A Review On: Fem Modelling of Cantilever Beam

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Abstract— During the last few decades, intense research on the detection of crack using the vibration based techniques has been done and various approaches have been developed by researchers. In the present paper, detection of the crack presence on the surface of beam-type structural element using natural frequency is presented. First two natural frequencies of the cracked beam have been obtained experimentally and used for detection of crack location and size. Detected crack locations and size are compared with the actual results and found to be in good agreement. Also, the effect of the crack location and the crack depth on the natural frequency is presented. Crack hinders the optimum performance of a machine. Presently most of the failures encountered by machines are due to material fatigue. Therefore crack detection and localization is the main topic of discussion for various researchers across the globe. The dynamic behaviour of a whole structure is affected due to the presence of a crack as the stiffness of that structural element is altered. The cracks in the structure change the frequencies, amplitudes of free vibration and dynamic stability areas to an inevitable extent. So a diagnosis of the changes allows the experimenter to identify the cracks without aborting the system applications. The effect of an open crack on the modal parameters of the cantilever beam subjected to free vibration is analysed and the results obtained from the numerical method i.e. finite element method (FEM) and the experimental method are compared. It is concluded that results obtained from experiment have a very good agreement with the results obtained from FEM and the structure vibrates with more frequency in the presence of a crack away from the fixed end.

Key words: ANSYS, Transversely Isotropic Piezoelectric, Stress Concentration, Complex Variation Method, Elliptic Cross bones, Rectangular Blocks, Poisson’s Ratio, Three-Dimensional Finite Element Method

I. INTRODUCTION

Beams are common elements of many architectural civil and mechanical engineering structures beams and the study of the bending of straight beams forms an important and essential part of the study of the broad field of mechanics of materials and structural mechanics. All under graduate courses on these topics include the analysis of the bending of beams, but only small deflections of the beam are usually considered. In such as case, the differential equation that governs the behavior of the beam is linear and can be easily solved. However, we believe that the motivation of students can be enhanced if some of the problems analyzed in more specialized books on mechanics of materials or structural mechanics are included in the undergraduate courses on these topics. However, it is evident that these advanced problems cannot be presented to undergraduate students in the same way as is done in the specialized monographs. The purpose of this paper is to analyze a simple laboratory experiment in order to introduce the concept of geometric nonlinearity in a course on mechanics or strength of materials. This type of nonlinearity is related to the nonlinear behavior of deformable bodies, such as beams, plates and shells, when the relationship between the extensional strains and shear strains, on the one hand, and the displacement, on the other, is taken to be nonlinear, resulting in nonlinear strain-displacement relations. As a consequence of this fact, the differential equations governing this system will turn out to be nonlinear. This is true in spite of the fact that the relationship between curvatures and displacements is assumed to be linear. The experiment will allow students to explore the deflections of a loaded cantilever beam and to observe in a simple way the nonlinear behavior of the beam. The experimental analysis is completed with a numerical analysis of the system using the ANSYS program, a comprehensive finite element package, which enables students to solve the nonlinear differential equation and to obtain the modulus of elasticity of the beam material. To do this, students must fit the experimental results of the vertical displacement at the free end to the numerically calculated values for different values of the modulus of elasticity or Young's modulus by minimizing the sum of the mean square root. Using the modulus of elasticity previously obtained, and with the help of the ANSYS program, students can obtain the elastic curves of the cantilever beam for different external loads and compare these curves with the experimental ones. ANSYS is a finite element modeling and analysis tool. It can be used to analyze complex problems in mechanical structures, thermal processes, computational fluid dynamics, magnetics, and electrical fields, just to mention some of its applications.

