A Problem on Decision Making Through Soft Approximation Space

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Abstract — This paper aims at defining the idea of soft rough set in a modified manner for strengthening the mechanism to handle the problem of uncertainty since it is an effective mathematical tool for approaching various types of unpredictability. The connection between rough set and soft set is established during the course of derivation developing related properties pertinent to this concept. In-place of an equivalence relation, a soft set is used to break the universe in to innumerable fragments for identifying each objects which leads to an approximation space called soft approximation space. In consequence soft lower approximation operator, soft upper approximation operator is introduced. Some examples are presented giving interesting properties of soft rough approximations. A comparison between the proposed method and the previous one is accomplished. The concept of soft rough approximate inclusion relation is presented and its properties are derived. Finally, four types of soft covering based rough sets have been defined by using soft neighborhoods, soft cohesion and soft inverse neighborhood along with examining related properties. In addition, soft rough set is defined in a new way without using full soft set.

Keywords: Soft Set, Rough Set, Soft Approximation Space, Soft Covering Based Rough Sets, Soft Neighborhoods, Soft Cohesion and Soft Inverse Neighbourhood

I. INTRODUCTION

Over the years Philosophers and Scientists have been showing keen interest in vague concepts as most of the practical problems in social sciences engineering, economics, medical science, artificial intelligence, environmental science, complex problems in biology and other fields promoting uncertain data.

Many researchers have proposed variety of mathematical hypothesis to deal various types of vague concepts such as probability theory, Fuzzy set (Zedah 1965), Rough set (Pawlak 1982), Soft set (Malodtsov 1999)etc to extract the useful hidden information from ambiguous data. In 1965 Zedah proposed the concepts of fuzzy sets by using partial membership function to deal uncertain data. Subsequently in 1982 Z.Pawlak introduced the concept of “Rough set theory” by using equivalence relation. But it is very difficult to find imprecise human knowledge by using equivalence relation in case of real-life problems.

The concept of Rough set was based on the presumption that in the universe, every object is associated with some knowledge. The data and knowledge are represented in a data table. The data table containing rows arelabeled by objects and columns labeled by attributes.

A new mathematical tool Soft set to deal with uncertainties was initiated by Malodtsov in 1999. Maji et al.(2002) applied soft set theory to decision mak- ing problem. W.Wu et al.(2002),Y. Yao (1998),and W. zhu (2007)made ex- tensions of rough sets over binary relation. Z. Bonikowski, Z. Zhao (2016) and W. Zhu et al. proposed covering based rough sets in different manners. The covering based rough sets drew attention of many researchers to study as covering based rough set is the extension of classical pawlak rough set. Jiang et al.(2011) proposed an adjustable approach to intuitionistic Fuzzy softsets in decision making. A novel approach to interval valued intuitionistic Fuzzy soft sets in decision making was proposed by Zhanget al.(2014).Feng et al.(2010,2011) introduced soft sets combined with Fuzzy sets and rough sets. Sabir et al. (2013) introduced the notion of modified soft rough set .J. Zhan et al. (2017) and J. Zhan; K.Zhu (2017) proposed soft rough sets over hemirings and applied in decision making. S. Yijksel et al. (2014) and Yijksal et al. (2015) proposed soft covering based rough sets by combining soft sets and covering based on rough sets. J. Zhan et al. (2018) introduced a novel SRC model by means of soft neighbourhood and applied its application to multi-criteria group through decision making. R. J. Kanwal et al.(2019) applied fuzzy soft sets on semigroup based on soft relations. T. Xie et al.(2019) introduced hesitant soft fuzzy rough set model and its applications.J. C. R. Alcantud et al. (2020) introduced multi-granular soft rough covering models. An Introduction to rough soft sets was proposed by D. Mohanty et al.(2012). This article proposes the idea of soft rough set in a new manner. The main contribution of the present work is to introduce another model to soft rough sets without any restrictions and satisfies the properties discussed by Pawlak in rough set theory. The properties of these approximations are studied and their relationship is examined with counter examples. The contribution of rough set and soft set may be utilized to enlarge practical application on both sets .This article is organized as follows. In Section 2 the paper presents fundamental definitions of rough set and soft set theories. Section 3 presents the notion of soft rough set in different way. In second part, two practical examples are presented to illustrate the importance of suggested approximations and the comparison between the proposed method and other methods has been established. In section 4 Soft rough approximate inclusion relation is introduced and explained along with its properties .In section 5, soft rough set is modeled in a new way without using full soft set.

Moreover, this paper defines four types of soft covering based rough sets by using soft neighborhoods, soft cohesion and soft inverse neighborhood and studies related properties.

II. PRELIMINARIES

Definition 2.1(Pawlak 1982):

A pair (U,A)is called Knowledge representation system (Information system) over an universe U where A is a set of attributes on U(finite set)and each elements in the set A is a mapping on U to the set of values of that elements.

Definition 2.2(Pawlak 1982):

Let R be an equivalence relation on non-empty set of objects U(called Universe). The pair (U,R)is called approximate space .U/R denote family of equivalence classes of R and [x]R denote the equivalence class of x ∈ U by an equivalence relation (knowledge) R, that is [x]R ∈ U/R For any X ⊆ U define two sets

$\underline{R}X = \{x \in U | [x]_R \subseteq X\}$

$\overline{R}X = \{x \in U | [x]_R \cap X \neq \Phi\}$

be known as the R-lower and R-upper approximations of X respectively, If $\underline{R}X = \overline{R}X$ then X is definable otherwise X is Rough.

The positive, negative and boundary region of X⊆ U are defined by

$POS_R(X) = \underline{R}X$

$NEG_R(X) = U - \overline{R}X$

$BND_R(X) = \overline{R}X - \underline{R}X$

The accuracy of the approximation of X by R (percentage of possible correct decisions when classifying objects employing the knowledge R) written as

$$Y_R(X) = \frac{|\underline{R}X|}{|\overline{R}X|} \; where \; |\overline{R}X| \neq 0$$

Proposition 2.1(Pawlak 1982):

Pawlak's rough set approximations have the following properties

1a.) $\underline{R}X \subseteq X$	1b.) $X \subseteq \overline{R}X$
2a.) $\underline{R}\varphi = \varphi$	2b.) $\overline{R}\varphi = \varphi$
3a.) $\underline{R}U = U$	3b.) $\overline{R}U = U$
4a.) $\underline{R}(X \cap Y) = \underline{R}X \cap \underline{R}Y$	4b.) $\overline{R}(X \cap Y) \subseteq \overline{R}X \cap \overline{R}Y$
5a.) $\underline{R}(X \cup Y) \supseteq \underline{R}X \cup \underline{R}Y$	5b.) $\overline{R}(X \cup Y) = \overline{R}X \cup \overline{R}Y$
6a.) If $(X \cup Y)$then $\underline{R}X \subseteq \underline{R}Y$	6b.) If $(X \cup Y)$then $\overline{R}X \subseteq \overline{R}Y$
7a.) $\underline{R}X^C = (\underline{R}X)^C$	7b.) $\overline{R}X^C = (\overline{R}X)^C$
8a.) $\underline{R}\underline{R}X = \underline{R}X$	8b.) $\overline{R}\overline{R}X = \overline{R}X$
9a.) $\underline{R}\overline{R}X = \overline{R}X$	9b.) $\overline{R}\underline{R}X = \underline{R}X$
10a.) If $X \in U|R \; then \; \underline{R}X = X$	10b.) If $X \in U|R \; then \; \overline{R}X = X$

