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The Development of a Plane from an Angular Vector

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Abstract— The technique of developments of a plane from a precise vector in a rectangular arranges framework has been depicted. The technique for making of a rakish vector from two rectilinear vectors has been thought of. The property of uniformity of precise vectors has been depicted.

Keywords: angular vector, cross product of vectors, equality of vectors, plane

I. INTRODUCTION

With the coming of rakish vectors in the vector hypothesis, there was a requirement for their utilization for demonstrating precise physical amounts, yet in addition for geometric developments. Rakish vectors can be utilized as arranged planes in the facilitate framework. These vectors permit us to perform activities like rectilinear vectors, for example, developing equal and opposite planes; making a plane opposite to the two unique planes; development of a rectilinear vector at the convergence of two planes, and so forth. One of the first and significant strides for these activities is to take care of the issue of developing a plane from a precise vector.

Prior to inferring a strategy for developments of a plane from a rakish vector and for a superior comprehension of the properties of vectors, we will consider:

1) Method of creating an angular vector from two rectilinear vectors,
2) Property of the equality between angular vectors.

II. CREATING AN ANGULAR VECTOR

The equation linking rectilinear and angular vectors is the cross product equation $\bar{a} \times \bar{b} = \bar{c}$.

This equation shows not only the relations between vectors, but also the method of creating an angular vector from two rectilinear vectors.

The equation shows that an angular vector is created from a sequence of two rectilinear (perpendicular to each other) vectors. The sequence of rectilinear vectors determines the direction of an angular vector.

In geometric form, when creating an angular vector for the equation $\bar{a} \times \bar{b} = \bar{c}$. It is convenient to imagine that rectilinear vector begins to be created b at the end of the rectilinear vector a and the position of these rectilinear (perpendicular) vectors, sets the direction of the angular vector c, Fig. 1. The direction sign of an angular vector, determines the angular direction in the selected coordinate system, just like the directions of the coordinate axes in the coordinate system determine the sign of rectilinear vector. The value of an angular vector is equal to the area of the rectangle formed from rectilinear vectors.

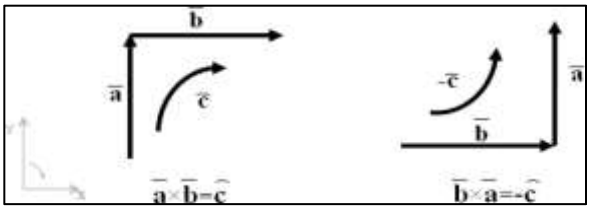

Fig. 1: Creating an angular vector of two rectilinear vectors

The same angular vector can be created from different combinations of rectilinear vectors in the cross product, Fig. 2.

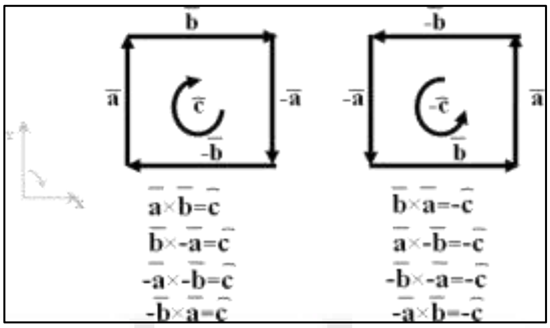

Fig. 2: Combinations of cross product of vectors

The choice (or derivation) of the equation for finding the projections of an angular vector in the coordinate-vector form depends on the selected coordinate system. These equations also depend on the sequence of rectilinear vectors in the cross product.

For all variants of rectangular coordinate systems (and there are 96 types = 8 octants of dividing the space * 6 variants of the location of the axis names in each octant * 2 variants of the angular direction for each version of the coordinate system), only two versions of the equations for finding the projections of angular vectors can be derived.