All structures are prone to damage, may be due to over-stressing in operation or due to extreme environmental conditions or due to any accidental event. Crack present in the component may grow during service and may result in the component failure once they grow beyond a critical limit. It is desirable to investigate the damage occurred in the structure at the early stage to protect the structure from possible catastrophic failures. There are various Non-destructive techniques (NDTs) available for the detection of the discontinuities in the structural components and the mechanical components. They are efficient but time consuming, expensive and laborious, particularly for slender beam like components. The crack present in the component imparts local flexibility to the component and reduces the natural frequency of free vibration of the component. The local damage also affects the mode shapes of the vibration of the component. Thus, Vibration Based Inspection (VBI) can be a potential method for crack detection. Though, there has been an intense study on crack detection through vibration based inspection, there is a need to develop an effective and economically appropriate approach. Vibration principles are the inherent properties of the physical science applicable to all structures subjected to static or dynamic loads. All structures again due to their rigid nature develop some irregularities in their life span which leads to the development of crack. The problem on crack is the basic problem of science of resistance of materials. Considering the crack as a significant form of such damage, its modeling is an important step in studying the behavior of damaged structures. Knowing the effect of crack on stiffness, the beam or shaft can be modeled using either Euler-Bernoulli or Timoshenko beam theories. The beam boundary conditions are used along with the crack compatibility relations to derive the characteristic equation relating the natural frequency, the crack depth and location with the other beam properties. Researches based on structural health monitoring for crack detection deal with change in natural frequencies and mode shapes of the beam. It have presented a method for determining the location and crack depth in double cracked beam. For diagnosing the crack, variation in natural frequency and anti-resonance properties are used by them. It have used mid frequency flexural wave to detect the presence of crack and its location in a cracked beam. They have proposed a method assuming an open and closed crack with triangular disk finite elements. He has analyzed the forced vibrations of the beam, the effects of the crack locations and sizes on the vibration behavior and discussed a basis for crack identification. They have developed a method for calculating the natural frequencies of continuous beams of uniform span length by vibration analysis. Bollinger have presented a method for analysis and prediction of the static and dynamic behavior of machine tool spindle systems using finite difference technique. Also have developed a transfer matrix method for the prediction of natural frequencies and normal modes of a row of skin-stringer panels. They have also presented few examples. Miles has carried out analysis of beams on many supports using vibration parameters. Also have briefly surveyed the use of transfer matrix method for analyzing the dynamic behavior of beam structures. It has considered the free vibration of a beam hinged at one end by a rotational spring (with a constant spring constant) and the other end free. In the theoretical section natural frequency is determined from the elastic property expressions by modeling the crack as mass less spring of cracked cantilever beam. The presence of crack changes the stiffness of the beam, so the vibration parameters of the beam. The theory is approved by performing experiments on cantilever beam specimen with crack and without crack. Natural frequencies of the cantilever beam specimen with crack and without crack are found out both numerically and experimentally for the same crack depth and different crack position with respect to the fixed end of the cantilever beam. Different natural frequencies are observed at different position of the crack.

Dynamical study of such components with presence of breathing crack is complicated due to repetitive opening and closing of crack surfaces leading to a highly nonlinear system. The crack surfaces open and close during each vibration cycle with contact happening between the two, leading to stiffness discontinuity. The other source of nonlinearity is geometric nonlinearity due to large deformation as compared to the geometric dimensions. Accurate analysis of this system, typically with Finite Element methods would generally lead to large number of degrees of freedom especially due to refined mesh near the crack tip and contact behavior between crack surfaces making the problem computationally intensive and costly. Hence, a reduced order scheme is formulated which reduces computational time and cost to a considerable amount. We study dynamics of a nonlinear cracked cantilevered beam as a simplified model for a cracked rotor blade. Subspace based projection method is used to reduce the modeling space of the system by projecting the dynamics onto the dominant nonlinear normal modes (NNM), hence preserving the nonlinearities in the system. Proper Orthogonal Modes (POM), which are the linear best fit to the NNM and can be obtained from a short but detailed FE analysis of the complete structure and are used as the basis function here. This results in a set of nonlinear ODE's that are solved simultaneously, using numerical means to predict the long term behavior of the system. MEMS cantilever gives quick and exact response based on bimolecular activities as compare to the conventional cantilever design. The small size and less amount of analyte required in detection compose MEMS cantilever more applicable in lower concentration biosensing. In such cantilever, the settling time of biomolecule, which is mainly taken by the analyte molecule to settle down on the sensor surface, decreases and provides rapid detection of biomolecules. There are two operating mode of cantilever: static and dynamic. In former, the deflection is measured by the pressure produced by the biomolecules on the cantilever surface. In later, the concentration of biomolecules is measured by detecting the change in oscillating frequency with respect to the change in mass, where applied sinusoidal voltage is kept constant. The deflection at the micro cantilever beam can be monitored by various methods namely, optical reflection, capacitive and electron tunneling, interferometric and piezoelectric based detection. The optical reflection is most effective method for deflection detection.

A. *Differential Equations for Displacement Functions*

A circularly curved beam carrying n + 1 sets of concentrated elements (CE's) with each set of CE's including a lumped mass (with eccentricity ei and mass moment of inertia, a radial linear spring with stiffness, a tangential linear spring with stiffness and a rotational spring (about axial y-axis) with stiffness (for i = 0 to n) and performing free vibrations in the vertical xz-plane. For convenience, one local coordinate system xyz(h) and one global coordinate system xyz with y-axis parallel to y-axis are introduced. If the radial x-axis is a symmetric axis for the cross-section of the arch, and the symbols ux, uh and wy represent the displacement in radial x-direction, the displacement in tangential z(h)-direction and the rotational angle about axial y-axis, respectively, then, with effect of shear deformation neglected, the relationships between the element forces (Fx,Fh,My) and the element displacements (ux,uh,wy).

Where the primes (0) denote differentiations w.r.t. the angular coordinate h, Fx and Fh are the radial and tangential forces in the positive x and z(h) directions, respectively, My is the bending moment about y-axis, A is the cross-sectional area, R is the average radius of curvature of the arch, E is Young's modulus, and Iy is the moment of inertia of the area A about the y-axis.