Definition 2.3(Molodtsov (1999))

A soft set over U is a pair like (F, A) where U be an universe of objects, A be the subset of certain parameter set E and F is a mapping from A to P(U) (power set of U)

Definition 2.4 (Feng et al. (2010))

A soft set (F, A) over U is called full soft set if

$$U = \bigcup_{a \in A} F(a)$$

Definition 2.5(Feng et al. (2011))

Let $S = (F, A)$ be a soft set over U. Then the pair $Q = (U, S)$ is called a soft approximation space. For any $X \subseteq U$, define two approximations

$\underline{Apr}_Q(X) = \{u \in U: \exists a \in A, u \in F(a) \subseteq X\}$

$\overline{Apr}_Q(X) = \{u \in U: \exists a \in A, u \in F(a) \cap X \neq \varphi\}$

for all $X \cup U$,are called lower Soft approximation and Upper lower Soft approximation of X. If $\underline{Apr}_Q(X) = \overline{Apr}_Q(X)$, X is called Soft definable on Q. Otherwise X is called Soft Rough Set on Q. Moreover the sets $POS_Q(X) = \underline{Apr}_Q(X), NEG_Q(X) = U - \overline{Apr}_Q(X)$and $BND_Q(X) = \overline{Apr}_Q(X) - \underline{Apr}_Q(X)$ are called soft positive region, soft negative region and soft boundary region of X respectively.

The accuracy of approximation

$$\gamma(X) = \frac{|\underline{Apr}_Q(X)|}{|\overline{Apr}_Q(X)|} \; where \; \overline{Apr}_Q(X) \neq \Phi$$

III. SOFT ROUGH APPROXIMATIONS

In this section the paper defines soft rough approximations and discusses its properties with many examples and counter examples .The paper has verified that the proposed method is an improvement to [7].

Definition 3.1:

Suppose S = (F, A) be a soft set over U. Then we say $Q = (U, S)$ is a soft approximation space. For any $X_1 \subseteq U$, define

$\underline{S}_Q(X_1) = \{v \in U: \exists e \in A, v \in F(e) \subseteq X_1\}$

$$\overline{S}_Q(X_1) = \left\{v \in U: \exists e \in A, v \in F(e) \; and \; F(e) \cap \left(X_1 - \underline{S}_Q(X_1)\right) \neq \varphi\right\} \cup \underline{S}_Q(X_1)$$

Two operators are called "Soft lower Q-approximation" and "Soft upper Q-approximation" of X_1 respectively.

Denote $\left(\underline{S}_Q(X), \overline{S}_Q(X)\right)$ as soft Q-rough approximation.

The soft Q-positive, Q-negative, Q-boundary regions and Q-accuracy of the "soft Q-approximations" are defined respectively by

$POS_Q(X_1) = \underline{S}_Q(X_1)$

$NEG_Q(X_1) = U - \overline{S}_Q(X_1)$

$BND_Q(X_1) = \overline{S}_Q(X_1) - \underline{S}_Q(X_1)$

If $\overline{S}_Q(X_1) = \underline{S}_Q(X_1)$ that is $BND_Q(X_1) = \varphi, \mu_Q(X_1) = 1$, then $X_1 \subseteq U$ is called Soft Q-definable or soft Q-exact set.

In general $\underline{S}_Q(X_1) \subseteq X_1, \underline{S}_Q(X_1) \subseteq \overline{S}_Q(X_1)$ for $X_1 \cup U$.

Example 3.1:

Let $U=\{v_1,v_2,\dots v_6\}, E=\{f_1,f_2,\dots,f_6\}, A=\{f_1,f_2,f_3,f_4\}$. Consider a soft set $S=(F,A)$ over U and $Q=(U,S)$ is a soft approximation space

	v_1	v_2	v_3	v_4	v_5	v_6
f_1	1	0	0	0	0	1
f_2	0	0	1	0	0	0
f_3	0	0	0	0	0	0
f_4	1	1	0	0	1	0

X	$\underline{S}_Q(X)$	$\overline{S}_Q(X)$	$BND_Q(X)$	$\mu_Q(X)$
$\{v_1,v_2,v_4\}$	φ	$\{v_1,v_2,v_5,v_6\}$	$\{v_1,v_2,v_5,v_6\}$	0
$\{v_3,v_4,v_5\}$	$\{v_3\}$	$\{v_1,v_2,v_3,v_5\}$	$\{v_1,v_2,v_5\}$	$\frac{1}{4}$
$\{v_1,v_2,v_5\}$	$\{v_1,v_2,v_5\}$	$\{v_1,v_2,v_5\}$	φ	1
$\{v_1,v_6\}$	$\{v_1,v_6\}$	$\{v_1,v_6\}$	φ	1

Table. 1: Soft set (F, A)

Remarks: We rewrite the lower and upper approximations for any $X\subseteq U$ as

$$\underline{S}_Q(X_1)=\bigcup_{e\in A}\{F(e)|F(e)\subseteq X_1\}$$

$$\overline{S}_Q(X_1)=\left(\bigcup_{e\in A}\left\{F(e)|F(e)\cap\left(X_1-\underline{S}_Q(X_1)\right)\neq\varphi\right\}\right)\cup\underline{S}_Q(X_1)$$

We note here that $X\subseteq\overline{S}_Q(X)$ may not true for some $X\subseteq U$.

Example 3.2:

Let $U=\{v_1,v_2,\dots v_8\}, E=\{f_1,f_2,\dots,f_6\}, A=\{f_1,f_2,f_3,f_4\}$. Consider a soft set $S=(F,A)$ over U and $Q=(U,S)$ is a soft approximation space

	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8
f_1	0	0	0	0	0	0	1	1
f_2	1	0	1	0	1	1	0	0
f_3	0	1	1	1	1	0	0	0
f_4	0	0	0	1	0	0	1	0

Comparison among the soft rough approximation [7] and generalized soft rough approximation (proposed method)

X	$\underline{Apr}_Q(X)$	$\underline{S}_Q(X)$	$\overline{Apr}_Q(X)$	$\overline{S}_Q(X)$
$\{v_4,v_7,v_8\}$	$\{v_4,v_7,v_8\}$	$\{v_4,v_7,v_8\}$	$\{v_2,v_3,v_4,v_5,v_7,v_8\}$	$\{v_4,v_7,v_8\}$
$\{v_1,v_2,v_3,v_4\}$	φ	φ	$\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$	$\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$
$\{v_2,v_3,v_4,v_5\}$	$\{v_2,v_3,v_4,v_5\}$	$\{v_2,v_3,v_4,v_5\}$	$\{v_1,v_2,v_3,v_4,v_5,v_6,v_7\}$	$\{v_2,v_3,v_4,v_5\}$
$\{v_4,v_7\}$	$\{v_4,v_7\}$	$\{v_4,v_7\}$	$\{v_2,v_3,v_4,v_5,v_6,v_7,v_8\}$	$\{v_4,v_7\}$

