Strong Semi Continuity in Topological Spaces

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Abstract— In this paper, we introduce and study strong semi continuity in topological spaces.

Key words: Strong Semi Continuity, Contra Strong Semi Continuity

I. INTRODUCTION & PRELIMINARIES

This section describes some weaker form sof continuous maps. Two of the oldest and most significant weaker forms of continuous maps are nearly continuous maps and quasi continuous maps.

In1922, Blumberg [9] introduced and studied the concept of nearly continuity for real valued functions on Euclidean spaces. This notion was generalized in 1958 to general topological spaces by Ptak[42]. Nearly continuity isalso known as almost continuity ([22], [23]) and pre continuity[30]. Even older is the notion of quasi continuity [24]. Several authors ([29], [33], [34], [10]) studied this notion. This notion appears under different names in literature, for example, the term neighborly continuity was used by Bledose[8],almost continuity by Frolik[16] and semi continuity by Levine[26]. Mashhour [31] introduced and studied α-continuity. Earlier to this, Noiri[37] introduced the same concept but under the name: strong semi continuity. Even Noiri, in his subsequent papers used the termα-continuity for this notion. Another weakerform of continuity is almostquasi continuity ([10], [36]). Recently Popa and Noiri[39] proved that the concept of almost quasi continuity coincides with the notion ofβ-continuity [1]. In recent literature the terms pre continuity, semi continuity andβ-continuity dominate. Biswas [7] introduced and studied the notion of simply continuity. Ewert [15] and Neubrun nova [35] also investigated the notion of simply continuity.

In recent years, several forms of generalized continuity have been introduced and studied by several authors ([3], [6], [11], [37], [39], [45]). Tong([46], [47]), Ganster and Reilly([17], [18]),Balachandran, Sundaram and Maki[5],Palaniappan and Rao[38], Maki([27], [28]), Arockiarani([2],[4]), and Gnanambal([19], [20]) introduced and studied A-continuous and B-continuous maps, LC-continuous and ic-continuous maps, generalized continuous maps, regular generalized continuous maps, $g\alpha$**-continuous maps andα- generalized continuous maps, generalized pre continuous maps and generalized pre regular continuous maps respectively. Now, werecallsomedefinitions.

A. Definition 1.1.

a) A map f : X $\rightarrow$Y is called
X for each open set V in Y,

a) weakly continuous[25]if for each point $x\in X$ and each open set V inY containing f(x), there exists an openset U in X containing x such that $f(U)\subseteq cl(V)$,
b) weak*continuous[25]if for each open set V in Y, $f^{-1}(Fr(V))$ is closed in X, where Fr(V) denotes the frontier of V,
c) simply continuous([7], [15], [35]) iff $^{-1}$(V)is simply open in X for each open set V in Y,.
d) α-continuous[31]if f $^{-1}$(V)isα-openinX,foreachopensetVinY,
e) A-continuous[46]if f $^{-1}$(V) isanA-setinXforeachopensetVin Y,
f) LC-continuous[17] if f $^{-1}$(V)isanLC setinXforeachopensetVinY,
g) B-continuous[47]if f $^{-1}$(V)isaB-setinXforeachopensetVinY,
h) C-continuous(Due to Sundaram[44])iff $^{-1}$(V)is a C-set(Due to Sundaram [44])in X for each open set V in Y,
i) C-continuous (Due to Hatir [21]) iff $^{-1}$(V) is a C-set (Due to Hatir[21]) in X for each open set V in Y,
j) ic-continuous [18]if f $^{-1}$(V)isanic-setin X for each open set V in Y,
k) strongly B-continuous [13] iff $^{-1}$(V)is a strong B-set in X for each open set V in Y.

B. Definition 1. 2.

A map f: X $\rightarrow$Yis called

a) pre continuous[30]iff $^{-1}$(F)is pre closed in X for each closed set F in Y,
b) β-continuous([5], [40]),iff $^{-1}$(F)is β-closed in X for each closed set F in Y,
c) generalized continuous(g-continuous)[5]iff $^{-1}$(F)is g-closed in X for each closed set F in Y,
d) regular generalized continuous(rg-continuous)[38]iff $^{-1}$(F)isrg closed in X for each closed set F in Y,
e) α-generalized continuous(αg-continuous) [28]iff $^{-1}$(F) is α g-closed in X for each closed set F in Y,
f) $g\alpha$**-continuous[27] if f $^{-1}$(F) is $g\alpha$**-closed in X for each closed set F in Y,
g) generalized pre continuous(gp-continuous) ([2],[4]) iff $^{-1}$(F) is gp-closed in X for each closed set F in Y,
h) Generalized pre regular continuous (gpr-continuous)[20]iff $^{-1}$ (F)is gpr-closed in X for each closed set F in Y.