B. *Theory & Operating Principle*

The principle of Bio-MEMS cantilever sensor is mainly based on the method of functionalization. In this method, bio receptors are coated on the micro cantilever surface to make it bio sensitive. The biochemical reaction between the applied analyse and the baroreceptor can be observed by the deformation of micro cantilever geometry. This deformation may be in the form of upward or downward deflection, which depends on the internal stress of the beam. For the rectangular micro cantilever beam the free end deflection increases with the increment in beam length and decrement in beam thickness. Materials which are used for the micro cantilever design also effect its free end deflection. More elastic material gives more deflection at the micro cantilever free end. According to Hook's law the free end deflection for micro cantilever beam is directly proportional to the applied load.

$$\delta \propto F$$
$$\delta = aF$$

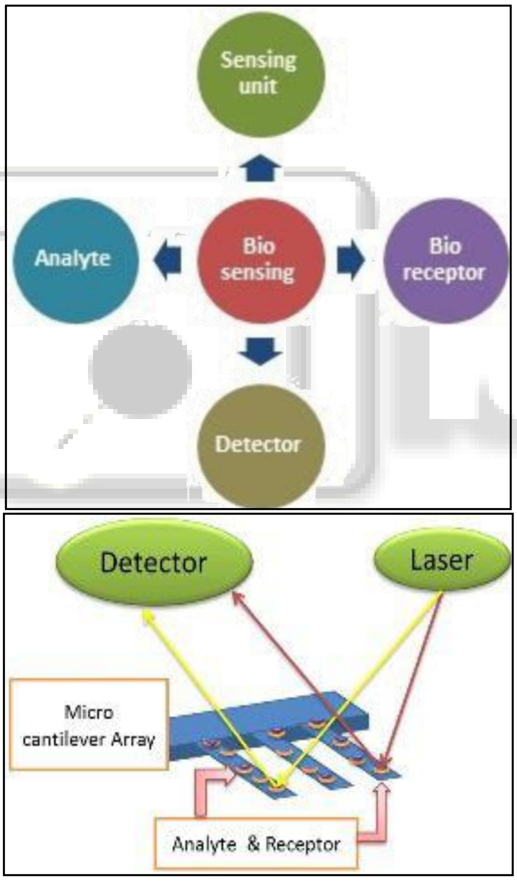

Fig. 1. (A): Basic Elements of Bio Sensing ; (B) Cantilever Sensor Setup for Bio Sensing

Bio-molecular analysis requires four elements: a target analyte, a specific bio receptor, a sensing unit and a detecting unit, as shown in Fig. 1(a). In "in vivo analysis", sensing unit need to be very sensitive. If the concentration of target analyte solution is small then it produces less pressure at the sensor surface. This less pressure generated by biomolecular event is unable to sense by the conventional rectangular micro cantilever based sensor. This paper focuses on the high sensitivity and better deflection at the micro cantilever free end for low mass loading. Based on this objective, new micro cantilever designs with better efficiency and sensitivity are proposed in this paper. For analyzing the deflection value, the surface area of all proposed designs is kept constant (5000µm2). The length and thickness of the proposed designs are also same as the conventional rectangular micro cantilever beam design.

Where 'p' is the pressure generated by the biomolecules on the micro cantilever surface. Parameter ' ' and ' ' represent the stress and Poisson ratio for the beam. In [10] the maximum surface stress generated by injection of myoglobin protein on eight cantilever array is 0.05 N/m. For the conventional rectangular micro cantilever beam, which is shown in Fig. 2(a), if the value of length (l), width (w) and thickness (t) are 500 µm, 100 µm and 50 µm,

Where, a =1/k, is the deflection at the cantilever free end & is the force produced by the biochemical reaction between the analyte and bioreceptor. Parameter k is the spring constant which mainly depends on flexural rigidity. In static mode, the sensitivity of micro cantilever beam can be defined as, the change in free end deflection for the change in mass loading on micro cantilever surface, then the pressure produced by the surface stress (0.05 N/m) is 19.2 Pa. The deflection at the free end of the rectangular cantilever beam for this pressure is 0.11×10-9.