Table. 2: Soft set (F, A)

Remarks: From table 1 and 2 we can notice the following

1. There are many subsets of U which are soft rough according to [7] but they are soft exact according to proposed method. The proposed approaches generalized soft rough approximation represent the best tools for approximating the sets as the boundary regions decreased and the accuracy of proposed approach is better than[7].
2. There are some subsets which are soft rough set to [7] but they are soft exact to our method. So we can say that the proposed approaches are useful in shortening the vagueness of objects. Therefore our approaches will be useful in decision making for extracting information and help in removing the vagueness of data in real life problems.

Proposition 3.1:

Consider a soft set $S=(F,A)$ over U and $Q=(U,S)$ is a soft approximation space, then for any $T,V\subseteq U$

(i) $\underline{S}_Q(\varphi)=\overline{S}_Q(\varphi)=\varphi$

(ii) $\underline{S}_Q(U)=\overline{S}_Q(U)=\cup_{e\in A}F(e)$

(iii) $T\subseteq V\Rightarrow\underline{S}_Q(T)\subseteq\underline{S}_Q(V)$

(iv) $\underline{S}_Q(T\cap V)\subseteq\underline{S}_Q(T)\cap\underline{S}_Q(V)$

(v) $\underline{S}_Q(T\cup V)\subseteq\underline{S}_Q(T)\cup\underline{S}_Q(V)$

(vi) $T\subseteq V\Rightarrow\overline{S}_Q(T)\subseteq\overline{S}_Q(V)$

(vii) $\overline{S}_Q(T\cap V)\subseteq\overline{S}_Q(T)\cap\overline{S}_Q(V)$

(viii) $\overline{S}_Q(T\cup V)\subseteq\overline{S}_Q(T)\cup\overline{S}_Q(V)$

Proof: (iii) Let $v\in\underline{S}_Q(T)$ then $v\in F(e)$ and $F(e)\subseteq T$ for some $e\in A$ but $F(e)\subseteq T\subseteq V$, so that $v\in F(e)$ and $F(e)\subseteq V$ for some $e\in A$, so $v\in\underline{S}_Q(V)$ this implies $\underline{S}_Q(T)\subseteq\underline{S}_Q(V)$. Similarly we can prove others.

Proposition 3.2:

Let $S=(F,A)$ be a soft set over U and $Q=(U,S)$ is a soft approximation space, then for any $T\subseteq U$,T is 'Soft Q-definable' if $\overline{S}_Q(T)\subseteq T$.

Proof: Suppose T is 'Soft Q-definable' then $\underline{S}_Q(T)=\overline{S}_Q(T)$. We know that $\underline{S}_Q(T)\subseteq T$ implies $\overline{S}_Q(T)\subseteq T$.

Conversely, suppose $\overline{S}_Q(T)\subseteq T$ and it's trivially true that $\underline{S}_Q(T)\subseteq\overline{S}_Q(T)$, so only we need to prove $\overline{S}_Q(T)\subseteq\underline{S}_Q(T)$. Let $v\in\overline{S}_Q(T)$, then $\left[v\in F(e) and\ F(e)\cap\left(T-\underline{S}_Q(T)\right)\neq\varphi\right]$ or $\left[v\in\underline{S}_Q(T)\right]$ for some $e\in A$. That means $v\in F(e)$ and $F(e)\subseteq T$. So $v\in\underline{S}_Q(T)$ which implies $\overline{S}_Q(T)\subseteq\underline{S}_Q(T)$. Therefore $\underline{S}_Q(T)\subseteq\overline{S}_Q(T)$ and T is 'Soft Q-definable'.

Proposition 3.3:

Let $Q=(U,S)$ is a soft approximation space for the soft set $S=(F,A)$ on U, then the following conditions are equivalent

1) S is a Full Soft Set
2) $\underline{S}_Q(U)=U$
3) $\overline{S}_Q=U$

Example 3.3:

Let $U=\{v_1,v_2,\dots v_6\}, E=\{f_1,f_2,\dots,f_6\}, A=\{f_1,f_2,f_3,f_4\}$. Consider a soft set $S=(F,A)$ over U and $Q=(U,S)$ is a soft approximation space, where $F(f_1)=\{v_1,v_6\}, F(f_2)=\{v_3\}, F(f_3)=\varphi$ and $F(f_4)=\{v_1,v_2,v_5\}$, Let $X = \{v_3, v_4, v_5\}$ then $\underline{S}_Q(X) = \{v_3\}, X - \underline{S}_Q(X) = \{v_4, v_5\}$ and $\overline{S}_Q(X) = \{v_1, v_2, v_3, v_5\}$

$\underline{S}_Q\left(\overline{S}_Q(X)\right) = \{v_1, v_2, v_3, v_5\} = \overline{S}_Q(X)$

$\underline{S}_Q\left(\underline{S}_Q(X)\right) = \{v_3\} = \underline{S}_Q(X)$

If $Y = \underline{S}_Q(X) = \{v_3\}$ then $\underline{S}_Q(Y) = \{v_3\} = \overline{S}_Q(Y)$

Also, when $Z = \overline{S}_Q(X) = \{v_1, v_2, v_3, v_5\}$ then $\underline{S}_Q(Z) = \{v_1, v_2, v_3, v_5\} = \overline{S}_Q(Z)$

Remarks: If $Y = \underline{S}_Q(X)$ then, Y is an exact set and hence $\underline{S}_Q(Y) = Y = \overline{S}_Q(Y)$

i. e. $\overline{S}_Q\left(\underline{S}_Q(X)\right) = \underline{S}_Q(X) = \underline{S}_Q\left(\underline{S}_Q(X)\right)$

Similarly for any $X \subseteq U, \overline{S}_Q(X)$ is an exact set thus $\underline{S}_Q\left(\overline{S}_Q(X)\right) = \overline{S}_Q(X) = \overline{S}_Q\left(\overline{S}_Q(X)\right)$

Example 3.4:

Let $U = \{v_1, v_2, \dots, v_6\}$; $E = \{f_1, f_2, \dots, f_4\}$ and $A = \{f_1, f_2, f_4\}$.

Consider a soft set $S = (F, A)$ over U and $Q = (U, S)$ is a soft approximation space where $F(f_1) = \{v_1, v_3, v_6\}$, $F(f_2) = \{v_2, v_4, v_5\}$.

Let $X = \{v_2, v_3, v_4, v_5\}$, then $\underline{S}_Q(X) = \{v_2, v_4, v_5\}$, $\overline{S}_Q(X) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$.