For a positive angular direction between the axes $O_y \to O_x \to O_z$(the direction can be selected both clockwise and counterclockwise, and depends on the selected coordinate system):

$$\hat{c} = \bar{a} \times \bar{b}$$

$$= \left(a_y b_x - a_x b_y\right)\hat{l} + \left(a_z b_y - a_y b_z\right)\hat{m} + \left(a_x b_z - a_z b_x\right)\hat{n}$$

$$= c_{xy}\hat{l} + c_{yz}\hat{m} + c_{zx}\hat{n}$$

Where, $c_{xy} = \left(a_y b_x - a_x b_y\right);\ c_{yz} = \left(a_z b_y - a_y b_z\right);\ c_{zx} = \left(a_x b_z - a_z b_x\right) \quad (1)$

For a negative angular direction between the axes $O_y \to O_x \to O_z$(the direction can be selected both clockwise and counterclockwise, and depends on the selected coordinate system):

$$\hat{c} = \bar{a} \times \bar{b}$$

$$= \left(a_x b_y - a_y b_x\right)\hat{l} + \left(a_y b_z - a_z b_y\right)\hat{m} + \left(a_z b_x - a_x b_z\right)\hat{n}$$

$$= c_{xy}\hat{l} + c_{yz}\hat{m} + c_{zx}\hat{n}$$

Where, $c_{xy} = \left(a_x b_y - a_y b_x\right);\ c_{yz} = \left(a_y b_z - a_z b_y\right);\ c_{zx} = \left(a_z b_x - a_x b_z\right) \quad (2)$

This option of coordinating the direction of angular vectors (and their projections) in various coordinate systems is slightly different from that proposed earlier in "Angular Vectors in the Theory of Vectors". This description is the best, since it completely solves the problem of choosing the angular direction in the coordinate system and the equations for finding projections of an angular vector.

For the cross product equation $\bar{a} \times \bar{b} = \bar{c}$ you do not need to use the right-hand rule (in mathematics) and the right-hand screw rule (in theoretical mechanics), which greatly simplifies the solution of problems and increases the choice of the coordinate system.

III. PROPERTY OF THE EQUALITY BETWEEN ANGULAR VECTORS

Let's remember that two rectilinear vectors $\bar{a}\,, \bar{b}$ are equal when they-

- have equal modules,
- are equally oriented relative to the coordinate axes (they have the same projection values on the corresponding coordinate axes),
- Directions of vectors coincide (signs of all projections of the vectors on coordinate axes coincide).

When writing of rectilinear vectors in algebraic form, vectors are equal $a(a_x, a_y, a_z), b(b_x, b_y, b_z)$ when their projections $a_x = b_x\,, a_y = b_y\,, a_z = b_z$ are equal. The equality of angular vectors $\hat{c}\,, \hat{d}, \hat{t}$ is very similar to the properties of rectilinear vectors-

- they have equal modules,
- they are equally oriented relative to the coordinate axes (they have the same projection values on the corresponding coordinate planes),
- Their directions of vectors coincide.

When angular moments are written in algebraic form, vectors are equal $c(c_{xy}, c_{yz}, c_{zx}) = d(d_{xy}, d_{yz}, d_{zx}) = t(t_{xy}, t_{yz}, t_{zx})$ when their projections are equal: $c_{xy} = d_{xy} = t_{xy}\,, c_{yz} = d_{yz} = t_{yz}\,, c_{zx} = d_{zx} = t_{zx}$. But, despite the similar properties of the equality of rectilinear and angular vectors, there are differences between them. They appear during the geometric construction of vectors, Equal rectilinear vectors are always parallel and co-directional in space (coordinate system).

For equal angular vectors, only the planes in which they are located are parallel. But the location of an angular vector in the plane (the location of the rectilinear vectors from which an angular vector is built) can be any. And the shape of the resulting the area figure (rectangles or squares) can be any. Moreover, the areas of equal angular vectors are equal.