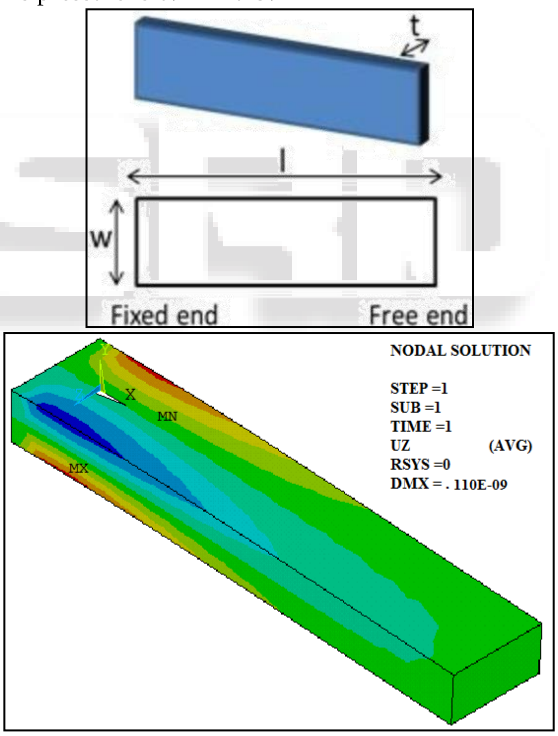

Fig. 2(A): Rectangular Microcantilever Beam Design; (B) It's Free End Deflection

C. *Proposed Micro Cantilever Designs*

We have proposed three new micro cantilever designs for the detection of biomolecules. These beam designs can be effectively used in 'in vivo analyses, where mass loading on the cantilever surface is very less. In such application these designs give better (nearly double) deflection at the free end of micro cantilever beam as compare to conventional rectangular one. A finite element method software ANSYS 12.1 is used for analyzing the deflection of the conventional rectangular and new proposed micro cantilever beam designs.

In this analysis element SOLID 20 node 186 is used for designing these beams. For defining the material properties, the density of material, Young's modulus and Poisson ratio are selected as 2.33 gm /cm3, 130 GPa and 0.28, respectively.

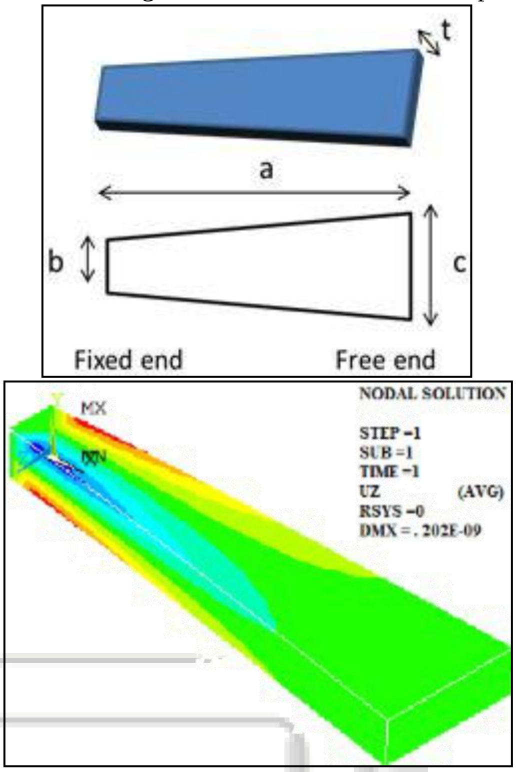

Fig. 3:

In this proposed design the width of beam increases linearly with the increment of beam length. The beam width at fixed end is narrow as compare to the width at the free end. This width variation from fixed end to free end gives the trapezoidal shape of micro cantilever beam shown in Fig. 3(a). Parameter 'a' and't' represent the complete length and thickness of this beam design. The width at fixed end and free end can be represented by the parameter 'b' and 'c', respectively. The value for parameter 'a', 'b' and 'c' are selected as 500 µm, 50 µm and 150 µm, respectively. The thickness 't' is 50 µm which is same as the rectangular beam.

A square step of size 50×50×50 is introduce at the fixed end of the trapezoidal shape design [discussed in previous section] such that the surface area and length are constant as the conventional rectangular beam. This gives the trapezoidal beam design with a square step at fixed end, as shown in Fig. 4(a). The length and thickness of this proposed beam are same as the rectangular beam. Parameter 'd', 'a' and 't' represent the length of square step, length of complete beam, and thickness of beam, respectively. The width of beam at the fixed end and free end of beam can be represented by parameter 'b' and 'c', respectively. The value for parameter 'a', 'b', 'c' and 'd' is selected as 500 µm, 50 µm, 161 µm and 50 µm, respectively. The thickness 't' is 50 µm which is same as the rectangular beam.

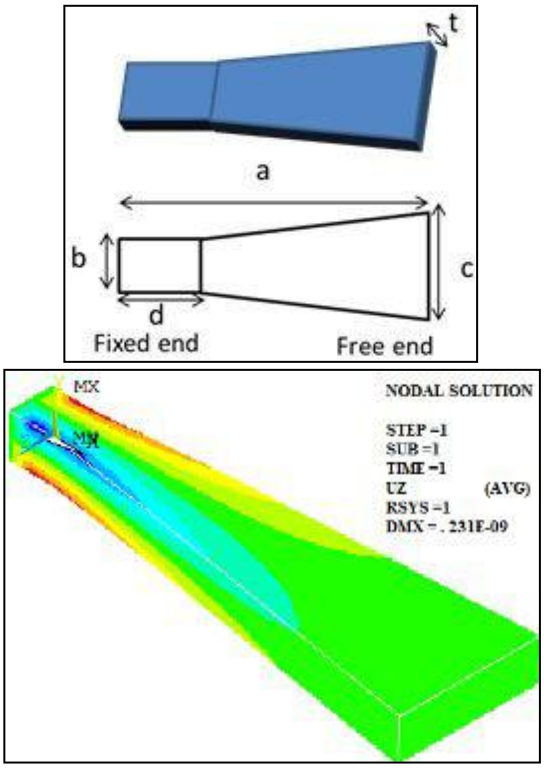

Fig. 4: Shows the Deflection at the Free End of This Proposed Micro cantilever Design with Constant Surface Area

(50,000µm2) and pressure (19.2Pa). The design gives the deflection of 0.231×10-9 m which is approximately twice as the rectangular microcantilever beam's deflection.