Here $-X = \{v_1, v_6\}$, therefore $\underline{S}_Q(-X) = \{v_1, v_2, v_3, v_6\}$ which implies $-\underline{S}_Q(X) \subseteq \overline{S}_Q(-X)$ and $-\overline{S}_Q(X) \subseteq \underline{S}_Q(-X)$.

The above relation is true if S is a full soft set over U. If S is not a full soft set then the relation is not true. So we have a proposition in the following.

Proposition 3.4:

Let$S = (F, A)$ be a full soft set over U and $Q = (U, S)$ is a soft approximation space, the for any $T \subseteq U$, we have

(i) $-\underline{S}_Q(T) \subseteq \overline{S}_Q(-T)$

(ii)$-\overline{S}_Q(T) \subseteq \underline{S}_Q(-T)$

Proof: (i) Let $v \in -\underline{S}_Q(T)$ then $v \notin \overline{S}_Q(T)$. This implies $v \notin \cup_{e\in A}\{F(e)|F(e) \subseteq T\}$. So $v \notin F(e) \subseteq T$ for $e \in A$. That implies $(v \in F(e) \subseteq -T)$ or $\left(v \in F(e)\ and\ F(e) \cap \left(-T - \underline{S}_P(T)\right) \neq \phi\right)$ which shows that for $v \in U$ there exist some $e_0 \in A$ such that $v \in F(e_0)$ and $F(e_0) \cap \left(\left(-T - \underline{S}_P(T)\right) \neq \phi\right)$ or $(v \in F(e_0) \subseteq -T)$ (Since S is a full soft set, there exists some $e_0 \in A$ such that $v \in F(e_0)$). Therefore $v \in \cup_{e\in A}\{F(e)|F(e) \cap (-T - \overline{S}_Q(T)) \neq \phi\} \cup \underline{S}_Q(-T)$. This implies $v \in \overline{S}_Q(-T)$ and hence $-\underline{S}_Q(T) \subseteq \overline{S}_Q(-T)$.

Remarks: The following results is the relationship between definition 2.5 and definition 3.1 that is for $X \subseteq U$

(i) $\underline{Apr}_Q(X) = \underline{S}_Q(X)$

(ii) $\overline{S}_Q(X) \subseteq \overline{Apr}_Q(X)$

Proof: Let $u \notin \overline{Apr}_Q(X)$ then $u \notin F(e), F(e) \cap X \neq \varphi$. Which implies $u \in F(e), F(e) \cap X = \varphi$ therefore $u \in F(e), F(e) \cap \left(X - \underline{S}_Q(X)\right) = \varphi$ and $u \notin \underline{S}_Q(X)$.

Therefore $u \notin \overline{S}_Q(X)$ we get $\overline{S}_Q(X) \subseteq \overline{Apr}_Q(X)$.

Also we conclude from this remark as

(a) $BND_Q(X) \subseteq Bnd_Q(X)$

(b) $\gamma_Q(X) \leq \mu_Q(X)$

The suggested approximations satisfied most properties of Pawlak's rough set which are given below.

Properties	$\underline{S}_Q(X)$	Properties	$\overline{S}_Q(X)$
1a	satisfied	1b	satisfied
2a	satisfied	2b	satisfied
3a	not satisfied	3b	not satisfied
4a	not satisfied	4b	satisfied
5a	satisfied	5b	satisfied
6a	satisfied	6b	satisfied
7a	not satisfied	7b	not satisfied
8a	satisfied	8b	satisfied
9a	satisfied	9b	satisfied
10a	satisfied	10b	satisfied

Table. 3: Properties of Pawlak's rough set.

Remarks: The validity of proposed approximation is determined as it satisfies most properties of Pawlak's rough set without any restrictions.

The proposed approach might provide a finer estimate than other soft rough approximation did.

A. Decision making for Information data

In this section we introduce practical example as an applications. We use the interesting example to illustrate the importance of suggested method.

In recruitment process, a person who has more characteristics (capabilities/skills) will have greater chance to get a job. So we want to choose those objects which meet the more conditions.

Suppose $S = (F, A)$ is full soft set where $F(e) \neq \varphi, \forall \in A$ and $A = \{e_1, e_2, e_3, \dots, e_n\} \subseteq E$; $e_1, e_2, e_3, \dots, e_n$are characteristics of applicants $U = \{u_1, u_2, u_3, \dots, u_n\}$ $(u_1, u_2, u_3, \dots, u_n)$ *are n applicants* and $Q = (U, S)$ be a soft approximation space. Suppose that $I = \{I_1, I_2, I_3, \dots, I_m\}$ are m experts. Let $S_1 = (M, I)$ be a soft set defined over U ; $M(I_i)$ represent the report of experts I_i.

Algorithm

An algorithm for decision making problems through soft rough approximations.

1. Input$U$, a full soft set $S = (F, A)$(where set A (choice of parameters) is asubset of parameter set E) and soft approximation space $Q = (U, S)$.
2. Input soft set $S_1 = (M, I)$

3. Compute soft lower Q-approximation and soft upper Q-approximation for every $M(I_i)$
4. Compute boundary region and accuracy of approximation for every $M(I_i)$
5. Find maximum value of $\mu_Q\big(M(I_i)\big)$
6. The final decision is I_i in step 5

Example 3.5:

A Company advertises for three posts and appointed three experts (I_1, I_2, I_3) for selection. Let $U = \{p_1, p_2, p_3, \dots, p_6\}$ be the set of six applicants to take part in the interview and $A = \{e_1, e_2, \dots, e_8\}$ is the set of characters of the candidates where e_1 represent for young , e_2 represent for educational background, e_3 represent for experience, e_4 represent for language proficiency, e_5 represent for strain capability, e_6 represent for technical knowledge, e_7 represent for diffuse skill and e_8 represent for tolerance.

Consider the soft set $S = (F, A)$ that describes characteristics of 6 applicants in table 3.The results in the form of soft set $S_1 = (M, I)$ by three experts I_1, I_2, I_3 are given in table 4.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
p_1	1	1	1	1	1	1	0	0
p_2	1	1	1	0	1	0	1	1
p_3	1	1	1	1	0	0	1	1
p_4	1	1	1	0	0	1	0	1
p_5	1	0	1	0	0	1	0	0
p_6	1	1	1	1	1	0	0	0

Table. 3: Tabular representation for soft set (F, A)

	I_1	I_2	I_3
p_1	1	1	0
p_2	0	1	0
p_3	1	1	1
p_4	0	1	1
p_5	0	0	1
p_6	1	0	1

Table. 4: Tabular representation for soft set $S_1 = (M, I)$ (we define 1 for yes and 0 for no)

Here, $M(I_1) = \{p_1, p_3, p_6\}$; $M(I_2) = \{p_2, p_3, p_4\}$; $M(I_3) = \{p_4, p_5, p_6\}$

