Mathematical Model for Inventory Optimization: A Fuzzy Integrated Approach

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Abstract— Inventory is always important for any type of business whether it comes for retails business or warehouses. Inventory is simply to keep a record of all the materials (Raw and furnished) inlet and outlets, and every kind of record which can be used in that business. Similarly optimization of this kind of inventory also plays an important role. If the inventory is not optimized then it may create difficulty to handle the business. Sometimes it may create un-ambiguities also. This paper is focused to analyze uncertainty in inventory management and optimization with the help of fuzzy integration. Optimization is achieved through total cost while the uncertainty is dealt with fuzzy sets. Fuzzy sets permit linguistic phrases handle in an accurate mathematical manner while crisp extents are essential for inventory management.

Keywords: Inventory Management, Uncertainty, Inventory Optimization. Fuzzy Set

I. Introduction

Manufacturing and supply [1] is completely related and both depend on each other. Whether it comes for automobiles, grocery, electronics, computers or any other business sector, there should be proper relationship between these two factors [2].

Uncertainty [3] regarding the control object [4] persists because the process of obtaining the essential information concerning the object is not always possible. There are different environments for the inventory [5]. Among them two have been discussed here:

Crisp Environment feasible for all the system parameters and the resources which are deterministic and precise. Fuzzy Environment which is a new artifact that has to associate through other features associated with the produce [6].

The Fuzzy logic scheme is competent of offering the most efficient explanation to intricate problems. The structure effortlessly used to enhance or adjust the process. The structure assists in handling uncertainties. "Fuzzy set theory" (FST) is principally fretful to quantitatively contract with indistinctness and ambiguity. Continuous fuzzy set 'A' is may be presented by below mentioned continuous membership functions (CMF) $\mu A\ (x)$.

A. Trapezoidal Membership Function (TMF):

A section sensible linear continuous function managed by four constraints $\{a, b, c, d\}$. Hence trapezoidal fuzzy number 'A' and its membership function are defined as follows:

$$\mu_A(x)=\begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & b \le x \le c \\ \frac{d-x}{d-c}, & c \le x \le d \\ 0, & x \ge d \end{cases} \qquad \boxed{x \in R} \tag{1}$$

B. Triangular Membership Function:

It is also section sensible linear function which is derived from the trapezoidal membership function. By considering $b = c$ whose membership function is defined as follows:

$$\mu_A(x)=\begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a \le x \le b \\ \frac{d-x}{d-b}, & b \le x \le d \\ 0, & x \ge d \end{cases} \qquad \boxed{x \in R} \tag{2}$$

Inventory verdicts are frequently completed separately by using Supply Chain (SC) [7] performers depending on the "local inventory status" (LIS) and "local performance objectives" (LPO) [8]. Uncertainty is a different essential concern to treaty in order to describe efficient SC inventory guidelines. Supply sometimes referred to as lead time (LT), insist or demand (D), sharing practices as well as inventory and backorder costs are typically uncertain [9]. In such cases uncertainty causes are sculpted by probabilistic allocations.

This paper addresses both integration of fuzzy logic uncertainty and inventory optimization through mathematical model to achieve total cost.

Uncertainty is resolved by modeling customer demand, holding and backorder costs [10] through ambiguous sets. Shorter and shorter production life cycles as well as rising innovation rates have made demand more variable and the collection of data required for random samples becomes more difficult. As a result, the ambiguous set theory by adapting administrative experience and judgment reveals that model uncertainty is more useful, more appropriate, and relatively more reliable [11]. Vague values should be converted to explicit numbers. This process is called diffusion. The ambiguous approach allows one to model uncertainty on the one hand and, on the other hand, to use the decisive equation proposed by Chen in 1998, until extended values are used in place of decisive values [12].

Finally, simulation analysis is performed to assess the benefits associated with rigorous integration between the SC phases.

Simulation requires the use of a probability distribution [13] to model an indeterminate variable.

However indefinite variables such as market demand and inventory-related costs are explained by membership rather than probability functions [14].

II. INVENTORY OPTIMIZATION

Inventory management and its optimization is imperative in all kind of businesses because it facilitate to prevent stock-out problems, control several sites, and guarantee precise documentations [15]. Inventory holding costs usually vary at different points in the supply chain, which requires an optimization model.