D. Natural Frequencies & Mode Shapes of A Cantilever Beam Carrying A Two-D.O.F. Spring} Mass System

A uniform cantilever beam carrying a two-d.o.f. spring}mass system at the free end. All the dimensions and material constants of the beam and the spring}mass system are exactly equal to those of the last exmple except that the boundary conditions of the beam were changed from clamped}clamped to clamped}free ones and the suspension positions of the two-d.o.f. spring}mass system were also moved from and where ¸ is the total length of the beam. The natural frequencies obtained from the three methods are shown in It is evident that the values of u6 *s* (*s*"1}5) obtained from the FEM2 (a) and ANCM(a) are very close to the corresponding ones obtained from FEM1. Besides, the value obtained from the FEM2 (b) are also in good agreement with those obtained from ANCM (b). The "rst "ve mode shapes obtained from FEM1, FEM2(a) and ANCM(a) are almost overlapped and are shown in Figure 6(b). For convenience of comparison, the mode shapes of the unconstrained (bare) uniform cantilever beam are placed.

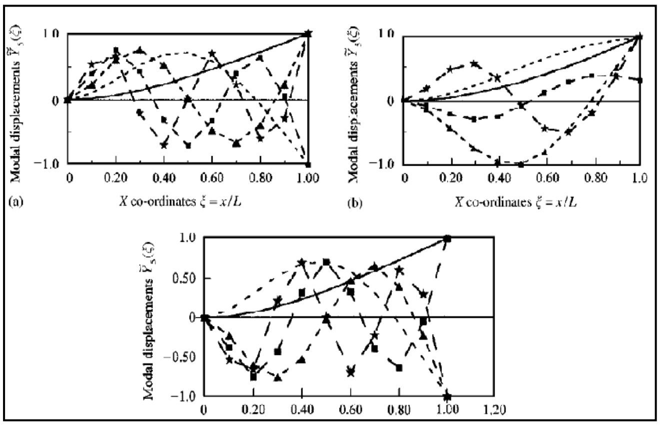

Fig. 5: The "st "ve Mode Shapes of a Uniform Cantilever Beam in the Conditions that: (a) Nothing Attached; (b) Carrying a Single two-d.o.f. spring}Mass System s; (c) Carrying Three two-d.o.f. Spring}Mass Systems , the 1st Mode , the 2nd Mode; -m-, the 3rd Mode, the 4th Mode, the 5th Mode

By comparing the present results with those of the last sections, one "nds that the in#uence on the dynamic characteristics of the constrained beam due to a single two-d.o.f. spring}mass system is much more than that due to the multiple two-d.o.f. spring}mass systems. This phenomenon seems to be like the e!ects on a beam due to the action of a concentrated load and the distributed loads.

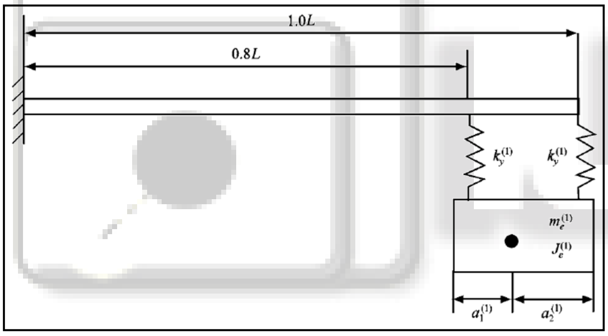

Fig. 6: A Uniform Cantilever Beam Carrying a two-d.o.f. spring}Mass System At The Free End.

II. Experimental details

In the laboratory it is possible to design simple experiments in order to analyse the deflection of a cantilever beam with a tip load applied at the free end. For example, Fig. 1 shows a photograph of a system made up of a flexible steel beam of rectangular cross-section built-in at one end and loaded at the free end with a mass. The beam is fixed to a vertical stand rod by means of a multi-clamp using two small metallic pieces, which provide a better support (Fig. 2). The length of the beam is L.0.40m and it has a uniform rectangular cross-section of width b.0.025m and height h.0.0004 m. The weight of the beam and the value of the load uniformly distributed over its entire length are W.0.3032N and w.W/L.0.758 N/m, respectively.

With this experimental set-up the students can, for instance, determine the vertical deflection of the end free as a function of the applied load, or the shape the beam adopts under the action of that force, by using vertical and horizontal rulers as can be seen in Fig. 3. This figure shows the procedure followed to obtain the experimental measurements of the elastic curve of the beam as well as of the horizontal and vertical displacements at the free end. The students can relate these measurements to geometric parameters of the beam (its length and the moment of inertia of its rectangular crosssection), as well as to the material of which it is made (using Young's modulus). This system is made up of very simple elements and only easy experimental measurements (basically lengths and masses) need be made. In addition, mathematical treatment of the equilibrium of the system does not involve great difficulty.