From the equality of angular vectors it follows that the same angular vector can be represented from various combinations of rectilinear vectors. Therefore, angular vectors are often displayed conditionally (without a geometric figure)

$$S(\hat{c}) = S(\hat{d}) = S(\hat{t}) = |\hat{c}| = |\hat{d}| = |\hat{t}|$$

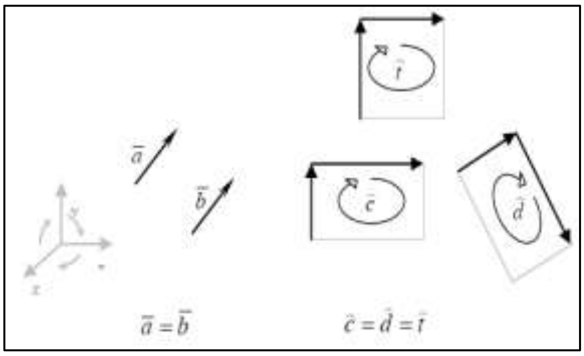

Fig. 3: Equality of rectilinear and angular vectors

It also follows from the equality of angular vectors that the projection of an angular vector onto any other plane (including the coordinate one) does not depend on the shape and location of an angular vector on the plane in which it is located.

IV. THE METHOD OF CONSTRUCTION OF A PLANE FROM AN ANGULAR VECTOR

It is hard to development of a precise vector and its projections onto the organize planes in a facilitate framework. All things considered, we just have the zone estimations of the figures on these planes (projections of a rakish vector), and we don't have the foggiest idea about their shapes and areas in the arrange planes. Also, to development of them, we need the first two rectilinear vectors. Their area, size, and heading decide the area, course, and state of a rakish vector.

Based on the property of the equality of angular vectors, we can represent an angular vector from any sequence of two rectilinear vectors.

Let's imagine that the angular vector $c(c_{xy}, c_{yz}, c_{zx})$ is construction of from two rectilinear vectors $\bar{a}(a_x, a_y, 0)$and located in the coordinate planes, Fig. 4. These vectors begin and end on the coordinate axes and construction of a plane in the form of a triangle in the coordinate system. Let's find the coordinates of the points $A(a_x, 0{,}0), B(0, b_y, 0)$ and $C(0{,}0, z_c)$ creating this plane.

As you can see, the vectors $\bar{a}, \bar{b}$ are not perpendicular. But this is not important because the cross product $\bar{a} \times \bar{b} = \hat{c}$ takes into account the location of the original rectilinear vectors. Only part of one of the vectors that can be represented perpendicular to the second vector will participate in the equation.

To find the projections of rectilinear vectors $\bar{a}, \bar{b}$ we need the condition $a_x = \pm b_x$. this condition determines the dependence of the projection values of two vectors $\bar{a}, \bar{b}$ which allow them to be placed so that they begin and end on the coordinate axes, and the signs allow us to solve the radical expression

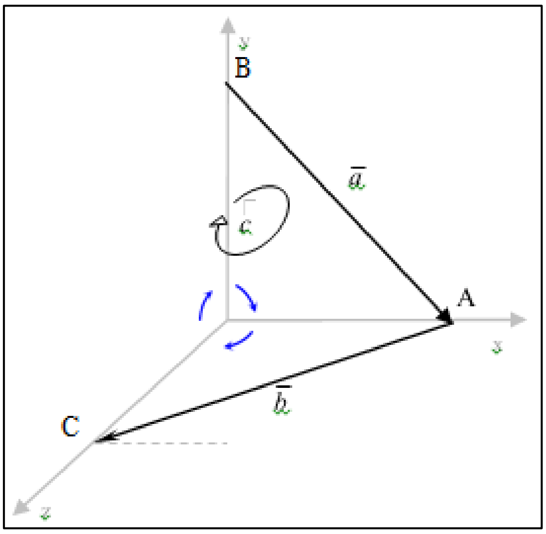

Fig. 4: Construction of a plane in the form of a triangle

For the coordinate system with an angular direction $O_y \rightarrow O_x \rightarrow O_z$. we find the equation for finding the projections of the angular vector from the cross product equation (1)

$$c_{xy} = (a_y b_x - a_x b_y);$$

$$c_{yz} = (a_z b_y - a_y b_z);$$

$$c_{zx} = (a_x b_z - a_z b_x)$$

We find the projections of rectilinear vectors from these equations.

