

# Harmonic Analysis of Symmetric Cross-Ply Laminated Composite Plate Using FEM

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**Abstract**— This analysis presents the vibration and Harmonic analysis of orthotropic laminated composite plate. The response of plate is determined using Finite Element Method. Laminate composites are increasing used in various mechanical structures and industrial applications, due to their higher stiffness and higher strength to weight ratio. The model has been developed and analyzed in ANSYS parametric design language code. In this analysis shell elements (SHELL8Node281) have been chosen to evaluate the elemental equations. Initially the model response for orthotropic plate and harmonic response for isotropic plate is verified with the available literature. The results are in good agreement with the available literature. Numerical results for the natural frequency and harmonic response amplitude are presented. Effects of boundary conditions, thickness to width ratio and number of layers on natural frequency and harmonic response of the orthographic plates are also investigated. The natural frequency, mode shape and harmonic analysis of laminated composite plate has been determined using finite element package ANSYS.

**Keywords:** vibration, Harmonic analysis, composite plate, boundary conditions, thickness to width ratio and number of layers on natural frequency

## I. INTRODUCTION

A composite is a structural material that consists of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibers, particles, or include concrete reinforced with steel and epoxy reinforced with graphite fibers, etc. flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibers, etc. [44].

It is known that fibers are stiffer and stronger than the same material in bulk form, whereas matrix materials have their usual bulk form properties. Geometrically fibers have near crystal size diameter and a very high length to diameter ratio. Short fibers, called whiskers, paradoxically exhibit better structural properties than long fibers. To gain a full understanding of the behaviour of fibers, matrix materials, agents that are used to enhance bonding between fibers and matrix, and other properties of fiber reinforced materials

Many industries use laminated plates with different size and thickness. It is widely used in various mechanical, civil and aero structures. These structures can be found easily in modern construction in the form of reinforced slabs and stiffened plates. Such structures are commonly used as floors in bridges, ship hulls, buildings and in the construction of wings, tails and fins of aircrafts and missiles. However, the composite laminates may severe dynamic loading during

their service life. Hence, for the designer's quest to model these complex structural problems precisely and study the effect of different geometries on the dynamic behaviour of composite laminates with less computational effort.

## II. METHODOLOGY

It is mentioned that for free vibration analysis, subspace method is used. The subspace iteration method was developed by Bathe [46] for the solution of frequencies and mode shapes of structures, and in particular for the earthquake analysis of buildings and bridges [47-49]. Originally in the 1970s, relatively few eigen pairs were sought in these solutions, like the lowest 10 to 20 frequencies and mode shapes, when the model contained a total of 1000 to 10,000 degrees of freedom. However, since its original development, the subspace iteration method has been used abundantly in research and commercial finite element programs for small and very large finite element systems, and the method has naturally attracted considerable attention for improvements. Enhancements as suggested by Wilson and Itoh [50] are also included as outlined subsequently.

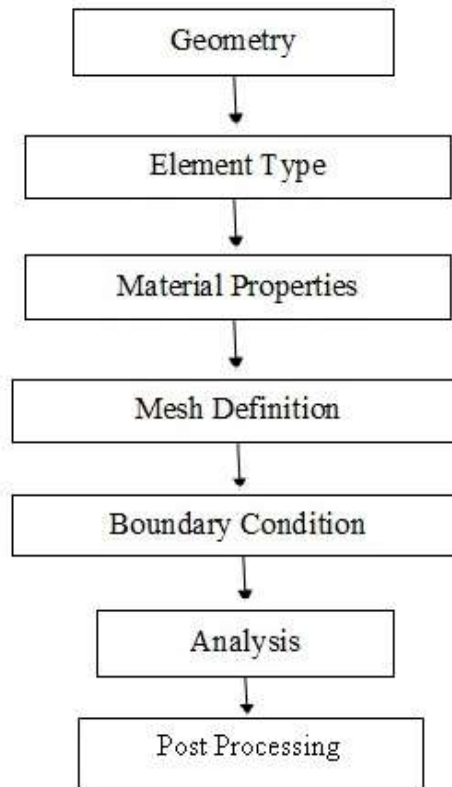
The Lanczos method can be very effective, in particular when solving for many frequencies and mode shapes [51]. Initially, the Lanczos method showed instabilities due to loss of orthogonality of the iteration vectors. However, these difficulties have been largely overcome and in good implementations the method can be very efficient [52, 53]. A particular asset of the method is that the computational effort may increase almost linearly with the number of eigen pairs sought. This asset can render the Lanczos method attractive compared to the original subspace iteration method if many eigen pairs need be calculated. Namely, in that case, the computational effort increases larger than linear in the original subspace iteration method, and this increase can be significant.