So,

$\underline{S}_Q\big(M(I_1)\big) = \{p_1, p_3, p_6\}$; $\underline{S}_Q\big(M(I_2)\big) = \{p_2, p_3, p_4\}$;

$\underline{S}_Q\big(M(I_3)\big) = \varphi$

$\overline{S}_Q\big(M(I_1)\big) = \{p_1, p_3, p_6\}$; $\overline{S}_Q\big(M(I_2)\big) = \{p_1, p_2, p_3, p_4, p_5, p_6\}$;

$\overline{S}_Q\big(M(I_3)\big) = \{p_1, p_2, p_3, p_4, p_5, p_6\}$;

$BND_Q\big(M(I_1)\big) = \varphi$; $BND_Q\big(M(I_2)\big) = \{p_1, p_5, p_6\}$

$BND_Q\big(M(I_3)\big) = \{p_1, p_2, p_3, p_4, p_5, p_6\}$;

$\gamma_Q\big(M(I_1)\big) = 1$; $\gamma_Q\big(M(I_2)\big) = \frac{1}{2}$; $\gamma_Q\big(M(I_3)\big) = 0$

Since, $\gamma_Q\big(M(I_1)\big) > \gamma_Q\big(M(I_2)\big) > \gamma_Q\big(M(I_3)\big)$

So we choose I_1 , i.e. p_1, p_3, p_6 are perfect candidates for that post. The expert I_1 is more capable than I_2 *and* I_3. He makes perfect judgment among three experts with a high degree of responsibility and judgment.

IV. SOFT ROUGH APPROXIMATE INCLUSION RELATION

Definition 4.1:

Consider a soft set $S = (F, A)$ over U and $Q = (U, S)$ is a soft approximation space, for $T, V \subseteq U$ define

$T \subseteq_Q$ V if $\underline{S}_Q(T) \subseteq \underline{S}_Q(V)$

$T \sqsubset_Q$ V if $\overline{S}_Q(T) \subseteq \overline{S}_Q(V)$

$T \sqsubseteq_Q V$ if T $\subseteq_Q V$ and T $\sqsubset_Q V$

The relations $\subseteq_Q, \sqsubset_Q$ and $\sqsubseteq_Q$ are called soft lower inclusion relation, soft upper inclusion relation and soft inclusion relation respectively.

Proposition 4.1:

$\subseteq_Q, \sqsubset_Q$ and $\sqsubseteq_Q$ are quasi ordering relation on Q.

Definition 4.2:

Consider a soft set $S = (F, A)$ over U and $Q = (U, S)$ is a soft approximation space,

For $T, V \subseteq U$ define

$T \eqsim_Q V$ if $\underline{S}_Q(T) = \underline{S}_Q(V)$

$T \simeq_Q$ V if $\overline{S}_Q(T) = \overline{S}_Q(V)$

$T \approx_Q V$ if $T \simeq_Q Y$ and $T \eqsim_Q V$

The relations $\eqsim_Q$, $\simeq_Q$ and $\approx_Q$ are called soft lower rough approximate equal relation, soft upper rough approximate equal relation and soft Q-rough approximate equal relation respectively.

Proposition 4.2:

Consider a soft set $S = (F, A)$ over U and $Q = (U, S)$ is a soft approximation space, For $T, V \subseteq U$ we have

1) If $T \subseteq_Q V$ then $T \subseteq_Q V, T \sqsubset_Q V, T \sqsubseteq_Q V$
2) If $T \subseteq_Q V$ and $V \subseteq_Q T$ then $T \eqsim_Q V$
3) If $T \sqsubset_Q V$ and $V \sqsubset_Q T$ then $T \simeq_Q V$
4) If $T \sqsubseteq_Q V$ and $V \sqsubseteq_Q T$ then $T \approx_Q V$
5) If $T \sqsubset_Q V$ if $T \cup V \simeq_Q V$
6) If $T \subseteq_Q V$ if $T \cap V \eqsim_Q V$
7) If $T \subseteq V, T \eqsim_Q T'$and $V \eqsim_Q V'$then $T' \subseteq_Q V'$
8) If $T \subseteq V, T \simeq_Q T'$and $V \simeq_Q V'$then $T' \sqsubset_Q V'$
9) If $T \subseteq V, T \approx_Q T'$and $V \approx_Q V'$then $T' \sqsubseteq_Q V'$
10) If $T' \sqsubset_Q T$ and $V' \sqsubset_Q V$ then $T' \cup V' \sqsubset_Q T \cup V$
11) If $T' \subseteq_Q T$ and $V' \subseteq_Q V$ then $T' \cap V' \sqsubset_Q T \cap V$
12) $T \cap V \subseteq_Q T \sqsubset_Q T \cup V$
13) If $T \subseteq_Q V$ and $T \eqsim_Q W$ then $W \subseteq_Q V$
14) If $T \sqsubset_Q V$ and $T \simeq_Q W$ then $W \sqsubset_Q V$
15) If $T \sqsubseteq_Q V$ and $T \approx_Q W$ then $W \sqsubseteq_Q V$

Definition 4.3:

Let $S = (F, A)$be a soft set over U,S is said to be total soft set if for any $e_1, e_2 \in A$ there exist $e_3 \in A$ such that $F(e_3) = F(e_1) \cap F(e_3)$ and $F(e_3) \neq \phi$.

Preposition 4.3:

$\simeq_P, \overline{\sim}_P$ and $\approx_P$ are equivalence relations over $P(U)$

Preposition 4.4:

Let $(Q = (U, S)$ is a soft approximation space, then for all $T, V\ T_1, V_1 \subseteq U$ the following condition holds

(i) $T \simeq_Q V$ if $T \simeq_Q T \cup V \simeq_Q V$
(ii) $T \simeq_Q T_1, V \simeq_Q V_1 \Rightarrow T \cup V \simeq_Q T_1 \cup V_1$
(iii) $T \simeq_Q V \Rightarrow T \cup (-V) \simeq_Q U$
(iv) $T \subseteq V; V \simeq_Q \phi \Rightarrow T \simeq_Q \phi$
(v) $T \subseteq V; T \simeq_Q U \Rightarrow V \simeq_Q U$
(vi) $T \subseteq V; V \overline{\sim}_Q \phi \Rightarrow T \overline{\sim}_Q \phi$
(vii) $T \subseteq V; V \overline{\sim}_Q U \Rightarrow V \overline{\sim}_Q U$

Proposition 4.5: Let $Q = (U, S)$is a soft approximation space, then for all $T, V\ T_1, V_1 \subseteq U$ we have

(i) $\underline{S}_Q(T \cap V) = \underline{S}_Q(T) \cap \underline{S}_Q(V)$ but if $\underline{S}_Q(T \cap V) = \underline{S}_Q(T) \cap \underline{S}_Q(V)$then S be a total soft set over U is not true.
(ii) $T \overline{\sim}_Q V$ if $T \overline{\sim}_Q T \cap V \overline{\sim}_Q V$
(iii) $T \overline{\sim}_Q T_1, V \overline{\sim}_Q V_1 \Rightarrow T \cap V \overline{\sim}_Q T_1 \cap V_1$
(iv) $T \overline{\sim}_Q V \Rightarrow T \cap (-V) \overline{\sim}_Q U$

