Finite Element Analysis of Gearing System

Rajnish Chandrakant[1] G. R Kesheorey[2]
[1,2]Vindhya Institute of Technology and Science, Bhopal, India

Abstract— This paper focuses on parametric stress analysis of a lathe gear is been done in order to optimize the performance parameters of gear during subjected to static and dynamic loads. For this an comparative analysis has been carried out in order to investigate the performance of machine gear and a proposed composite gear. The bending stress, contact stress and contact ratio for a gear and several case studies are considered within the investigation.

Keywords: Bending Stress, MATLAB, Contact Stress, Pressure Angle

I. Introduction

In engineering and automobile, the overall efficiency of any kind of power transmission machine depends on the amount of power loss in the process.

The best way of transmitting power between the shafts is gears. Gears are mostly used to transmit torque and angular velocity. The design of gear is a complex process generally it needs large number of iteration and data sets. In many cases gear design is traditional and specified by different types of standards [1], [2]. Spur shaped gears found to be almost everywhere because of the contact forces act along a straight line.

II. Literature Review

Proper tooth design can usually prevent gear tooth bending fatigue. However, gear tooth or rim fatigue failures may occur even when the tooth design itself is adequate. Possible causes of such failures are insufficient rim thickness in the design, improperly processed material containing inclusions where cracks can start, severe operating conditions such as overload or misalignment, operation near the resonant frequency of a gear structure, or localized wear such as fretting at a gearshaft connecting joint which could initiate a crack (McFadden, [3]; Albrecht, [5]; Couchan et al., [6])

Chang and Tsai [7,8] proposed a mathematical model of parametric tooth profiles for spur gears using pressure angle as a parametric variable. Tsai and Tsai [9] proposed a method of designing high-contact-ratio spur gears using quadratic parametric tooth profiles for the shorter addendum without undercut. Lee, Lin, Oswald and Townsend [12] studied the effects of the linear profile modification on the dynamic tooth load and stress for high-contact-ratio gearing. Costopoulos, Kanarachos and Pant&s [13] proposed an optimum tooth profile of spur gear rotary pumps method to reduce the delivery fluctuation. Tavakoli and Houser [14] developed a procedure for computing static transmission errors and tooth load sharing for low- and high-contact ratio internal and external spur gears. Kahraman and Blankenship [15] investigated the influence of involute contact ratio on the torsional vibration behavior of a spur gear pair. Most fundamental mechanics of gears can be found in Buckingham~s book [16].

III. Mathematical Model

The geometry of the problem herein investigated is depicted in Fig. 1.

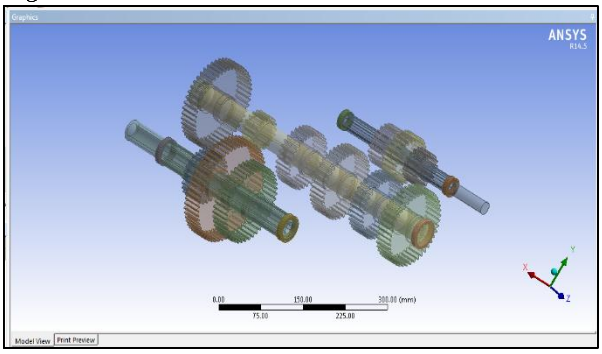

Fig. 1: Model Configuration

The FEM Formulation

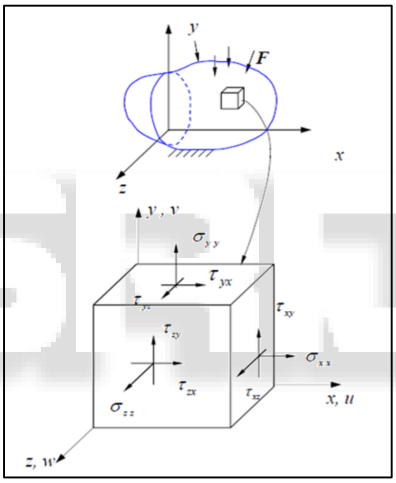

Fig. 2: Infinitesimal element showing stress state [17].