The Lanczos method and Bethe's subspace iteration method (or variants of these two iterative schemes) are two techniques that, at present, are very widely used for the solution of large eigenvalue problems in finite element analysis. Any noteworthy improvements to these methods are therefore of interest.

An important step in the subspace iteration method is to establish effective starting iteration vectors, which also implies to, ideally, use the optimal number of iteration vectors. Lately much effort has been spent on using parallel processing in finite element analysis, in shared memory and distributed memory processing modes. Whereas the Lanczos method (working on individual vectors) can intrinsically only be parallelized in the factorization of the stiffness matrix and the forward reduction and back-substitution of the individual vectors, the subspace iteration method allows in addition the parallel solution of multiple iteration vectors, which can result in a large computational benefit.

Solution technique and steps

The equations are solved by using Block Lanczos algorithm. Been followed for the solution.  
The following steps have



Solution steps in ANSYS

### III. RESULT AND DISCUSSION

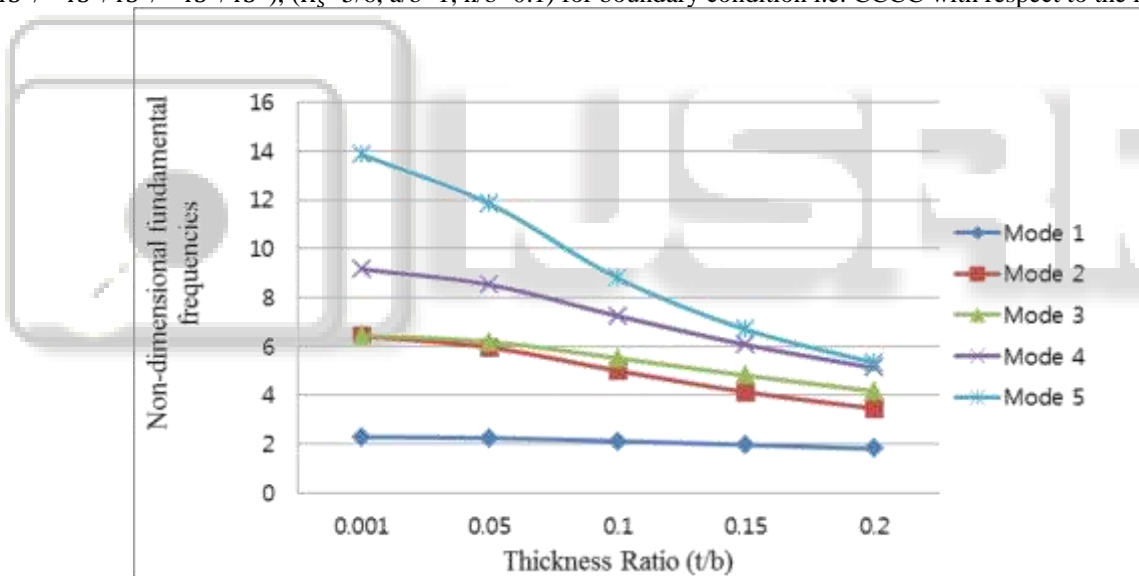
In the present study, Harmonic analysis of plate without uncertainty is solved in ANSYS, FEA software using the full method. Harmonic Analysis is done to compute the first natural frequencies for various boundary conditions with different thickness to length ratios. The results are compared for increasing layers for symmetrical laminated orthotropic plate

Reference	Mode							
M=N	1	2	3	4	5	6	7	8
7	5.1341	8.9086	21.3748	30.7758	33.2154	41.0802	42.6495	58.0033
Ref.[43]	5.1894	9.0006	21.5817	31.2121	33.6750	41.6003	43.0237	58.6416
9	5.1339	8.9084	21.3356	30.7655	33.2030	40.9883	42.3604	57.5666
Ref.[43]	5.1891	9.0017	21.5432	31.2015	33.6634	41.5090	42.7359	58.1986
11	5.1338	8.9082	21.3242	30.7624	33.1989	40.9656	42.2665	57.4541
Ref.[43]	5.1890	9.0017	21.5311	31.1978	33.6603	41.4853	42.6429	58.0878
13	5.1338	8.9082	21.3201	30.7614	33.1979	40.9584	42.2293	57.4128
Ref.[43]	5.1890	9.0017	21.5264	31.1968	33.6587	41.4768	42.6055	58.0456
15	5.1338	8.9081	21.3180	30.7604	33.1968	40.9553	42.2117	57.3942
Ref.[43]	5.1890	9.0017	21.5232	31.1957	33.6576	41.4716	42.5796	58.0193
17	5.1338	8.9080	21.3170	30.7604	33.1958	40.9532	42.2025	57.3849
Ref.[43]	5.1890	9.0017	21.5232	31.1957	33.6576	41.4716	42.5796	58.0193
19	5.1338	8.9080	21.3160	30.7604	33.1958	40.9522	42.1983	57.3798
Ref.[43]	5.1890	9.0017	21.5227	31.1957	33.6571	41.4705	42.5744	58.0140
Ref.[27]	5.191							
Ref.[32]	5.191							