Let us consider the case of a long, thin, cantilever beam of uniform rectangular cross section made of a linear elastic material, whose weight is W, subjected to a tip load F as shown in and 5. In this study, we assume that the beam is nonextensible and the strains remain small. Firstly, we assume that Bernoulli-Euler's hypothesis is valid, that is, plane cross-sections which are perpendicular to the neutral axis before deformation remain plane and perpendicular to the neutral axis after deformation. Next, we also assume that the planesections do not change their shape or area.

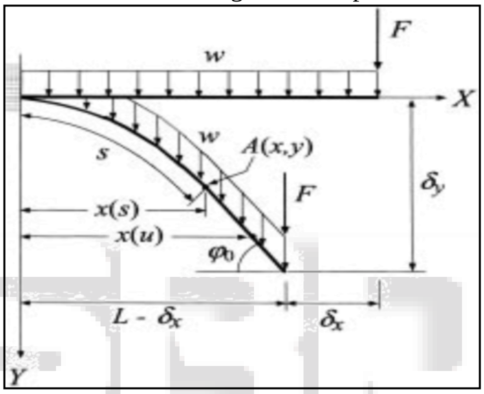

Fig. 7: Scheme of the Cantilever Beam under the Action of a Uniformly Distributed load and a Vertical Concentrated load at the Free End, and Definition of Parameters.

In Fig x and y are the horizontal and vertical displacements at the free end, respectively, and '0 takes into account the maximum slope of the beam. We take the origin of the Cartesian co-ordinate system at the fixed end of the beam and let (x, y) be the co-ordinates of point A, and s the arc length of the beam between the fixed end and point A. The bending moment M at a point A with Cartesian co-ordinates (x, y) can be easily obtained from a free-body diagram of the segment from u.s to u.L.

$$M(s) = \int_s^L w[x(u) - x(s)]\, du + F(L - \delta_x - x)$$

Where Lÿ_x ÿx is the distance from the section of the beam at a point A to the free end where force F is applied, and u is a dummy variable of s.

The nonlinear differential equation that governs the deflections of a cantilever beam made of a linear material under the action of a uniformly distributed load and a vertical concentrated load at the free end.

III. Results

We shall now study the large deflections of a cantilever beam using the ANSYS program, a comprehensive finite element package. We use the ANSYS structural package that simulates both the linear and nonlinear effects of structural models in a static and a dynamic environment. Firstly we have to obtain the Young's modulus of the material. To do this, we obtain experimentally the values of the vertical displacements at the free end, _y, for different values of the concentrated load F applied at the free end of the beam. We consider seven values for F: 0, 0.098, 0.196, 0.294, 0.392, 0.490 and 0.588N and we obtain the theoretical value of _y for different values of E around the value of E.200GPa (the typical value of Young's modulus for steel) using the ANSYS program. We obtain the value of Young's modulus E by comparing the experimentally measured displacements at the free end _y,exp(Fj), where j.1, 2, . . . , J; J being the number of different external loads F considered (in our analysis J.7), with the numerically calculated displacements y(E, Fj). We obtain the value of E for which the sum of the mean square root _2 is minimum, where _2 is given by the following equation.

$$\chi^2(E) = \sum_{j=1}^{J} [\delta_y(E, F_j) - \delta_{y,\exp}(F_j)]^2$$

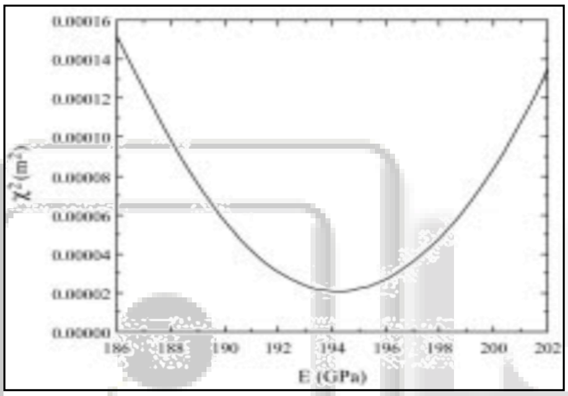

Fig. 8: We Have Plotted the Calculated Values of 2 as a Function of E

The displacement _y at the free end of the cantilever beam as a function of the external vertical concentrated load F applied at the free end. The dots correspond to the experimental data and the continuous lines to the displacements numerically calculated with the help of the ANSYS program, using the value obtained for Young's modulus, E.194.3 GPa. Comparing the experimentally measured displacements with the calculated values, we can see that the agreement is satisfactory.