V. SOFT COVERING BASED ROUGH SET

In this section, we have established the definition of soft rough set in a new manner given in subsection 5(A). We claim here the model provides in 5.1 be the most appropriate definition of soft rough set to capture the uncertain concepts. In next subsection 5(B),four types of soft covering based rough sets are established ,for clarity see ([38],[39],J. Zhan et al and [40]J. C. Alcantud et al)

A. Soft rough set

Definition 5.1:

Let $S = (F, A)$ be a soft set over U and $Q = (U, S)$ is a soft approximation space, then for $X_1 \subseteq U$ define $LS_Q(X_1) = \{v \in U: \exists e \in A, v \in F(e) \subseteq X_1\}$ and

$$HS_Q(X_1) = \begin{cases} J & if\ X_1 \subseteq J \\ J \cup K & if\ X_1 \nsubseteq J \end{cases}$$

Where

$$J = \{v \in U: \exists e \in A, u \in F(e) \cap (X_1 - LS_Q(X_1) \neq \phi\} \cup LS_Q(X_1)$$

$$K = \bigcap_{e \in A} \{(U - F(e)): e \in A\}$$

$LS_Q(X_1)$ and $HS_Q(X_1)$ are called "lower Soft Q-approximation" and "upper Soft Q-approximation" of X_1 respectively.

If $LS_Q(X_1) = HS_Q(X_1)$, then X_1 is called soft Q-definable (or soft Q-exact set) otherwise $X_1$is called soft Q-rough set.

Here $LS_Q(X_1) \subseteq X_1; LS_Q(X_1) \subseteq HS_Q(X_1)$and $X_1 \subseteq HS_Q(X_1)$ for all $X_1 \subseteq U$.

Example 5.1:

In example 3.1 if $T = \{v_3, v_4, v_5\}$ then $LS_Q(T) = \{v_3\}, J = \{v_1, v_2, v_3, v_5\}$ and $K = (U - F(f_1)) \cap (U - F(f_2)) \cap (U - F(f_3)) \cap (U - F(f_4)) = \{v_2, v_3, v_4, v_5\} \cap \{v_1, v_2, v_4, v_5, v_6\} \cap \{v_1, v_2, v_3, v_4, v_5, v_6\} \cap \{v_3, v_4, v_6\} = \{v_4\}$

$HS_Q(T) = J \cup K = \{v_1, v_2, v_3, v_4, v_5\}$.

Since $T \nsubseteq J$, so $LS_Q(T) \subseteq T, LS_Q(T) \subseteq HS_Q(T)$ and $T \subseteq HS_Q(T)$ but $\underline{Apr}_Q(T) = \{v_3\}, \overline{Apr}_Q(T) = \{v_1, v_2, v_3, v_5\}$ and $T \nsubseteq \overline{Apr}_Q(T)$.

Remarks: According to previous definition of soft rough set, if S is a full soft set then $\underline{Apr}_Q(T) \subseteq T \subseteq \overline{Apr}_Q(T)$hold. But according tour new definition 5.1 of soft rough set model, $\underline{LS}_Q(T) \subseteq T \subseteq \overline{HS}_Q(T)$ holds for all $T \subseteq U$, there is no requirement of full soft set. Hence we claim the definition 5.1 for soft rough set be the one of the best model to handle the uncertainty.

B. Roughness on soft covering

Now we write the covering of the universe U and then softness on covering of U .Also we introduce soft approximation space on covering of U. We extend the concept of soft covering based rough set written earlier.

Definition 5.2:

[41] Let U be a nonempty finite set called universe and $S = \{p_1, p_2, \ldots, p_n\}$ be a collection of subsets of U. If $p_1 \cup p_2 \cup p_3 \cup \ldots \ldots \cup p_n = U$then S is said to be covering of U. The pair (U, S) is called covering approximation space.

Definition 5.3:

[41] Let E be the set of parameters, $A \subseteq E$ and F is a mapping from A to P(U) (power set of U) such that $F(e) = \cup p_i$ for some i and $e \in A$. If $\cup_{e \in A} F(e) = U$ then (F, A) is called soft set on covering of U and $(U, C, F, A)$is called soft approximation space on covering of U.

Definition 5.4:

Let (F, A) be a soft set on covering of U and $Q = (U, C, F, A)$ be a soft approximation space, for any $X \subseteq U$ define soft covering lower approximation and soft covering upper approximation as

$$\underline{TC}_Q(X) = \{u \in U: \exists e \in A, u \in F(e) \subseteq X\}$$

$$\overline{TC}_Q(X) = \Big\{u \in U: \exists e \in A, u \in F(e) and\ F(e) \cap \big(X - \underline{TC}_Q(X)\big) \neq \phi\Big\} \cup \underline{TC}_Q(X)$$

If $\underline{TC}_Q(X) = \overline{TC}_Q(X)$then X is said to be soft covering based rough set, otherwise X is called soft covering based definable.

Example 5.2:

Let $U = \{u_1, u_2, u_3, \ldots \ldots, u_8\}$ be the universe consisting of 8 models of smart phone for sale.

Suppose $S = \{p_1, p_2, p_3, \dots\dots, p_6\}$be a covering of U where $p_1 = \{u_1, u_3\}, p_2 = \{u_4, u_5\}, p_3 = \{u_2, u_6\}, p_4 = \{u_1, u_7, u_8\}, p_5 = \{u_3, u_4, u_5\}, p_6 = \{u_8\}$ and $p_1 \cup p_2 \cup p_3 \cup \dots\dots \cup p_6 = U$. Let $A = \{e_1, e_2, e_3, \dots\dots, e_6\} \subseteq E$ be a set of parameters where e_1 stands for infrared remote control, e_2 stands for crystal clear display, e_3 stands for plenty of storage space, e_4 stands for long lasting battery, e_5 stands for high power camera, e_6 stands for multiple windows.

Let us consider a mapping $F: A \to P(U)$be such that $F(e_1) = p_1 = \{u_1, u_3\}, F(e_2) = p_2 \cup p_3 = \{u_2, u_4, u_5, u_6\}$, $F(e_3) = p_4 = \{u_1, u_7, u_8\}, F(e_4) = p_1 \cup p_2 = \{u_1, u_3, u_4, u_5\}, F(e_5) = p_6 = \{u_8\}$, $F(e_6) = p_1 \cup p_6 = \{u_1, u_3, u_8\}$. If $X = \{u_1, u_3, u_7\}$ then $\underline{TC_Q}(X) = \{u_1, u_3\}$ and $\overline{TC_Q}(X) = \{u_1, u_3, u_7, u_8\}$.