Displacement

$$U = \{u(x,y,z), v(x,y,z), w(x,y,z)\}$$

Cauchy's Stress tensor =

$$\sigma = \begin{bmatrix} \sigma_{xx}\tau_{xy}\tau_{xz} \\ \tau_{yx}\sigma_{yy}\tau_{yz} \\ \tau_{zx}\tau_{zy}\sigma_{zz} \end{bmatrix}$$

The strain-stress relations (Hooke's law) for isotropic materials are given by:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu & 0 & 0 & 0 \\ -\nu & 1 & -\nu & 0 & 0 & 0 \\ -\nu & -\nu & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2(1+\nu) & 0 & 0 \\ 0 & 0 & 0 & 0 & 2(1+\nu) & 0 \\ 0 & 0 & 0 & 0 & 0 & 2(1+\nu) \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

Strain-Displacement relations are:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x}, \varepsilon_{yy} = \frac{\partial v}{\partial y}, \varepsilon_{zz} = \frac{\partial w}{\partial z}, \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y},$$

$$\gamma_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + X = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + Y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + Z = 0$$

$$(\lambda + G)\frac{\partial e}{\partial x} + G\nabla^2 u + X = 0$$

$$e = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$$

$$b = \sqrt{\frac{4F\left\{\frac{\left[1-\mu_1^2\right]}{E_1} + \frac{\left[1-\mu_2^2\right]}{E_2}\right\}}{\pi l\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}}$$

Stresses internal to the cylinder are given by

$$\sigma_x = -P_{\max}\left\{\left[1+\left(\frac{y}{b}\right)^2\right]^{\frac{1}{2}}\left[2-\left(1+\left(\frac{y}{b}\right)^2\right)^{-1}\right] - 2.\frac{y}{b}\right\}$$

$$\sigma_y = -P_{\max}\left[1+\left(\frac{y}{b}\right)^2\right]^{\frac{-1}{2}}$$

Von-mises stress is given by

$$\sigma_{von} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2}$$

The equation below is the AGMA bending stress equation for S.I specification of gears.

$$\sigma_b = \frac{F_t k_v k_m k_o}{bmj}$$

$$\sigma = \frac{F_t}{bpy}$$

Lewis,

According to Shigley [28], the fundamental equation for pitting resistance (contact stress) is

$$S_c = C_p\sqrt{\frac{W_t k_a k_s k_m k_f}{k_v dFI}}$$

$$CR = \frac{\sqrt{(r_p+\phi)^2 - r_p^2\cos^2\phi}}{\pi m\cos\phi} + \frac{\sqrt{(r_g+\phi)^2 - r_g^2\cos^2\phi} - (r_p+r_g)\sin\phi}{\pi m\cos\phi}$$

$$C_p = \sqrt{\frac{1}{\pi\left(\frac{\left[1-\mu_1^2\right]}{E_1} + \frac{\left[1-\mu_2^2\right]}{E_2}\right)}}$$

In figure 1 the lathe gearing system configuration has been shown. The gearing system is discretisized in 3x 105 nodes and 7x103 elements and the boundary conditions are applied and the Von-mises (Bending) stress is evaluated.

IV. RESULTS AND DISCUSSIONS

This section gives results on Stress analysis of lathe gearing system with using ANSYS and Analytical calculation. The parametric study of effect of variation in number of teeth, pressure angle and comparison between composite and machined gearing system is carried out.

The MATLAB results are validated with literature and by Analytical calculation for a few cases are also illustrated.

No of teeth(N)	MATLAB Stresses(MPA)	3D Stresses (ANSYS)(MPA)
23	126.8841	126.99
25	122.2941	122.6

Table 1: Validation of Von-Mises Stresses for gearing Models

$$\sigma_b = \frac{F_t}{bmJ}k_v k_o k_m = \frac{3000}{43\times 4\times 0.38}\times 1.2\times 1.25\times 1.8 = 126.8841MPa$$

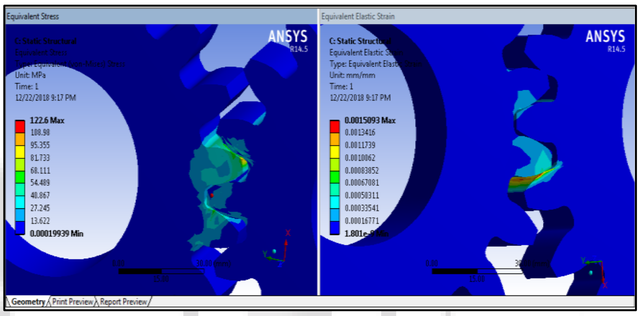

Fig. 2: 3-D Von-Mises Stress and deformation for Gear with 19 Teeth

From Table 1, and Figure 2 shows the stress distribution in spur gear and Shows the comparison of results for different 3-D models and the corresponding MATLAB stress values and Present FEM values. From this it can be revealed that on comparing Analytical result with computational result shows good agreement.