Optimization model designs can be created as:

$$\text{OP: min}TC = \sum_{i=0}^{m} C_i\, E[H_i] \quad (3)$$

$$\text{Solution to} \begin{cases} f(R) \geq fr, \\ R_i \in Z+,\ \ i = 0,1,2,3 \ldots .m \end{cases} \quad (4)$$

Here OP is a multidimensional discrete nonlinear optimization, fr represent required fill rate [16] and Z+ represents set of nonnegative integers, and f(R) represents achieved fill rate given the base-stock vector B.

The notation to be used in the subsections below.

1) Unit holding costs vector: $c= (c_0, c_1, c_2, \ldots c_m)$
2) Base-stock vector: $R =(R_0, R1, R_2, \ldots R_m)$
3) WIP vector: $H= (H_0, H_1, H_2, \ldots H_m)$
4) Achieved fill rate: $f(R)$
5) Upstream base-stock vector:
   $\tau_i= (R_0, R1, R_2, \ldots Ri)$; $0\leq i \leq m$
6) Downstream base-stock vector
   $\gamma_i= (R_i, R_{i+1}, R_{i+2}, \ldots Rm)$; $0\leq i \leq m$
7) Invariant upstream base-stock vector
   $\tau_i^0 = (R_0^0, R_1^0, R_2^0, \ldots . R_i^0)$; $0\leq i \leq m-1$
8) Downstream partial optimal expected total cost

$$h_i(\tau_{i-1})= \min \gamma_i \sum_{j=i}^{m} cj\ E[Hj\ (\tau i-1), \gamma i\ ]; \ i = 0,1,2,3 \ldots .m \quad (5)$$

Minimum total expected holding cost associated with the segment from node *i* through node *m*, given the base-stock levels from node 0 through node $i-1$.

Since the inventory held at stage *i* does not depend on $,\gamma_{i+1}$, we have

$$h_i(\tau_{i-1})= \min R_i; (E[Hi\ (\tau i-1), Ri] + h_{i+1}(\tau_{i-1}, R_i)) \ ; \ i = 0,1,2,3 \ldots .m-1 \quad (6)$$

9) Workload ρ

III. NUMERICAL VALUES OF INTERMEDIATE INVENTORY CONTROL

Demand (D) can be put up from the conventional demand term [17] which is based on numerous terms as mentioned below:

Demand Estimation (D) = Conventional Demand + Information' Impact+ Demand Vacillation + Random Error (Miscellaneous)

- Conventional demand is used for any inventory model.
- Information' Impact connects two significant features:
  1) Using correct information, and
  2) Using correct estimating model.
- Daily/weekly demand vacillation has negative impact on initial feature.
- Random error is hard to keep away.

Let us consider diverse arrangements of workloads (ρ) and cost parameters [18] as mentioned above. For every arrangement estimate the optimized "total inventory holding cost" without intermediary inventory control defined as *NOP* and denoted by $\boldsymbol{TC_{wo}^*}$. After that evaluate $\boldsymbol{TC_{wo}^*}$ with $\boldsymbol{TC_w^*}$.

It is noticeable that $\boldsymbol{TC_{wo}^*} \geq \boldsymbol{TC_w^*}$ in all cases.

Since the inventory holding cost and the fill rate are increasing in the base-stock level *R*, the cost optimization leads to the minimal base-stock level that meets the fill-rate requirement. Let R^* be the optimal base-stock level [19]. Because $f = (1\text{-}\rho^R)$, we have $f\,r \approx (1\text{-}\rho^{R^*})$. Hence, the optimal total inventory is

$$E[H] = E[R^*\text{-}N]^+ \quad (7)$$

$$= \sum_{i=0}^{R*-1} (\mathbf{R*-i})\mathbf{P}(\mathbf{N=i})$$

$$= \sum_{i=0}^{R*-1} (\mathbf{R*-i})\ (\mathbf{1}-\rho)\,\rho^i$$

$$= (\mathbf{1}-\rho)\ \rho^{R*}\rho^{-1} \quad (8)$$

Hence, the optimal total cost is proportional to $1/(\mathbf{1}-\rho)$ and it is quite similar to EOQ model.