$$c_{xy} = a_y \cdot b_x - a_x \cdot 0$$

$$a_y = \frac{c_{xy}}{b_x} = \frac{c_{xy}}{\pm a_x}$$

$$c_{yz} = 0 \cdot 0 - a_y \cdot b_z$$

$$b_z = \frac{c_{yz}}{-a_y} = -\frac{c_{yz} \cdot b_x}{c_{xy}} = \mp \frac{c_{yz} \cdot a_x}{c_{xy}}$$

$$c_{zx} = a_x \cdot b_z - 0 \cdot b_x$$

$$a_x = \frac{c_{zx}}{b_z} = \mp \frac{c_{zx} \cdot c_{xy}}{c_{yz} \cdot a_x}$$

$$a_x = \sqrt{\mp \frac{c_{zx} \cdot c_{xy}}{c_{yz}}}$$

For the coordinate system with an angular direction $O_x \rightarrow O_y \rightarrow O_z$ the equation for finding the projections of the angular vector is as follows:

$$c_{xy} = 0 - a_y \cdot b_x$$

$$a_y = -\frac{c_{xy}}{b_x} = \frac{c_{xy}}{\mp a_x}$$

$$c_{yz} = a_y \cdot b_z - 0$$

$$b_z = \frac{c_{yz}}{a_y} = -\frac{c_{yz} \cdot b_x}{c_{xy}} = \mp \frac{c_{yz} \cdot a_x}{c_{xy}}$$

$$c_{zx} = 0 - a_x \cdot b_z$$

$$a_x = -\frac{c_{zx}}{b_z} = \pm \frac{c_{zx} \cdot c_{xy}}{c_{yz} \cdot a_x}$$

$$a_x = \sqrt{\pm \frac{c_{zx} \cdot c_{xy}}{c_{yz}}}$$

The choice of the upper or lower sign in the three equations for (3), (4) depends on whether the radical a_x is a real number (whether the radical expression is positive). If there are signs in the projections of an angular vector $\hat{c}(-,-,-)$, $\hat{c}(-,+,+)$, $\hat{c}(+,-,+)$, $\hat{c}(+,+,-)$, the "minus" sign is selected in the sub-radical equation. If there are signs in the projections of an angular vector $\hat{c}(+,+,+)$, $\hat{c}(-,-,+)$, $\hat{c}(-,-,+)$, $\hat{c}(+,-,-)$, $\hat{c}(-,+,-)$ then the "plus" sign is selected. Accordingly, the upper or lower sign in the equations (3, 4) of finding the other two projections of rectilinear vectors is selected. The radical solution for the projection ax gives two solutions. As a result, we get two pairs of oppositely directed $\bar{a}$, $\bar{b}$ and $-\bar{a}, -\bar{b}$ vectors located on the same plane.

In order to find a pair of vectors, we used the cross product equation $\bar{a} \times \bar{b} = \hat{c}$. In fact; we do not know the sequence of rectilinear vectors in the cross product that suits us (for the same angular vector $\bar{c}$)

If we chose the cross product $\bar{b} \times \bar{a} = \hat{c}$, we would get pairs of vectors: $\bar{b}, -\bar{a}$; $-\bar{b}, \bar{a}$ Let's choose from the obtained four vectors a suitable pair of vectors by an additional condition $a_x = -b_x$, Next, we find the co-ordinates of the points A,B,C,

$$\bar{a} = (x_B - x_A, y_B - y_A, 0);$$

$$\bar{b} = (x_A - x_C\, , 0, z_A - z_C);$$

$$x_B - x_A = a_x\, , y_B - y_A = a_y;$$

$$x_B = 0\, , y_A = 0$$

$$x_A = -a_x\, , y_B = a_y$$

$$x_A - x_C = b_x\, , z_A - z_C = b_Z$$

$$x_C = 0\, , z_A = 0$$

$$x_A = b_x\, , z_C = -b_Z$$

$$A(x_A, 0,0); B(0, y_B, 0); C(0,0, z_C)$$

Example 1. Construction of a plane from an angular vector

Solution: Let us be given an angular vector $\hat{c}(10, -5,8)$ and a co-ordinate system. Fig. 5 it is necessary to find points $A(x_A, 0,0), B(0, y_B, 0); C(0,0, z_C)$ on the coordinate axes to construct a plane in which the angular vector is located.

