-3D hybrid type HSDT, the fully 3D material matrix is employed in the stress-strain relations, and the governing equations for the dynamic problem are derived through Hamilton's principle. The validity of the present quasi-3D isogeometric approach is investigated by testing several numerical examples in the open literature and comparing the predicted results with the available reference solutions. It can be concluded that the proposed analysis method is accurate and effective in solving free vibration behaviour of FGM plates with complex cutouts. Illustrative examples are also given to examine the effects that the ingredient fraction, plate geometric parameter, and boundary condition have on the free vibration behaviour. Results demonstrate that there exists a critical plate length-to-thickness ratio above which the natural frequency of the perforated FGM plate does not increase anymore even though the ratio becomes larger [6].

According to H.S.Yang and C.Y.Dong, the vibration and buckling analyses of the FGM (functionally graded material) plates with multiple internal cracks and cutouts under thermal and mechanical loads are numerically investigated using the combined XIGA-PHT (extended isogeometric analysis based on PHT-splines) and FCM (finite cell method). Material properties are graded only in the thickness direction. The effective material properties are estimated by using either the rule of mixture or the Mori-Tanaka homogenization technique. The plate displacement field is based on the HSDT (higher-order shear deformation plate theory) without any requirement of the SCF (shear correction factor). The HSDT model can exactly represent the shear stress distribution and improve the accuracy of solutions. The PHT-splines can naturally fulfill the C1-continuous requirement of the HSDT model. The representation of internal defects is mesh-independent. The discontinuous and singular phenomena induced by the cracks are captured using the enrichment pattern in the XIGA, and

the influence of cutouts is implemented by the FCM. The geometries of cutouts are captured by means of adaptive quadrature procedure based on a simple unfitted structural mesh, which avoids the need for multiple patches to describe the complex geometry and eliminates the enforcement of C1-continuity patch-coupling across the patch boundaries. The initial mesh density around the cracks and cutouts can be controlled flexibly utilizing the local refinement property of the PHT-splines. After validating the results of the developed approach with those available in the literature, the effects of material gradient index, side to thickness ratio, boundary conditions, cutout size, and crack length on the normalized frequency and the critical buckling parameter are investigated. Numerical results illustrate the effectiveness and accuracy of the present approach [7].

Agya pal Singh worked on a three-dimensional elasticity based approximate analytical solution for free vibration analysis of composite functionally graded Levy-type rectangular plates having an in-plane gradation of material properties. The density and elastic properties of the composite plates are considered to vary linearly along the x-direction. Modified Hamilton's principle has been applied to derive the weak form of governing equations in which all stresses and displacements consider primary variables to ensure the exact point-wise satisfaction of all interlaminar continuity and edge support conditions. The extended Kantorovich method, in conjunction with the Fourier series and power series approach, has been used to obtain the approximate solution in analytical form. New benchmark numerical results are presented for single-layered and multi-layered in-plane functionally graded rectangular plates. The influence of the in-plane heterogeneity, thickness ratio, and support conditions on the flexural frequencies and mode shapes of the plate are presented in detail. The present study shows that the free vibration response of the rectangular plate is affected significantly by the in-plane gradation of material properties, but the extent of influence largely depends on the edge support conditions of the plate as well as on configuration. The present benchmark solution can be used to validate other 3D numerical approaches and two-dimensional (2D) plate theories [8].

According to RoshanLal, free axisymmetric vibrations of functionally graded circular plates subjected to a non-linear temperature distribution along the thickness direction have been studied on the basis of classical plate theory. The top and bottom surfaces of the plate are under uniform thermal environment. The mechanical properties of the plate material are assumed to be temperature-dependent and the plate is graded in the thickness direction. The equations for thermo-elastic equilibrium as well as the axisymmetric motion for such a plate model have been derived using Hamilton's principle. Employing generalized differential quadrature rule, the numerical values of thermal displacements from thermo-elastic equilibrium equation and frequencies from the equation of motion for the lowest three modes of vibration have been computed for clamped, simply supported, and free plates. The effects of the temperature difference at the surfaces together with the material-graded index and thickness of the plate on the frequencies have been investigated. The benchmark results for uniform and linear temperature distributions have been computed. For the

validity of present considerations and the technique, the frequency parameter has been compared for some special cases with published results obtained from other approximate methods [9].