IV. CONCLUSION:

Fuzzy integration is an approach to patchy dispensation which permits for various potential certain values to be practiced through the identical parameters. Fuzzy logic endeavors to resolve complexities with unlock, inexact range of information's and heuristics that put together it to attain exact outcomes. With this model the cost structure obtained is identical to standard EOQ model and it was assessed by fuzzy set. Fuzzy integrated model can be appropriately use in enterprise resource planning (ERP) structure as well as in inventory management if vendor can get the pertinent information for the every fuzzy function.

REFERENCES:

[1] Axsäter, S. 1990. Simple solution procedures for a class of twoechelon inventory problems. *Oper. Res.* 38 64–69.
[2] Axsäter, S. 1993. Continuous review policies for multi-level inventory systems with stochastic demand. S. Graves, A. H. G. R.
[3] Kan, P. H. Zipkin, eds. *Handbook in Operations Research and Management Science: Logistics of Production and Inventory*. North-Holland, Amsterdam, The Netherlands, 175–198.
[4] Brito Mp, Dekker R & Flapper Sdp. 2005. Reverse Logistics: A Review of Case Studies. In Distribution Logistics: Advanced Solutions To Practical Problems, Lecture Notes in Economics and Mathematical Systems, Bernhard Fleischmann, Andreas Klose Editors. 544: 243–272, Springer.
[5] Helmrich Mjr, Jans R, Heuvel Wvd& Wagelmans Apm. 2014. Economic lot sizing with remanufacturing: complexity and efficient formulations. *IIE Transactions, Special Issue: Scheduling & Logistics*, 46(1).
[6] Lage Juniorm & Godinho Filho M. 2012. Production planning and control for remanufacturing: literature review and analysis. *Production Planning & Control*, 23(6): 419–435.
[7] Reimannm & Zhang W. 2013. Joint optimization of new production, warranty servicing strategy and secondary

market supply under consumer returns. *Pesquisa Operacional*, 33(3), Rio de Janeiro, Setp/Dec 2013.
[8] Gotzel C & Inderfurth K. 2002. Performance of MRP in Product Recovery Systems with Demand, Return and Leadtime Uncertainties. In Quantitative Approaches to Distribution Logistics and Supply Chain Management, Lecture Notes in Economics and Mathematical Systems, 519: 99–114, Springer.
[9] Towill D.R., Supply chain dynamics. International Journal of Computer Integrated Manufacturing 4:4, pp 197–208. 1991.
[10] Li, Y., Tan Gek Woo., Information sharing in asupply chain with dynamic consumer demand pattern. Proceedings of the 37th Hawaii International Conference on System Sciences. 2004.
[11] Karwowski W., Evans G.W. (1986), “Fuzzy concepts in production management research: a review”, *International Journal of Production Research*, 24(1), 129-147.
[12] Haque, L., L. Liu, Y. Zhao. 2002. Tail asymptotics of a two-stage inventory-queue model.Working paper, School of Mathematics and Statistics, Carleton University, Ottawa, Canada.
[13] Zimmermann HJ. Description and optimization of fuzzy mathematical programming. Int. J General Syst. 1976; 2(4):209-215.
[14] Yao JS, Chiang J. Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. European Journal of Operational Research. 2003; 148:401-409.
[15] Salameh MK, Jaber MY. Economic production quantity model for items with imperfect quality. Int. Journal of Production Economics. 2000; 64:59-64.
[16] Nirmal Kumar Mandal, ET. al. (2005). Multi-objective fuzzy inventory model with three constraints: a geometric programming approach. Fuzzy sets and Systems, 150, 87-106.
[17] K. S. Park (1987). Fuzzy set theoretic interpretation of economic order quantity. IEEE Transactions on Systems and Cybernetics, 17 (6) 1082-1084.
[18] E. A. Silver, R. Peterson (1985). Decision Systems for Inventory Management and Production Planning, John Wiley, New York.
[19] H. Tanaka, T. Okuda and K. Asai (1974). On Fuzzy Mathematical Programming. Journal of Cybernetics, 3(4), 37-46.
[20] Patel, P.C., Sharma, B.K. (2019). An Advance Approach of Aggregation and Archival of Digital Forensic Information on Windows-Based Environment. Third International Congress on Information and Communication Technology. Advances in Intelligent Systems and Computing, vol-797. Springer, Singapore