C. Definition 1.3.

A map f:X $\rightarrow$Y is called A continuous if for each open set V in Y,f $^{-1}$(V) belongs to A, Where A is a collection of subsets of X.

D. Remark 1.4.

a) Definition1.3.contains the definitions of D(c,p)-continuity[41],D(c,α) continuity[41],D(α,p)-continuity[41], D(c,s)–continuity[14], D(c,ps) continuity[14], D(α,s)-continuity[14]and D(α,ps) -continuity[14].

b) Munshi and Basan[32] called a mapf:X $\to$Yg-continuous if f $^{-1}$(F)isg-closed in X for every g-closed set F in Y. Balachandran, Sundaram and Maki[5] and Sundaram[43] used the termgc-irre solute for this notion. In literature g-continuity of Balachandran, Sundaram and Maki[5] dominates. In the present work, the termg-continuity means g-continuity of Balachandran, Sundaram and Maki[5] . Also, in this work, the terms C-sets and C-continuity shall mean C-sets and C-continuity of Hatir, Noiriand Yuksel[21] and the terms
c) C(S)-sets and C(S)-continuity shall mean C-sets and C-continuity of Sundaram[44].

II. Strong Semi Continuity & Contra Strong Semi Continuity in Topological Spaces

In this section, we introduce and study the notions of strong semi continuity and contrast rong semi continuity in topological spaces.

A. *Definition 2.1.*

A map f:X $\to$Y is said to be strong semi continuous if f $^{-1}$(F)isstrong semi closed in X for every closed set F in Y.

B. *Definition2.2.*

A map f:X $\to$Y is said to be contra strong semi continuous if f $^{-1}$(V) is strong semi closed in X for every open set V in Y.

C. *Remark 2.3.*

The concepts of strong semi continuity and contra strong semi continuity are independent as seen from the following example.

D. *Example 2.4.*

LetX={a,b,c,d},τ={ϕ,{c},{d},{c,d},{a,c},{a,c,d},X},Y={x,y }and σ={ϕ,{x},Y}.
Define f:(X,τ)$\to$ (Y,σ) by f(a)=y and f(b)=f(c)=f(d)=x. Then f is strong semi continuous but not contrast rong semi continuous. Now, define g:(X,τ)$\to$ (Y,σ) by g(b)=g(c)=g(d)=y and g(a) =x. Then g is contrast rong semi continuous but not strong semi continuous.

E. *Proposition 2.5.*

Let S and T be subsets of X.
a) If S is strong semi closed in X, then S is semi closed in X,
b) If S is strong semi closed in X, then S is a D(c,p)-set in X,
c) If S is strong semi closed in X, then S is an LC-set in X.

F. *Theorem 2.6.*

A map f:X $\to$Y is said to be semi continuous if f $^{-1}$(F) is semi closed in X for every closed set Fin Y.

G. *Proposition 2.7.*

Iff: X $\to$Y is strong semi continuous, then it is semi continuous.

1) *Proof*

Let F be a closed set in Y and let f :X $\to$Ybe strong semi continuous. Then f $^{-1}$(F)is strongsemi-closedinX. By Proposition2.5 (a), f $^{-1}$(F) is is semi closed in X and the proof follows from Theorem 2.6.However, the converse need not be true as seen from the following example.

H. *Example 2.8.*

Let X={a,b,c} and τ={ϕ,{a},X}. Define f: (X,τ)$\to$(X,τ) by f (a) = f(b) =a and f(c) = c. Then f is semi continuous but not strong semi continuous, because f $^{-1}$({b,c})={c}is semi closed but not strong semi closed(X,τ).
Now, we give examples to show that strong semi continuity is independent of fifteen weaker forms of continuity.

I. *Example 2.9.*

Let X = {a,b, c} and τ={ϕ, {a}, {b}, {a, b}, X}. Define f:(X, τ) $\to$ (X,τ) b y f(a)=c and f(b)=f(c)=a. Then f $^{-1}$({b,c})=f $^{-1}$({c})={a}and f $^{-1}$({a,c})=X . The set{a}is strong semi closed but is neither closed nor g-closed nor rg-closed nor pre closed in(X,τ). Thus f is strong semi-continuous but is neither rg-continuous nor g-continuous nor continuous nor pre continuous.

J. *Example 2.10.*

Let X={a,b, c} and τ={ϕ,{a},X}. Define f:(X,τ)$\to$ (X , τ)by f(a)=b, f(b)=a and f(c)=c. Then f is rg-continuous and g-continuous but not strong semi continuous.