The natural frequency is greatly affected by crack depth and crack location. To illustrate this, the condition *A* 0is incorporated for different crack depth, crack location and frequency. It shows the effect of the crack location on the first natural frequency for various crack depths whereas shows the effect of the crack depth ratio on the first natural frequency for various crack locations. It can be observed that the crack of a particular size present near the fixed end reduces the natural frequency significantly higher than the crack of the same size present closer to the free end. It can also be observed that the higher crack depth ratio has more effect on the normalized natural frequency than that of smaller ratio. A contour diagram of natural frequency for each crack location and depth is prepared and presented in the Fig. 9. From this diagram, the depth of the crack can be read from its cracked natural frequency, if the position of the crack is known.

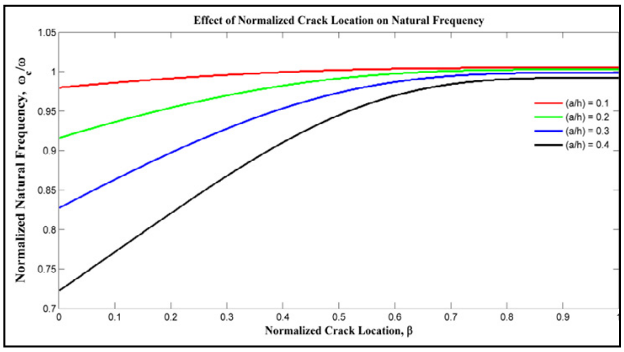

Fig. 9: Effect of Normalized Crack Location on the Normalized Natural Frequency

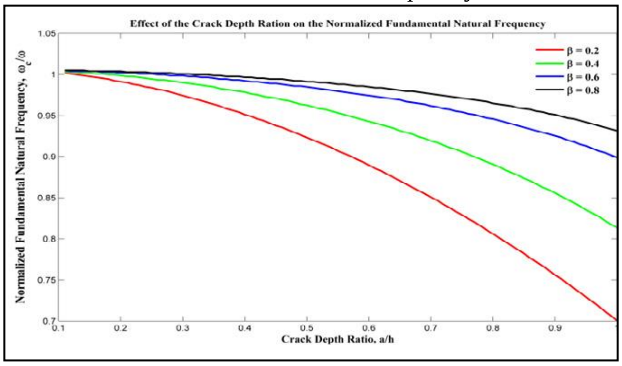

Fig. 10: Effect of the Crack Depth Ratio on the Normalized Natural Frequency

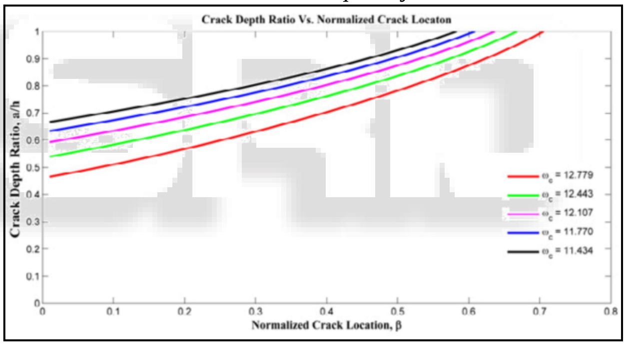

Fig. 11: Contours of Crack Depth Ratio to Normalized Crack Locations for Various Normalized Natural Frequency of Cantilever Beam with Single Crack

The experimental setup is developed for obtaining natural frequencies of cracked and uncracked beaM. The beam dimensions and its material properties are as follows: Length (*L*) = 0.78 m, width (*b*) = 0.04 m and depth (*h*) =0.01 m, mass density ( ) = 7860 kg/m3 and modulus of elasticity (*E*) = 210 GPa. The cracks are developed using wire-cut electro-discharge machining and natural frequencies are measured using LMS make FFT analyzer.

The experimental natural frequencies of the different cantilever beam are then utilized in the mathematical formulation presented here, to predict the crack location and size. Corresponding to first and second natural frequencies, the variation of normalized crack depth (a/h) with normalized location (x/L) is obtained, as shown in the Fig. 6. The intersection of these two graphs will precisely predict the crack location and the size. The Fig. 6 is prepared for the natural frequencies corresponding to *a* / *h* 0.5 and 0.5. The predicted location and the crack size are 0.52 and *a* / *h* 0.48, respectively. For different crack locations, natural frequencies are obtained and crack locations and the crack sizes are predicted as mentioned.

A. Results on Mems System

In triglyceride detection [4], the reaction between lipid and lipase alter the density of the de-ionized solution. The change in density causes change in pressure produced by the biomolecules and effects the free end deflection of the beam. The pressure 19.2 Pa which is equivalent to the surface stress 0.05N/m produces the free end deflection of 0.110×10-9 m for the conventional rectangular micro cantilever beam. Same pressure produces 0.202×10-9 m, 0.231×10-9 m and 0.213×10-9 m deflection at the free end of first, second and third proposed micro cantilever beam designs, respectively. Hence, the deflections for the proposed micro cantilever designs are nearly 2× then the conventional rectangular micro cantilever beam. Table 1 represents the deflection for the conventional rectangular and the proposed micro cantilever beams with the same surface area (5,000µm2) and constant pressure 19.2 Pa.

