Bulk Viscous FRW Cosmological Models with Decaying Vacuum Energy

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Abstract— We have investigated FRW cosmological models with constant Ratio of bulk viscosity and matter density in general relativity. Functional relation on Hubble parameter is assumed, which yield constant value of deceleration parameter. The solution of field equations has been solved the best value for the Hubble parameter, energy density, isotropic pressure and cosmological term at present and at recombination. The physical and kinematical behavior of the model is also studied.

Keywords: FRW Model, Deceleration Parameter, Bulk Viscosity, Hubble Parameter

I. INTRODUCTION

The Einstein field equations are solved and analyzed separately for different epochs, although some author have given unified solutions. For instance, Carvalho [1] considered Friedmann–Robertson–Walker (FRW) model for perfect fluid in general relativity and presented a unified solution using variable adiabatic parameter 'gamma' of 'gamma-law' equation of state. In the present paper we consider the effects of bulk viscosity on the early evolution of universe for flat FRW model. The matter filling the cosmological (isotropic and homogeneous) background is functional relation on Hubble parameter discussed by a bulk viscous fluid having $\Lambda = 3\beta H^2$ and whose viscosity coefficient ζ is function of matter energy density ρ of the form $\zeta = \zeta_0 \rho$, where ζ_0 and β are positive numerical constant. We study the evolution of the universe as it goes from an inflationary phase to a radiation-dominated era. Our approach is based on Carvalho's [1] that studied the FRW models using functional form of gamma, depends on scale factor and presented a unified solution for two early phases of universe. Barrow [2] and Santos *et al* [3] have investigated FRW models with bulk viscous term. But both of them have given the solutions using equation of state with constant gamma.

Furthermore, there are several processes which are expected to give rise to viscous effects. These are the decoupling of neutrinos during the radiation era and the decoupling of radiation and matter during the recombination era. Bulk viscosity is associated with the GUT phase transition and string creation. Misner [4–6] and Weinberg [7, 8] have considered the effect of viscosity on the evolution of cosmological models. Due to such assumption, dissipative processes are supposed to play a fundamental role in the evolution of the early Universe. The theory of relativistic dissipative fluids, created by Eckart [9] and Landau and Lifshitz [10] has many drawbacks, and it is known that it is incorrect in several aspects mainly those concerning causality and stability. Israel [11] formulates a new theory in order to solve these drawbacks. This theory was later studied by Israel and Stewart [12] into what is called transient or extended irreversible thermodynamics. The best currently available theory for analyzing dissipative processes in the universe is the full causal thermodynamics studied by Israel and Stewart [10], Hiscock and Lindblom [13] and Hiscock and Salmonson [14]. The full causal bulk viscous thermodynamics has been extensively used to study the evolution of the early Universe and some astrophysical processes [15, 16]. However, due to the complicated nature of the evolution equations, very few exact cosmological solutions of the gravitational field equations are known in the framework of the full causal theory [17].

The paper paper, we studied the main components of bulk viscous fluid FRW cosmological model with cosmological term Λ, and introduce the assumptions. These assumptions bring us to study to origin and evolution of universe.

II. METRIC AND FIELD EQUATION:

We consider homogeneous and isotropic spatially flat Rabertson-Walker line element of the form

$$ds^2 = -dt^2 + s^2(t)(dx^2 + dy^2 + dz^2) \tag{1}$$

where S(t) is the scale factor.

The energy-momentum tensor for bulk viscous fluid is taken as

$$T_{ij} = (\rho + p)v_i v_j + \bar{p} g_{ij} \ , \tag{2}$$

where ρ is proper energy density and $\bar{p}$ is the effective pressure given by

$$\bar{p} = p - \xi v^i_{;i} \tag{3}$$

satisfying equation of state.

In the above equation p is the isotropic pressure and v^i is the four-velocity vector satisfying $v^i v_i = -1$. The Einstein field equations (in gravitational units 8 π G = c = 1) and varying cosmological constant Λ (t), in comoving system of coordinates to

$$\bar{p} - \Lambda = (2q-1)H^2 , \tag{4}$$

$$\rho + \Lambda = 3H^2 . \tag{5}$$

In the above equation, H is the Hubble parameter and q is the deceleration parameter defined as

$$H = \frac{\dot{S}}{S} , \tag{6}$$

$$q = -1 - \frac{\dot{H}}{H^2} = -\frac{S\ddot{S}}{\dot{S}^2} \tag{7}$$

where an overhead dot (.) represents ordinary derivative with respect to t. The vanishing divergence of Einstein tensor gives rise to

$$\dot{\rho} + 3(\rho + \bar{p})H + \dot{\Lambda} = 0 \tag{8}$$

III. Solution and Discussion:

The equations (4) – (5) are two equations involving five unknown terms S, ρ, Λ, p and ξ. Therefore, three more equation connecting these variables.

First, we choose functional relation on Hubble parameter

$$H = a S^{-m}, a > 0, m \geq 0 \tag{9}$$

From equations (6) and (9), we get

$$S = (maT)^{1/m} \tag{10}$$

where T = t + t'. (t' is constant of integration).

From equations (10) and (7), we get

$$q = m - 1 \tag{11}$$

Secondly, we consider

$$\Lambda = 3\beta H^2 \tag{12}$$

where β is constant [18, 19].

Next, we assume

$$\xi = \xi_0 \rho \tag{13}$$

ξ_0 being constant [20].

The line-element (1) becomes

$$ds^2 = -dt^2 + (maT)^{\frac{2}{m}}(dx^2 + dy^2 + dz^2) \tag{14}$$

Spatial volume V, Expansion scalar θ, Matter density ρ, Cosmological term Λ, Isotropic pressure p, Bulk viscosity ξ and the ratio Ω = Λ / ρ for the model (14) are given by

$$V = S^3 = (maT)^{3/m} \tag{15}$$

$$\theta = \frac{3}{mT} \tag{16}$$

$$\rho = \frac{3(1-\beta)}{m^2 T^2} \tag{17}$$

$$\Lambda = \frac{3^{\beta}}{m^2 T^2} \tag{18}$$

$$p = \frac{1}{m^2 T^2}\left[\frac{9(1-\beta)\xi_0}{nT} + (2m-1)\right] \tag{19}$$

$$\xi = \frac{3\xi_0(1-\beta)}{m^2 T^2} \tag{20}$$

$$\Omega = \frac{\beta}{1-\beta} \tag{21}$$

We notice that the scale factor S is zero and expansion scalar θ is infinite at T = 0, which shows that universe starts evolving with zero volume at T = 0 with big-bang. At T = 0, physical quantity ρ, Λ, V, p, ξ are all diverge. In the limit of large value of T i.e. T → ∞, the physical quantity ρ, Λ, θ, p, ξ becomes zero.

We observe that ρ / θ^2 and Ω are constant throughout the evolution of universe. vacuum energy is decreasing function of time.

IV. Conclusion:

In this paper, we have investigated homogeneous and isotropic spatially flat FRW cosmological models with functional relation on Hubble parameter and constant ratio matter density and bulk viscosity in the context of general relativity. We notice that q = m – 1 > 0, the universe is decelerating throughout the evolution. When m = 1, we obtain H = 1 / T and q = 0. Therefore, galaxies move with constant speed.

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