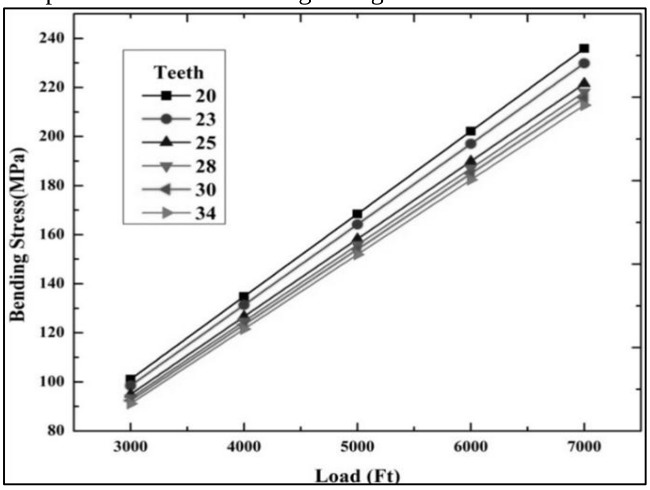

Fig. 3: Variation of Bending Stress with respect to Tangential load and no. of Gear Teeth

In figure 3 shows the Variation of Bending Stress with respect to Tangential load and no. of Gear Teeth. It can be conclude that on increasing no. of teeth with respect to tangential load the bending stress increases linearly. As more the load more will be the bending. Such bending stress can be overcome by increasing no. of teeth.

Therefore during design of gear no. of teeth plays a crucial role in selection of gear performance parameters such dynamic factor kv, over load factor ko and j geometry factor.

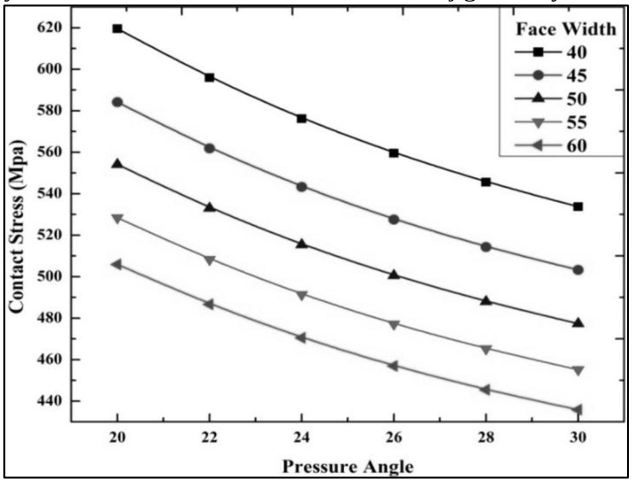

Fig. 4: Variation of Contact Stress with respect to Pressure Angle and Face Width

Figure 4 shows the Variation of Contact Stress with respect to Pressure Angle and Face Width. From this it is seems that on increasing pressure angle along with Face width the contact stress significantly decreases.

It can also be revealed that pressure angle and face width are inversely proportional to contact stress. And the load carrying capacity becomes more.

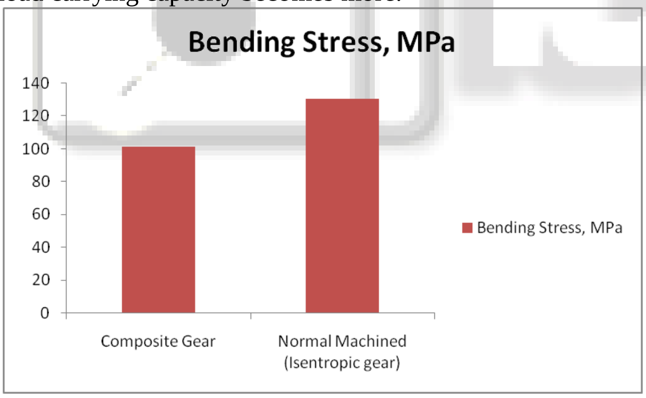

Fig. 5: Comparison of bending stress of composite gear and Normal Machined gear

Figure 5 shows the Comparison of bending stress of composite gear and Normal Machined gear. From this it is seems that the composite gear has lower level of bending stress as compared with normal machined gear. From this it can be concluded that composite gear can resist more bending stress then machined gear.

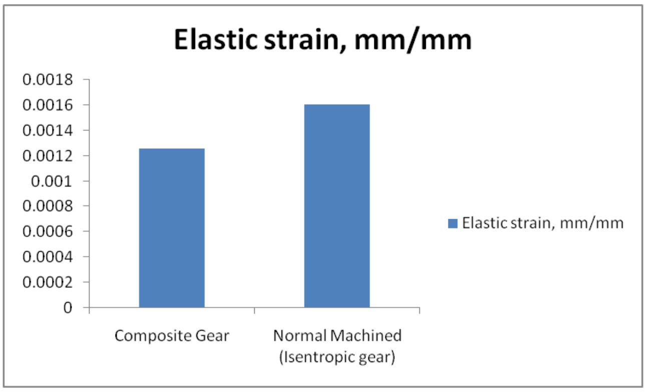

Fig. 6: Comparison of elastic strain of composite gear and Normal Machined gear

Figure 6 shows the Comparison of elastic strain of composite gear and Normal Machined gear. From this it is seems that the composite gear has lower level of elastic strain as compared with normal machined gear. From this it can be concluded that composite gear can has lower tendency of failure due to lower elastic strain as compared to machined gear.