If $Y = \{u_2, u_5, u_6\}$ then $\underline{TC_Q}(Y) = \phi$ and $\overline{TC_Q}(X) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$

If $Z = \{u_1, u_7, u_8\}$ then $\underline{TC_Q}(Z) = \{u_1, u_7, u_8\}$ and $\overline{TC_Q}(Z) = \{u_1, u_7, u_8\}$

(1) The definition of soft covering on U and soft covering approximation space is given from which we derive three types of neighborhood operators. Based on neighborhood operations we define three numbers of soft covering rough sets. Feng et al.2010, 2011[29] and yuksel et al. 2014, 2015 (1) A full soft set S=(F,A) over U is called soft covering if $F(e) \neq \phi\ \forall e \in A$. It is denoted by symbol C_S.

(2) If $S = (F, A)$ be a soft cover over U, the order pair $P = (U, C_S)$ is called soft covering approximation space.

(3) Suppose $S = (F, A)$be a soft covering over U, S is called partial soft covering (PSC) over U=if for each a,$a' \in A$ with a $\neq a'$,F(a) $\cap F(a) = \phi$

Definition 5.5:

Let $P = (U, C_S)$ be a soft covering approximation space. For any $u \in U$, define $A_P(u) = \{x \in U: u \in F(e_i) and\ x \in\cap F(e_i)\ \forall e_i \in A\}$ is called soft neighborhood of u.

Definition 5.6:

Let $P = (U, C_S)$ be a soft covering approximation space. For any $u \in U$, define $A_P(u) = \{x \in U: C_P(u) = C_P(x)\}$ is called soft cohesion of u.

Definition 5.7:

Let $S = (F, A)$ be a soft covering over U and $\psi: U \to P(A)$ be a mapping defined as $\psi(u) = \{e: u \in F(e)\}$ and $P = (U, C_S)$ be a soft covering approximation space, for any $u \in U$; define $IN_P(u) = \{t \in U: \psi(t) \subseteq \psi(u)\}$ is called soft inverse neighborhood of $u \in U$.

Example 5.3:

Let $U = \{p_1, p_2, p_3, \dots\dots, p_6\}$ be the set of six candidates to take part for an interview of a company and $B = \{e_1, e_2, e_3, \dots\dots, e_6\}$ denotes six characters of the candidates where e_1 represent for educational background; e_2 represent for experience; e_3 represent for technical knowledge; e_4 represent for smart; e_5 represent for managerial skill and e_6 represent for young.

Consider the soft set $S = (F, B)$ be the descriptions of the 6 candidates in following table.

	e_1	e_2	e_3	e_4	e_5	e_6
u_1	1	1	1	1	1	1
u_2	1	1	1	0	1	0
u_3	1	1	1	1	0	0
u_4	1	1	1	0	0	1
u_5	1	0	1	0	0	1
u_6	1	1	1	1	1	0

Tabular representation for soft set (F, B)

Here

$F(e_1) = \{u_1, u_2, u_4, u_5, u_6\}$; $F(e_2) = \{u_1, u_3, u_5, u_6\}$;

$F(e_3) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$; $F(e_4) = \{u_1, u_2, u_3, u_5, u_6\}$;

$F(e_5) = \{u_2, u_5, u_6\}$; $F(e_6) = \{u_1, u_3, u_4, u_5, u_6\}$;

$\psi(u_1) = \{e_1, e_2, e_3, e_4, u_6\}$; $\psi(u_2) = \{e_1, e_3, e_4, u_5\}$;

$\psi(u_3) = \{e_2, e_3, e_4, u_6\}$; $\psi(u_4) = \{e_1, e_2, e_3, u_6\}$;

$\psi(u_5) = \{e_1, e_3, e_4, e_5, u_6\}$; $\psi(u_6) = \{e_1, e_2, e_3, e_4, u_5, u_6\}$;

$C_P(u_1) = \{u_1, u_6\}$; $C_P(u_2) = \{u_2, u_5, u_6\}$; $C_P(u_3) = \{u_1, u_3, u_6\}$; $C_P(u_4) = \{u_1, u_4, u_6\}$; $C_P(u_5) = \{u_5, u_6\}$;

$C_P(u_6) = \{u_6\}$;

$IN_P(u_1) = \{u_1, u_3, u_4\}$; $IN_P(u_2) = \{u_2\}$;$IN_P(u_3) = \{u_3\}$;

$IN_P(u_4) = \{u_4\}$;$IN_P(u_5) = \{u_2, u_5\}$;$IN_P(u_6) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$;

$A_P(u_1) = \{u_1\}$; $A_P(u_2) = \{u_2\}$;$A_P(u_3) = \{u_3\}$; $A_P(u_4) = \{u_4\}$; $A_P(u_5) = \{u_5\}$; $A_P(u_6) = \{u_6\}$;

Definition 5.8:

Let $P = (U, C_S)$ be a soft covering approximation space. For any $X \subseteq U$, the soft covering lower approximation and upper approximations are defined as

$$\underline{FC_P}(X) = \{u \in U: C_P(u) \subseteq X\}$$

$$\overline{FC_P}(X) = \{u \in U: C_P(u) \cap \left(X - \underline{FC_P}(X)\right) \neq \phi\} \cup \underline{FC_P}(X)$$

If $\underline{FC_P}(X) \neq \overline{FC_P}(X)$ then X is said to be the first type of soft covering based rough set otherwise X is called soft covering based definable.

Definition 5.9:

Let $P = (U, C_S)$ be a soft covering approximation space. For any $X \subseteq U$, the soft covering lower approximation and upper approximations are defined as

$$\underline{SC_P}(X) = \{u \in U: C_P(u) \subseteq X\}$$

$$\overline{SC_P}(X) = \{u \in U: C_P(u) \cap \left(X - \underline{SC_P}(X)\right) \neq \phi\} \cup \underline{SC_P}(X)$$

If $\underline{SC_P}(X) \neq \overline{SC_P}(X)$ then X is said to be the second type of soft covering based rough set otherwise X is called soft covering based definable.

Definition 5.10:

Let $P = (U, C_S)$ be a soft covering approximation space. For any $X \subseteq U$, the soft covering lower approximation and upper approximations are defined as

$\underline{IC_P}(X) = \{u \in U : A_P(u) \subseteq X\}$

$\overline{IC_P}(X) = \{u \in U : A_P(u) \cap \left(X - \underline{IC_P}(X)\right) \neq \phi\} \cup \underline{IC_P}(X)$

If $\underline{IC_P}(X) \neq \overline{IC_P}(X)$ then X is said to be the third type of soft covering based rough set otherwise X is called soft covering based definable.