Table 1: Convergence study of non-dimensional frequencies ( $\bar{\omega} = \frac{\omega b^2}{h} \sqrt{\frac{\rho}{E_{22}}}$ ) for a cross-ply laminate ( $0^0/90^0/0^0$ ), ( $K_s=5/6$ ,  $a/b=1$ ,  $h/b=0.01$ ) for boundary condition i.e. CSFF with respect to the results given by

Reference				Mode				
M=N	1	2	3	4	5	6	7	8
7	2.2051	3.6155	3.8205	5.0037	5.5057	5.6206	6.5150	6.7627
Ref.[43]	2.2483	3.6744	3.9060	5.0836	5.6111	5.7462	6.6012	6.9140
9	2.2045	3.6123	3.8173	4.9976	5.4876	5.6028	6.4963	6.7441
Ref.[43]	2.2476	3.6713	3.9026	5.0770	5.5928	5.7284	6.5821	6.8937
11	2.2042	3.6112	3.8162	4.9956	5.4817	5.5970	6.4903	6.7381
Ref.[43]	2.2474	3.6702	3.9015	5.0749	5.5867	5.7225	6.5759	6.8876
13	2.2041	3.6108	3.8158	4.9946	5.4793	5.5946	6.4878	6.7356
Ref.[43]	2.2472	3.6697	3.9010	5.0739	5.5842	5.7200	6.5733	6.8849
15	2.2040	3.6106	3.8155	4.9942	5.4782	5.5934	6.4866	6.7344
Ref.[43]	2.2482	3.6695	3.9008	5.0735	5.5830	5.7189	6.5722	6.8836
17	2.2040	3.6104	3.8153	4.9940	5.4776	5.5928	6.4860	6.7338
Ref.[43]	2.2471	3.6694	3.9007	5.0733	5.5825	5.7183	6.5715	6.8829
19	2.2040	3.6104	3.8153	4.9938	5.4773	5.5925	6.4856	6.7335
Ref.[43]	2.2470	3.6693	3.9005	5.0731	5.5822	5.7179	6.5712	6.8822
Ref.[21]	2.2857	3.7393	3.9813	5.1800	5.7019	5.8455	6.7167	7.0452
Ref.[32]	2.2857	3.7392	3.9813	5.1799	5.7019	5.8545	6.7166	7.0449

Table 2: Convergence study of non-dimensional frequencies ( $\bar{\omega} = \frac{\omega b^2}{\pi^2 h} \sqrt{\frac{\rho}{E_{22}}}$ ) for an angle-ply laminate ( $45^\circ/-45^\circ/45^\circ/-45^\circ/45^\circ$ ), ( $K_s=5/6$ ,  $a/b=1$ ,  $h/b=0.1$ ) for boundary condition i.e. CCCC with respect to the results given by



#### IV. CONCLUSION

In this study, vibration and harmonic analysis of orthotropic symmetric laminated plate is analysed. The following conclusion has been made:

- Convergence tests and comparison studies have been carried out using the Finite Element Method.
- The obtained thickness results have illustrated a good agreement with those available in the literature for different ratios, aspect ratios and different support conditions.
- It is clear from the results that the convergence is achieved at the mesh size of  $13 \times 13$ .

- The maximum variation in the non-dimensional fundamental frequency limited to 0 to 6%.
- It is observed that the non-dimensional fundamental frequency increases as the number of layers increases but the variation is negligible beyond five layers.
- It is observed that there is a significant variation in the resonance condition for CCCC boundary conditions for three, five and seven number of layers.
- It is found that as the thickness ratio increases the fundamental frequency increases but the value of resonance amplitude decreases for all boundary conditions.

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