K. *Example 2.11.*

LetX={a,b,c},τ={ϕ,{a},X}andσ ={ϕ,{a},{a,b},{a,c},X}
Let f:(X,τ)$\to$ (X,σ) be the identity map. Then f $^{-1}$({b})={b},f $^{-1}${c}={c}andf^{-1}({b,c})={b,c} are α-closed in(X,τ) and the sets{b} and{c} are not strong semi closed in(X,τ).
Thus f isα-continuous but not strong semi continuous.

L. *Example 2.12.*

Let X={a,b,c,d} and τ={ϕ,{c}, {d}, {c,d}, {a,c}, {a,c,d},X}.
LetY={x,y}andσ={ϕ,{x},Y}.Definef:(X,τ)$\to$(Y,σ)byf(a)= yand f(b)=f(c)=f(d)=x.
Then f is strong semi continuous but not D(c,α)-continuous.

M. *Example 2.14.*

Let X= {a,b,c,d}, τ= {ϕ, {a},{b},{a, b}, {a,b,c}, {a,b,d},X}, Y={x,y} and σ={ϕ,{x},Y}. Define f:(X,τ) $\to$(Y,σ) by f(a)=f(d)=y and f(b) =f(c)=x. Then f is A-continuous but not strong semi continuous, because f $^{-1}$({x})={b,c}isanA-setin(X,τ)and f $^{-1}$({y})={a,d}is not strong semi closed in(X,τ).

N. *Example 2.15.*

Letf:(X, τ)$\to$(Y,σ) be as in Example 2.12. Then f is strong semi continuous and weakly continuous but is neither weak*continuous nor continuous.

O. *Example 2.16.*

Let X = {a, b, c, d}, τ = {ϕ,{c}, {d}, {c, d}, {a, c}, {a, c, d}, X},Y = {x,y,z} and σ= {ϕ, {z}, {x, y}, Y}.Define f: (X,τ)$\to$(Y,σ) by f(a) = f(b)= f(c) = x and f(d) = z. Then f is strong semi continuous and weak*continuous but is neither continuous nor weakly continuous.

P. Example 2.17.

Let X={a,b,c}, Y={a,b}, τ ={ϕ,{a},X} and σ={ϕ,{a},Y}. Define f:(X,τ) →(Y, σ) by f(a) = f(b) = a and f(c) = b. Then f is weakly continuous, rg-continuous and g-continuous but is neither continuous nor strong semi continuous.

Q. Example2.18.

Let X={a,b,c},Y={a,b},τ={ϕ,{a},X},and σ be the discrete topology on Y.

Define f:(X,τ) →(Y,σ) by f(a)=f(b)=a and f(c)=b. Then f is weak* continuous, rg-continuous and g-continuous but is neither continuous nor strong semi continuous.

R. Example 2.19.

LetX= Y ={a,b,c},τ={ϕ,{a},{b}, {a,b}, {b, c}, X} and σ = {ϕ,{a, c},Y}, Let f:(X,τ) →(Y, σ) be the identify map. Then f^{-1}({a, c}) = {a, c} is a strong B-set in(X, τ) and f^{-1} ({b}) = {b} is not strong semi closed in (X, τ). Hence, f is strongly B-continuous but not strong semi continuous.

S. Example 2.20.

LetX= Y ={a,b,c},τ ={ϕ,{a},{b}, {a,b},X} and σ = {ϕ,{b, c},Y}. Let f:(X,τ) →(Y, σ) be the identity map. Then f is strong semi continuous but not strongly B-continuous, because {a} is strong semi closed and {b,c} is not a strong B-set in (X, τ).

T. Theorem 2.21.

Let S be a subset of X. Then the following are equivalent.

a) S is an ic-set in X,
b) s int(S) = int(S).

U. Note 2.22.

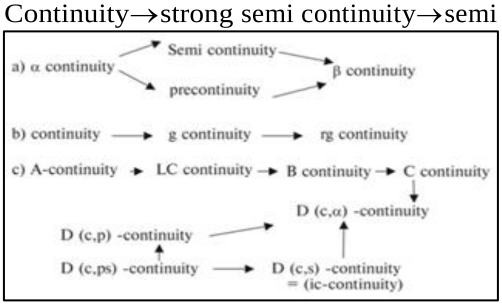
In the light of the above theorem, we see that the notions of D (c,s)-set and D(c,s)-continuity coincide with the notions of ic-set and ic-continuity respectively.

III. REMARK

From[14],[21]and[38],we have the following implications and none can be reversed.