V. Conclusion

- It can highly be recommended that in order to increase load carrying capacity of tooth pressure angle such be increased.
- It can be conclude that on increasing no. of teeth with respect to tangential load the bending stress increases linearly.
- As more the load more will be the bending. Such bending stress can be overcome by increasing no. of teeth.
- Composite gear can resist more bending stress then machined gear.
- It can be concluded that composite gear can has lower tendency of failure due to lower elastic strain as compared to machined gear.

References

[1] Vasilios Spitas, et.al," Fast modelling of conjugate gear tooth profiles using discrete presentation by involute segments", Mechanism and Machine Theory 42 (2007) 751–762.

[2] B. Math, Satish Chand, "An Approach to the Determination of Spur Gear Tooth Root Fillet", ASME 340 Vol. 126, MARCH 2004

[3] P.D. McFadden, Analysis of the Vibration of the Input Bevel Pinion in RANWessex Helicopter Main Rotor Gearbox

[4] WAK143 Prior to Failure, Aeronautical Research Laboratories Report No. AR004049, 1985

[5] C. Albrecht, Transmission design using finite element method analysis techniques, Journal of American Helicopter Society 33(2) (1988) 3–14.

[6] D.C. Couchan, G.K. Barnes, and R.W. Cedoz, ShotPeened Gear Failures Due to Operation in a Misaligned Condition, AIAA Paper No. AIAA932147, presented at the 29th Joint Propulsion Conference, Monterey, CA, 1993.

[7] H.L. Chang and Y.C. Tsai, A mathematical model of parametric tooth profiles for spur gears, ASME Journal of Mechanzcal Design 114, 8-16 (1992).
[8] H.L. Chang and Y.C. Tsai, A study of linear parametric tooth profiles using pressure angle, Journal of the Chinese Soczety of Mechanical Engineers 13 (l), 17-25 (1992).
[9] M.H. Tsai and Y.C. Tsai, Design of high-contact-ratio spur gear using quadratic parametric tooth profiles, Mech. Mach. Theory 33 (5), 551-564 (1998).
[10] C. Lee, H.H. Lin, F.B. Oswald and D.P. Townsend, Influence of linear profile modification and loading conditions on the dynamic tooth load and stress of high-contact-ratio spur gears, ASME Journal of Mechanical Deszgn 113, 473-480 (1991).
[11] C. Lee, H.H. Lin, F.B. Oswald and D.P. Townsend, Computer-aided design of high-contact-ratio gears for minimum dynamic load and stress, ASME Journal of Mechanics Design 115, 171-178 (March 1993).
[12] C. Lee, H.H. Lin, F.B. Oswald and D.P. Townsend, Effect of contact ratio on spur gear dynamic load with no tooth profile modifications, ASME Journal of Mechanzcs Design 118, 439-443 (September 1996).
[13] T.H. Costopoulos, A. Kanarachos and E. Pantazis, Reduction of delivery fluctuation and optimum tooth profile of spur gear rotary pumps, Mech. Mach. Theory 23 (2), 141-146 (X988).
[14] M.S. Tavakoli and D.R. Houser. Optimum profile modificatrons for the minimization of static transmission errors of spur gears, ASME Journal of Mechanzcal Design 108, 86-95 (March 1986).
[15] Kahraman and G.W. Blankenship, Effect of involute contact ratio on spur gear dynamics, ASME Journal of Mechanzcal Design 121, 112-118 (1999).
[16] E. Buckingham, Analytzcal Mechanzcs of Gears, Dover, (1988).
[17] F.L. Litvin, Gear Geometry and Applzed Theory, PTR Prentice Hall, New Jersey, (1994)
[18] Yahaya, S. H., and Ali, J.M., “Spur Gear Design with an S-Shaped Transition Curve Application Using MATHEMATICA and CAD Tools”. Proc of 2009 International Conference on Computer Technology and Development; 426-429
[19] Muhammad Abbas, S.H. Yahaya, Ahmad Abd Majid, Jamaludin Md. Ali ,“Spur Gear Tooth Design with S-Shaped Transition Curve using T-Bézier Function”, Procedia Engineering, Volume 50, 2012, Pages 211-221)
[20] Shigley, J.E and Charles R.M., 2003,”Mechanical Engineering Design”, Tata McGraw-Hill, New Delhi, 6th edition.