For our condition $\hat{c}(10, -5,8)$ and with a positive angular direction $O_y \rightarrow O_x \rightarrow O_z$. we choose the upper sign for equations (3).

$$a_x = \sqrt{-\frac{c_{zx} \cdot c_{xy}}{c_{yz}}} = \sqrt{-\frac{8 \cdot 10}{-5}} = \pm 4\,.$$

$$a_y = \frac{c_{xy}}{b_x} = \frac{c_{xy}}{+a_x} = \frac{10}{\pm 4} = \pm \frac{5}{2}$$

$$b_z = -\frac{c_{yz} \cdot a_x}{c_{xy}} = -\frac{-5 \cdot (\pm 4)}{10} = \pm 2$$

And we get two pairs of rectilinear vectors:

$$\overline{a_1}\left(4, \frac{5}{2}, 0\right), \overline{a_1}(4,0,2); \overline{a_2}\left(-4, -\frac{5}{2}, 0\right), \overline{a_2}(-4,0,-2)$$

Each pair of rectilinear vectors obtained can be used to construct an angular vector. But we, by the condition of the problem, need to construction of a plane with points located on the coordinate axes. To do this, we can re-find rectilinear vectors using the cross product $\bar{b} \times \bar{a} = \hat{c}$ and get the pairs of vectors.

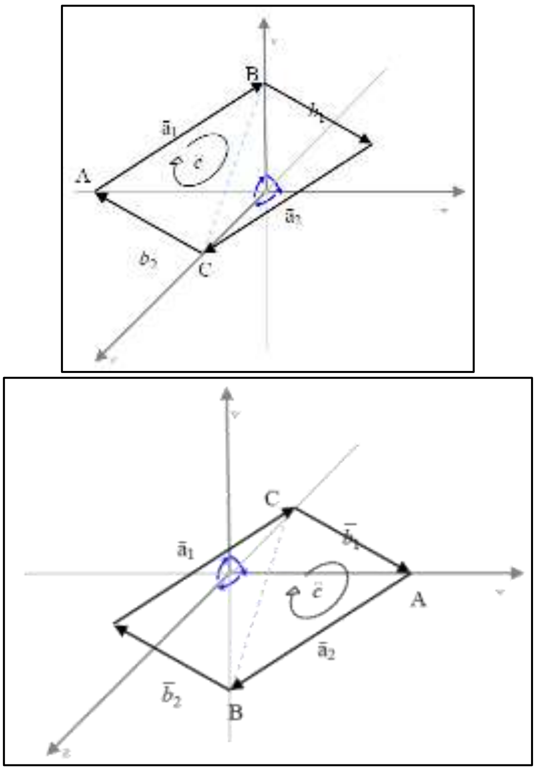

Fig. 5: Construction of a plane from angular vector.

$$\overline{b_2}(-4,0,2), \overline{a_1}\left(4,\frac{5}{2},0\right)\ ;\ \overline{b_1}(4,0,2)\ , \overline{a_2}\left(-4,-\frac{5}{2},0\right)$$

Or, we choose a pair $\bar{a}, \bar{b}$ satisfying the condition $a_x = -b_x$, from four vectors.

$$\overline{a_1}\left(4,\frac{5}{2},0\right), \overline{b_2}(-4,0,2)$$

Then,

$$x_A = -a_x = b_x = -4,$$
$$y_B = a_y = \frac{5}{2}\ , z_C = -b_z = 2$$

Answer 1: $A(-4,0,0); B\left(0,\frac{5}{2},0\right); C(0,0,2)$

Or

$\overline{a_2}\left(-4,-\frac{5}{2},0\right), \overline{b_1}(4,0,2)$; then

$$x_A = -a_x = b_x = 4,$$
$$y_B = a_y = -\frac{5}{2},$$
$$z_C = -b_z = -2$$

Answer 2: $A(4,0,0); B\left(0,-\frac{5}{2},0\right); C(0,0,-2)$

Conclusion: Both solutions are fair and create two parallel planes

V. CONCLUSION

The depicted straightforward technique shows that development of a plane from a rakish vector permits us to discover a few arrangements on the double (to make a few equal planes) that grow the capacities of designers in geometric developments.

The considered method of forming an angular vector from two rectilinear vectors shows a geometric relationship between these vectors. This explanation of the logical dependence of rectilinear and angular vectors is the missing link in proving the existence of the cross product equation for vectors $\bar{a} \times \bar{b} = \hat{c}$.