Ömer Civalek used two different numerical approaches for the frequency response of annular sector and sector plates with functionally graded materials and laminated composite cases. First-order shear deformation (FSDT) and Love's conical shell theories are used for obtaining the annular sector plate equations via some suitable angles and geometric parameters. The method of harmonic differential quadrature (HDQ) and discrete singular convolution (DSC) has been used for the numerical solution of the resulting governing equations for modal analysis. Simple power-law and four-parameter power-law distributions in terms of the volume fractions of constituents have been used for FGM composites. Comparison and convergence studies for the present numerical methods have been made via existing results available in the literature for the sector and annular sector plates. Frequency values for annular sector/sector plates have been obtained for different material and geometric parameters, boundary conditions, and sector angles [10].

Ankur Gupta presented an analytical model for nonlinear vibration analysis of variable thickness, thin isotropic, and functionally graded rectangular micro-plate containing a partial crack located within the center line of the plate. Linear and parabolic thickness variation is considered in one or both the in-plane directions of the plate. The thickness variation is such that the volume of the plate is equal to that of the uniform thickness plate. The continuous line crack is parallel to one of the edges of the plate. The equations of motion are derived using the equilibrium principle based on Classical Plate Theory while the size effect is incorporated using the modified couple stress theory. The partial crack is represented by bending moment and in-plane force according to the simplified line spring model. The effect of in-plane forces is considered by employing Berger's formulation. The derived governing equation is converted into a cubic nonlinear Duffing equation by employing Galerkin's method. The effect of nonlinearity is established by deriving the frequency response equation for the cracked variable thickness plate using the method of multiple scales. The nonlinear frequency response curves show the phenomenon of bending hardening or softening. The influence of crack length, crack location, internal material length scale parameter, boundary conditions, gradient index, and unidirectional and bidirectional taper constant on the fundamental frequency of the square plate is demonstrated. When compared to a uniform thickness plate, the effect of crack can be reduced by varying the thickness of the plate. Thus, it is concluded that in a given volume, it is better to employ a variable thickness plate as far as the vibration characteristics are considered [11].

In the paper of Nguyen Dinh Duc, the nonlinear dynamic response and vibration of a plate made of piezoelectric functionally graded materials (FGM) and reinforced by eccentrically outside stiffeners with piezoelectric actuators resting on Pasternak-type elastic foundations subjected to mechanical and electrical loads in the thermal environment are investigated. One of the salient

features of this work is the consideration of temperature on the piezoelectric layer, and stiffeners, and all of them (the plate, outside stiffeners, and piezoelectric layers) are deformed under temperature. The material properties of the FGM plates are assumed to be temperature dependent. The governing equations for the plates are derived by using the first-order shear deformation theory of plates, taking into account both the geometrical nonlinearity in the von Karman sense, initial geometrical imperfections, and different types of boundary conditions. The effects of geometrical parameters, material properties, imperfections, elastic foundations, stiffeners, thermal stress in stiffeners, mechanical and electrical loads, the temperature on the nonlinear dynamic response, and vibration of the piezoelectric FGM plates are studied simultaneously for the first time [12].

K. Swaminathan worked on a comprehensive review of the various methods employed to study the static, dynamic, and stability behavior of Functionally Graded Material (FGM) plates. Both analytical and numerical methods are considered. The review is carried out with an emphasis to present stress, vibration, and buckling characteristics of FGM plates predicted using different theories proposed by several researchers without considering the detailed mathematical implication of various methodologies. The effect of variation of material properties through the thickness, type of load case, boundary conditions, edge ratio, side-to-thickness ratio, and the effect of nonlinearity on the behavior of FGM plates are discussed. The main objective of this paper is to serve the interests of researchers and engineers already involved in the analysis and design of FGM structures [13].