Beam type	Free end deflection
Rectangular design	0.110×10^{-9}m
First proposed design	0.202×10^{-9}m
Second proposed design	0.231×10^{-9}m
Third proposed design	0.213×10^{-9}m

Table 1: Comparison Table

In all these proposed designs, the deflection at the micro cantilever free end increases because of large surface area at the free end of beam as compare to fixed end of beam. The bio receptors attached at the micro cantilever surface are more at the free end and bind almost all analyse molecules at the free end of beam. The biochemical reaction relatively large due to large number of binding at the micro cantilever free end, which exhibits more deflection at the free end of all proposed designs as compare to conventional rectangular micro cantilever beam.

While in the proposed micro cantilever beam designs width increases linearly or non-linearly with respect to length. This gives the change in centre of mass of the beam. Because of the increment of width from fixed end to free end, the centre of gravity shifted toward the free end. Hence, small loading of biomolecules give measurable deflection at the micro cantilever free end.

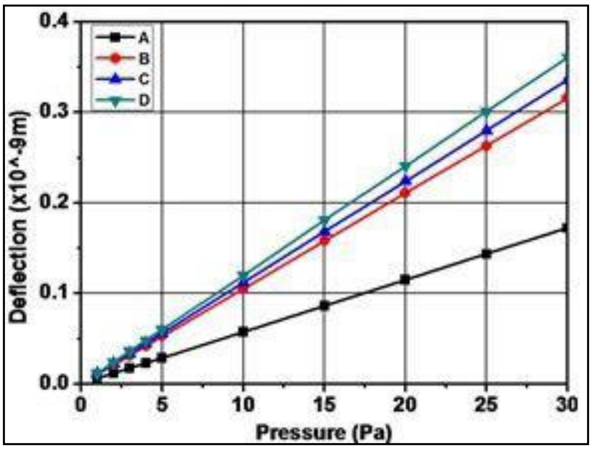

Fig. 11: Free End Deflection Verses Applied Molecular Pressure

This concluded that, with the same surface area (5,000µm2) and same molecular pressure (19.2Pa) generated by analyte-receptor molecules, the proposed designs exhibit more deflection at free end and for the variation in molecular pressure the proposed designs are more sensitive than the conventional rectangular micro cantilever beam

IV. Conclusion

- We have studied the deflections of a cantilever beam theoretically, experimentally and numerically. Firstly, we obtained the equations for large deflections, and by analysing this equation students can see that, although they are dealing with a simple physical system, it is described by a differential equation with a nonlinear term.
- The present method to detect crack location and size is fast and efficient.
- Crack with larger crack depth ratio (a/h) imparts greater reductions in natural frequency than that of the smaller.
- Crack depth ratio. Hence, the accuracy of results improves as crack depth increases. Crack present near to fixed end imparts greater reductions in natural frequency than that to present at away from the fixed end.
- For a given elliptic shape factor t, the ratio of the maximum SCF $(Kt)_{max}$ to the $(Kt)_{surf}$ on the free surface is a monotonic increasing function of the thickness to root radius ratio B/q; and the ratio of the $(Kt)_{surf}$ to the corresponding plane stress value (Kt)p-6 is a monotonic decreasing function of B/q; they will approach to different constants with increasing B/q. The ratio $(Kt)_{max}$/(Kt)p-6 is not a monotonic function of B/q, it increases from its plane stress value at B/q = 0 to its peak value and then decreases gradually to a constant with increasing B/q.
- The corresponding empirical formulae among the interior maximum SCFs, the SCFs on the free surfaces and the corresponding planar solutions for 3D elastic isotropic plates, each containing an elliptic hole, were obtained, which are the functions of B/q and t. The errors of the empirical formulae to the corresponding FE results are less than 1.5% in all cases studied in this paper. The proposed formulae are useful for strength and fatigue design of practical engineering structures with notches and holes, and similar defects.
- It should be noted that large plates with width and height 20 times of the major axis of the elliptic hole are used, so that in-plane size effect cannot be considered. It also should be aware that the Poisson's ratio u can influence the out-of-plane stress constraint and u = 0.333 is used in this paper. When u is much higher than 0.333, the present formulae for $(Kt)_{max}$/ $(Kt)_{surf}$ and $(Kt)_{max}$/(Kt)p-6 may be lower and dangerous in application, while for u < 0.333, the present solutions will be safety.
- The relative size of a laminated plate W/d has a strong effect on the stress concentration. In general, the smaller the relative size, the larger the stress concentration. When W/D $\geq$ 10.0, it is reasonable that the finite plate can be treated as an infinite plate in engineering analysis.

- The increase of numbers $\pm 45^0$lamina is beneficial to the decrease of stress concentration because it reduces extent of anisotropy of laminates.
- With the increase of ellipticity, the stress concentration becomes more serious. But for a specific laminated plate under a specific loading there exists a hole of specific ellipticity which causes the smallest stress concentration, and in general, it is not a circular hole.

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