FromExamples2.9. – 2.20., and from the above implications, it is clear that strong semi continuity is independent of α-continuity, pre continuity, g-continuity, rg-continuity, A-continuity, LC-continuity, B-continuity, C-continuity, D(c,α)-continuity, D(c, p)-continuity, D(c,ps)-continuity, D(c, s)-continuity (=ic-continuity, by Note 2.22.), weakly continuity, weak*continuity and strongly B-continuity. Also, we have the following implications. None is reversible.

Continuity→strong semi continuity→semi



continuity→β-continuity

Dontchev studied the notion of contra continuity [12].Now let us recall the definition of contra continuity.

A. Definition3.1.

A map f: X→Y is said to be contra continuous if f^{-1}(V)is closed in X for every open set V in Y.

B. Proposition 3.2.

Let S be a subset of X. If S is closed in X, then S in strong semi closed in X.

C. Proposition 3.3.

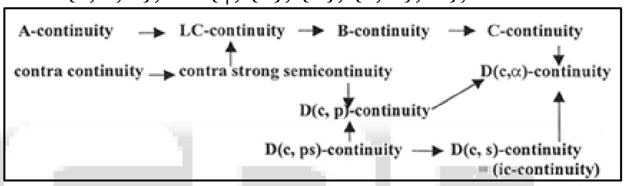
If f:X→Y is contra continuous, then f is contra strong semi continuous.

1) Proof.

Let f:X→Y be contra continuous and let V be an open set in Y. Then f^{-1}(V) is closed in X. By Proposition3.2.f^{-1}(V)is strong semi closed.Therefore f is contra strong semi continuous. However, the converse need not be true as seen from the following example.

D. Example 3.4.

LetX= {a, b, c}, τ = {ϕ, {a}, {b}, {a, b}, X},



Y = {x, y} andσ= {ϕ, {x},Y}.Define f:(X,τ)→(Y,σ)by f(a) = xandf(b)=f(c) = y. Thenf $^{-1}$({x})={a} is strong semi closed but not closed in (X,τ). Therefore f is contra strong semi continuous but not contra continuous.

E. Remark 3.5.

The concept of A-continuity is independent of contra continuity and contra strong semi continuity as seen from the following example.

F. Example 3.6.

Let X={a,b,c}, τ={ϕ,{a},X},Y={x,y} and σ={ϕ,{x},Y}. Define f:(X,τ)→(Y,σ) by f(a)=x and f(b)=f(c)=y. Then f is continuous, A-continuous, LC-continuous, B-continuous, C-continuous, D(c,α)-continuous, D(c,p)-continuous, D(c,s)-continuous and D(c,ps)-continuous but is neither contra continuous nor contra strong semi continuous. Also define g:(X,τ)→(Y,σ)by g(a)=yandg(b)=g(c)=x. Then g is contra continuous and contra strong semi continuous but not A-continuous.

G. Proposition3.7.

If f:X→Y is contra strong semi continuous, then fis LC-continuous.

1) Proof

Let f:X→Y be contra strong semi continuous and let V be an open set in Y. Then f^{-1}(V) is strong semi closed in X. By Proposition 2.5.(c),f $^{-1}$(V) is an LC-set. Therefore f is LC-continuous.

However, the converse need not be true as seen from the following example.

H. Example 3. 8.

Let X={a,b,c}, τ={ϕ,{a},{a,b},X}, Y={x,y} and σ={ϕ,{x},Y}. Define f:(X,τ)→ (Y,σ) by f(b)=x and f(a)=f(c)=y.Then f is LC-continuous but not contra strong semi continuous.

I. Proposition 3.9.

Let f:X→Y be contra strong semi continuous. Then f is D(c,p)-continuous.

1) Proof

The proof follows from Proposition 2.5. (b) The converse need not true as seen from Example3.6.

J. Remark 3.10.

The notion of contra strong semi continuity is independent of D(c,s)-continuity and D(c,ps)-continuity as seen from Example3.6. and from the following example.

K. Example3.11.

Let X={a,b,c,d}, τ ={ϕ,{c}, {d},{c,d},{a,c},{a,c,d},X},Y ={x,y} and σ={ϕ,{x},Y}. Define f:(X,τ)→(Y,σ) by f(a) =f(c)=y and f(b)=f(d)=x. Then f is contra strong semi continuous but is neither D(c,s)-continuous nor D(c,ps) -continuous, for,{b,d}is strong semi closed but is neither a D(c,ps)-set nora D(c,s)-set in(X, τ).

L. Remark 3.12.

From Tong([46],[47]), Hatir[21], Dontchev and Przemski[14] and from the above results and examples related to contra strong semi continuity, we have the following implications. None of the implications can be reversed.

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