In Ankit Gupta's paper, displacement-based new hyperbolic higher-order shear and normal deformation theory (HHSNDT) is introduced for the geometrically nonlinear vibrations response of FGM plates. The proposed theory accounts for nonlinear in-plane and transverse displacement through the plate thickness. Unlike any other theory, the number of unknown functions involved in the present theory is only four, as against five or higher in the case of other well-known shear deformation theories. It also accounts for the stretching effects across the thickness and does not require any shear correction factor. The fundamental equations of the FGM plate are obtained using variational principle and von Karman theory is employed for large transverse deflection. Voigt and Mori-Tanaka model is used with the conjunction of exponential law and power law to estimate the graded material properties. The accuracy of the present theory is ascertained by comparing it with various available results in the literature. A comprehensive numerical study is carried out based on the present theory to examine the influence of the homogenization techniques, geometrical parameters, amplitude ratio, and boundary conditions on the vibration response of the graded plate [14].

Hui-CuiLi presents a free vibration analysis of functionally graded material (FGM) plates that are partially submerged in an incompressible, inviscid fluid. The FGM plates with four gradient types of continuously varying material properties along the thickness direction, including the power law, exponential, sinusoidal, and cosine forms are studied to examine various distributions of material properties. The plate is modeled based on the Mindlin Plate

Theory (MPT) and the fluid loading effect on the FGM plates is modeled using the method of added mass. The variational principle is applied to derive the governing equations of this fluid-plate interaction system. The differential quadrature (DQ) method is used to solve this problem by converting the governing equations into a system of linear equations. The fundamental frequency and the corresponding mode shape are obtained using an iterative procedure. Numerical results for several examples are obtained and presented to investigate the vibration characteristics of the submerged FGM plates in terms of the gradient index, gradient type, immersed depth, fluid density, aspect ratio, and slenderness ratio. Results indicate that the larger aspect ratio and immersed depth increase the fundamental frequency of the FGM plate, while larger gradient index, fluid density, and slenderness ratio decrease the fundamental frequency. Among four different material gradient types, the FGM plate with power law type gradient has the smallest fundamental frequency, while the one with sinusoidal form has the largest value. The mode shape in fluid deviates from that in a vacuum and shows an unsymmetrical shape for CCCC and SSSS FGM plates [15].

Loc V. Tran proposed a novel and effective formulation that combines the eXtended IsoGeometric Approach (XIGA) and higher-order shear deformation theory (HSDT) to study the free vibration of cracked functionally graded material (FGM) plates. Herein, the general HSDT model with five unknown variables per node is applied for calculating the stiffness matrix without needing a shear correction factor (SCF). In order to model the discontinuous and singular phenomena in the cracked plates, IsoGeometric Analysis (IGA) utilizing the Non-Uniform Rational B-Spline (NURBS) functions is incorporated with enrichment functions through the partition of unity method. NURBS basis functions with their inherent arbitrary high-order smoothness permit the C1 requirement of the HSDT model. The material properties of the FGM plates vary continuously through the plate thickness according to an exponent function. The effects of gradient index, crack length, crack location, length to thickness on the natural frequencies, and mode shapes of simply supported and clamped FGM plate are studied. Numerical examples are provided to demonstrate the performance of the proposed method. The obtained results are in close comparison with other published solutions in the literature [16].

In the paper of Mohammad Talha, an extensive study of the free vibration and static analysis of square and rectangular functionally graded plates is presented, which is based on the higher order shear deformation theory with a special modification in the transverse displacement in conjunction with finite element models. The systems of algebraic equations are derived using variational approach for the free vibration and static problem. A C0 isoparametric Lagrangian element with 117 degrees of freedom per element is developed and implemented for both the problems. Convergence and validation studies have been carried out to inculcate the accuracy of the present formulation [17].

III. CONCLUSION

An overview of advanced developments in thermal stress, vibration, and buckling analysis of single / multi-layer FGM

and sandwich plates is presented. Most theories/computational models used for the analysis of composite laminates / isotropic plates are extended for FGM plates. Three-dimensional elasticity solutions proved to be the most accurate solutions for the thermal analysis of FGM plates. Due to the mathematical complexity, the investigations are limited to analytical methods with only simply supported edges or clamped edges. Most of the three-dimensional solutions reported on the stress analysis of FGM plates have assumed a three-dimensional distribution of temperature variations for thermal analysis, while vibration and kink analyzes indicate the one-dimensional variation of temperature distribution.

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