The considered property of the equality of angular vectors allows us to expand the methods of solving problems and when solving them, visually represent the behavior of the angular vector and its projections.

VI. REFUTATION

In the paper "*Angular Vectors in the Theory of Vectors"*, the projections of an angular vector onto the coordinate planes were connected by a sequence of indices $b_{xy} = -b_{yx}\ , b_{yz} = -b_{zy}\ , b_{zx} = -b_{xz}$.

This rule will be superfluous, because it adds excessive attention to the indexes, and the positive direction of the projections of an angular vector is determined by the selected coordinate system in which there is an angular direction.

The sequence of indices should not affect the direction of the projection of an angular vector.

REFERENCES

[1] Amel'kin, N. I. (2000). Kinematics and dynamics of a rigid body. M. MFTI. (Russian)

[2] Andraphanova, N. V. (2015). Geometrical similarity transformations in dynamic geometry environment GEOGEBRA.

[3] European Journal of Contemporary Education, 12(2), 116-128. https://doi.org/10.13187/ejced.2015.12.116

[4] Baumgart, B. G. (1974). Geometric modeling for computer vision (No. STAN-CS-74-463). STANFORD UNIV CA DEPT OF COMPUTER SCIENCE.

[5] Dorst, L., Fontijne, D., & Mann, S. (2010). Geometric algebra for computer science: an object-oriented approach to geometry. Elsevier.

[6] Faas, F. G., & van Vliet, L. J. (2003, June). 3D-orientation space; filters and sampling. In Scandinavian conference on image analysis (pp. 36-42). Springer, Berlin, Heidelberg. https://doi.org/10.1007/3-540-45103-X_6

[7] Glynn, D. G. (2010). Theorems of points and planes in three-dimensional projective space. Journal of the Australian Mathematical Society, 88(1), 75-92. https://doi.org/10.1017/S1446788708080981

[8] Kharchenko, Y. M. (2017). Angular Vectors in the Theory of Vectors. Journal of Mathematics Research, 9(5), 71. https://doi.org/10.5539/jmr.v9n5p71

[9] Kharchenko, Y. M. (2019). Unresolved problems in mechanics using vectors. 20th International Scientific and Technical Conference "The Progressive Technics, Technology and Engineering Education". Ukraine. Kherson. NTUU "KPI-Sikorsky", 42-45. (Ukrainian). Retrieved from http://conf.mmi.kpi.ua/proc/article/view/180905/180895

[10] Kortenkamp, U. H., & Richter-Gebert, J. (1998). Geometry and Education in the Internet Age.

[11] Maresch, G. (2014). Strategies for Assessing Spatial Ability Tasks. Journal for Geometry and Graphics, 18, 125-132.

[12] Marinkovic, V., Janicic, P., & Schreck, P. (2014). Solving geometric construction problems supported by

theorem proving. In Proceedings of the 10th International Workshop on Automated Deduction in Geometry (ADG 2014) (pp. 121-146).

[13] Pottmann, H., Liu, Y., Wallner, J., Bobenko, A., & Wang, W. (2007). Geometry of multi-layer freeform structures for architecture. ACM Transactions on Graphics (TOG), 26(3), 65. https://doi.org/10.1145/1276377.1276458

[14] Schreck, P. (2001, July). Robustness in cad geometric constructions. In Proceedings Fifth International Conference on Information Visualisation (pp. 111-116). IEEE.

[15] Wallner, J., Krasauskas, R., & Pottmann, H. (2000). Error propagation in geometric constructions. Computer-Aided Design, 32(11), 631-641. https://doi.org/10.1016/S0010-4485(00)00053-1

[16] Wang, K., Tong, Y., Desbrun, M., & Schröder, P. (2006). Edge subdivision schemes and the construction of smooth vector fields. ACM Transactions on Graphics (TOG), 25(3), 1041-1048. https://doi.org/10.1145/1141911.1141991

[17] Zadorozhnyj, V. N. (2010). Higher mathematics for technical universities. Part II. Analytic geometry: Tutorial. Tomsk. (Russian).