Theorem 5.1:

Let $P = (U, C_S)$ be a soft covering approximation space, for any $X, Y \subseteq U$ then

i) $\underline{FC_P}(\phi) = \phi; \overline{FC_P}(\phi) = \phi$
ii) $\underline{FC_P}(U) = U; \overline{FC_P}(U) = U$
iii) If $X \subseteq Y$ then $\underline{FC_P}(X) \subseteq \underline{FC_P}(Y)$ and $\overline{FC_P}(X) \subseteq \overline{FC_P}(Y)$
iv) $\underline{FC_P}(X) \subseteq X \subseteq \overline{FC_P}(X)$
v)
   a. $\underline{FC_P}(X \cap Y) = \underline{FC_P}(X) \cap \underline{FC_P}(Y)$
   b. $\underline{FC_P}(X \cup Y) \supseteq \underline{FC_P}(X) \cup \underline{FC_P}(Y)$
   c. $\overline{FC_P}(X \cap Y) \subseteq \overline{FC_P}(X) \cap \overline{FC_P}(Y)$
   d. $\overline{FC_P}(X \cup Y) = \overline{FC_P}(X) = \overline{FC_P}(Y)$
vi) $-\underline{FC_P}(X) = \overline{FC_P}(-X)\ and -\overline{FC_P}(X) = \underline{FC_P}(-X)$
vii)
   a. $\underline{FC_P}\left(\underline{FC_P}(X)\right) = \underline{FC_P}(X)$
   b. $\overline{FC_P}\left(\overline{FC_P}(X)\right) = \overline{FC_P}(X)$
   c. $\underline{FC_P}\left(\overline{FC_P}(X)\right) = \overline{FC_P}(X)$
   d. $\overline{FC_P}\left(\underline{FC_P}(X)\right) = \underline{FC_P}(X)$

Instead of the operator FC_P if we replace the operators SC_P or IC_P then all the properties from (i) to (vii) holds good.

Example 5.4:

Let $S = (F, A)$ be a soft covering on U in example 5.3. The selection committee selected $X = \{u_3, u_5, u_6\}$ as the judgment in the interview. Now from definition 5.8, 5.9 and 5.10; we obtain the upper and lower approximation is small, then choice is better.

$\underline{FC_P}(X)) = \{u_5, u_6\}; \overline{FC_P}(X)) = \{u_1, u_3, u_5, u_6\};$

$\underline{SC_P}(X)) = \{u_3\}; \overline{SC_P}(X)) = \{u_1, u_2, \dots \dots, u_5, u_6\};$

$\underline{IC_P}(X)) = \{u_3, u_5, u_6\}; \overline{IC_P}(X)) = \{u_3, u_5, u_6\};$

Proposition 5.1:

Let $P = (U, C_S)$ be a soft covering approximation space, for any $u, v \in U$; if $v \in IN_P(u)$ then $IN_P(v) \subseteq IN_P(u)$

Proof: Let $t \in IN_P(v)$ then $\psi(t) \subseteq \psi(v)$ by definition so $\psi(t) \subseteq \psi(v)$ as $v \in IN_P(u)$ therefore $t \in IN_P(u)$ which implies $IN_P(v) \subseteq IN_P(u)$

C. Relation between soft rough sets and soft covering based rough sets

In this section, we establish relation between soft rough sets and soft covering based rough sets.

1) Relation between soft rough sets and first type of soft covering based rough sets.

Proposition 5.2:

Let $P = (U, C_S)$ be a soft covering approximation space, then for any $X \subseteq U, \underline{S_P}(X) \subseteq \underline{FC_P}(X)) \subseteq \overline{FC_P}(X)) \subseteq \overline{S_P}(X)$

Proof: We have already proved $\underline{FC_P}(X)) \subseteq X \subseteq \overline{FC_P}(X))$

let $u \in \underline{S_P}(X)$then there exist $e \in A$ such that $u \in F(e) \subseteq X$. So we have $u \in C_P(u) = \{x \in U : u \in F(e_i)\ and\ x \in \cap F(e_i) \forall e_i \in A\} \subseteq F(e) \subseteq X$, that is $u \in \underline{FC_P}(X))$. Therefore $\underline{S_P}(X) \subseteq \underline{FC_P}(X))$.

For any $u \in \overline{FC_P}(X)), C_P(u) \cap (X - \underline{FC_P}(X))) \neq \phi$. According to definition of $C_P(u)$, for any $e \in A$ and $u \in F(e) \cap (X - \underline{FC_P}(X))) \neq \phi$. So $u \in \overline{S_P}(X))$ and $\overline{FC_P}(X)) \subseteq \overline{S_P}(X)$.

Example 5.5:

In example 5.4 if $X = \{u_3, u_5, u_6\}$ then $\underline{FC_P}(X)) = \{u_5, u_6\}; \overline{FC_P}(X)) = \{u_1, u_3, u_5, u_6\}$

$\underline{S_P}(X) = \phi, \overline{S_P}(X) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$. So $\underline{S_P}(X) \subseteq \underline{FC_P}(X)) \subseteq \overline{FC_P}(X)) \subseteq \overline{S_P}(X)$

2) Relation between soft rough sets and second type of soft covering based rough sets

Proposition 5.3:

Let $P = (U, C_S)$ be a soft covering approximation space, than for any $X \subseteq U, \underline{S_P}(X) \subseteq \underline{SC_P}(X)) \subseteq \overline{SC_P}(X)) \subseteq \overline{S_P}(X)$

Example 5.6:

In example 5.4, if $X = \{u_3, u_5, u_6\}$, then $\underline{SC_P}(X)) = \{u_3\}; \overline{SC_P}(X) = \{u_1, u_2, \dots \dots, u_5, u_6\}$; $\underline{S_P}(X) = \phi, \overline{S_P}(X) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$. So $\underline{S_P}(X) \subseteq \underline{SC_P}(X)) \subseteq \overline{SC_P}(X)) \subseteq \overline{S_P}(X)$

3) Relation between soft rough sets and third type of soft covering based rough sets

Proposition 5.4:

Let $P = (U, C_S)$ be a soft covering approximation space, than for any $X \subseteq U, \underline{S_P}(X) \subseteq \underline{IC_P}(X)) \subseteq \overline{IC_P}(X)) \subseteq \overline{S_P}(X)$

Example 5.7:

In example 5.4, if $X = \{u_3, u_5, u_6\}$, then $\underline{IC_P}(X)) = \{u_3, u_5, u_6\}; \overline{IC_P}(X) = \{u_3, u_5, u_6\}; \underline{S_P}(X) = \phi, \overline{S_P}(X) = \{u_1, u_2, u_3, u_4, u_5, u_6\}$.

So $\underline{S_P}(X) \subseteq \underline{IC_P}(X)) \subseteq \overline{IC_P}(X)) \subseteq \overline{S_P}(X)$

VI. CONCLUSION

The goal of this article is to present Soft rough set model in a new manner. It introduces soft rough set approximations and satisfies most properties of Pawlak's rough set without any restrictions. The paper infers salient properties of soft rough approximations with examples. Comparison between these approaches and other works has been made. The focus of the paper is to increase the accuracy of approximation. It is found from the result that, the proposed methods are more accurate than other methods in decision making. Hence these methods are very useful in real-life applications. Soft rough approximate equal relations are discussed. We hope, the purposed model will benefit for further investigation in the research field.

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B. *Ethical Approval:*

This article does not contain any studies with human participants or animals performed by any of the authors.

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