Studies on the Effect of CTOD (Crack Tip Opening Displacement) on the Stress Triaxiality under Mixed Mode Fracture (I/II) in Plane Strain Condition

CH. Durga Jaya Teja[1] B.V.S. Raghu Vamsi[2] Dr. P. Nageswara Reddy[3]
[1]P.G Student [2]Assistant Professor [3]Professor
[1,2,3]Department of Mechanical Engineering
[1,2,3]Gudlavalleru Engineering College, Gudlavalleru, Andhra Pradesh, India

Abstract— Triaxiality is the ratio of hydro static pressure to the von-mises effective stress. As the triaxiality of the stress state influences greatly the amount of plastic strain, a material may undergo the ductile failure before actual failure occurs. This study gives the mathematical model for crack tip opening displacement (CTOD) based M-criterion model and used to study the effect of CTOD on stress Triaxiality and compare with simple M-criterion. Finite element analysis of single edge cracked rectangular plate is carried out in order to support the relations developed and validate the work. Optimization is carried out in order to determine maximum stress triaxiality and corresponding CTOD and crack initiation angle.

Nomenclature

E	young's modulus of the material
μ	poisson's ratio
β	crack inclination angle w.r.t horizontal
δ	crack tip opening displacement
σ_1, σ_2 , σ_3	principal stresses
σ_{12}	shear stress at the crack tip in the xy plane
K_1,K_2	mode- I and mode-II Stress intensity factors respectively
r	radius of the crack tip
σ11, σ22 , σ33	normal stress components at the crack tip along x, y and z directions respectively.
σ_{ys}	yield strength of the material
θ	orientation at the crack tip

Key words: CTOD (Crack Tip Opening Displacement), Stress Triaxiality

I. Introduction

Different techniques, analytically and theoretically like boundary element method (BEM), finite element method (FEM) etc. and experimentally like photo elasticity, uniaxial and biaxial tension tests and impact tests were implemented successfully to study the behaviour of cracked objects under various loading conditions. Many criterions were developed based on various parameters like stress, strain, energy and CTOD to predict the onset of crack growth. At low crack initiation angles, no criterion is in agreement but at higher angles, M-criterion and T-criterion were in good agreement. At the smaller angles the M-Criterion and modified M-Criterion are in good agreement, but at higher angles Modified M-Criterion gives better results.

II. Literature review

To study the onset of fracture in a member with existing crack, many theories were suggested by researchers in the past considering different parameters like stress, strain and energy. When a parameter approaches its critical value under mixed mode loading conditions, then occurs the onset of crack initiation. Firstly, Griffith (1921) proposed a criterion based on energy, which states that the fracture will occur when the energy stored in a structure overcomes the surface energy of the material. Later, Hussain et al. (1974) proposed G-criterion, which states that the crack initiation occurs in the direction of maximum energy release rate. First stress based criterion (MTS-criterion) was proposed by Erdogan and Sih (1963), which states that the crack initiation occurs in the direction of maximum tangential stress along a constant radius around the crack tip. They have assumed that the material is ideally brittle. This criterion was modified by Shafique and Marwan (2000) to make it applicable to ductile materials.

Another stress based criterion (M-criterion) was proposed by Kong et al. (1995), which states that the direction of crack initiation coincides with the direction of maximum stress triaxiality ratio along a constant radius around the crack tip. It is the ratio of hydrostatic stress to the equivalent von Mises stress. Sih (1974) proposed another criterion based on strain energy density (S-criterion), which states that the direction of crack initiation will coincides with the direction of minimum strain energy density along a constant radius around the crack tip. This is the first criterion which involves material properties, Poisson's ratio (μ) and shear modulus of elasticity in studying the onset of crack initiation. Theocaris et al. (1982) modified the S-criterion with a variable radius at the core region as plastic around the crack tip and proposed maximum dilatational strain energy criterion (T-criterion). Ukadgoanker and Awasare (1995) modified the T-criterion to determine the critical radius and angular initiation of crack in ductile materials. Shafique and Marwan (2004, 2008) proposed R-criterion to predict the crack initiation in ductile materials, which was later extended to anisotropic materials. S. S. Bhadauria et al. (2010) by comparative study of crack initiation angles S-criterion predicts the minimum crack initiation angle while M-criterion and T-criterion predicts the maximum initiation angle. Sutton et al. (1999, 2000).proposed that the crack will initiate in the direction of either Mode-I or Mode-II, depending on the maximum in either of the opening component or shearing component of CTOD, measured behind the crack tip, attains maximum value. Liu et al. (2009) under mixed mode fracture loading (I/II). shows that affect of the thickness of specimen to the stress and strain distribution is significant and the model is efficient in numerical analysis.

Out of eight criterions suggested in the literature for finding crack initiation angle, only two criterions namely, M-criterion and T-criterion are agreeing with experimental results at higher inclinations of crack for all types of loading. Hence M-criterion is adopted in this work to study the behavior of crack under mixed-mode fracture

In this work, two new mathematical models are developed for stress Triaxiality relating with mixed mode stress intensity factors, crack tip opening displacement (CTOD) and energy release rate(ERR). These models are optimized to determine the critical stress Triaxiality and the direction of crack initiation using genetic algorithm in MATLAB. Also finite element analysis has been carried out to validate the results.

Mathematical modeling for the stress field, Energy release rate, crack tip opening, ratio of principal stresses and stress triaxiality under mixed mode loading for plane strain condition Finite element analysis is carried out for plane strain conditions to find the stress intensity factors using ANSYS. Implementing Modified M-criterion to find the crack initiation angle in plane strain condition using SIF's obtained in ANSYS. Optimization of minimum of the using Genetic Algorithm in MATLAB to compare the values in plane strain condition.

III. MATHEMATICAL MODELING

The stress field equations in the mixed mode fracture at the crack tip in Cartesian co-ordinates is defined as:

$$\sigma_{11} = \frac{1}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - K_{II}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right] \qquad (2.1)$$

$$\sigma_{11} = \frac{1}{\sqrt{2\pi r}}\left[K_I\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) - K_{II}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)\right] \qquad (2.2)$$

$$\sigma_{33} = (\sigma_{11} + \sigma_{22}) \qquad (2.3)$$

Where σ_{11} and σ_{22} are normal stress components at the crack tip along the x, y directions and σ_{33} are shear stress at the crack tip in the xy plane.

Simplifying the above equations we can get the principal stress are

$$\sigma_1 = \frac{1}{\sqrt{2\pi r}}\left[\left(k_I\cos\frac{\theta}{2} - k_{II}\sin\frac{\theta}{2}\right) + \frac{1}{2\sqrt{2}}\sqrt{(k_I^2+5k_{II}^2) - (k_I^2 - 3k_{II}^2)\cos 2\theta + 4k_Ik_{II}\sin 2\theta}\right] \qquad (2.4)$$

$$\sigma_2 = \frac{1}{\sqrt{2\pi r}}\left[\left(k_I\cos\frac{\theta}{2} - k_{II}\sin\frac{\theta}{2}\right) - \frac{1}{2\sqrt{2}}\sqrt{(k_I^2+5k_{II}^2) - (k_I^2 - 3k_{II}^2)\cos 2\theta + 4k_Ik_{II}\sin 2\theta}\right] \qquad (2.5)$$

In the case of plane strain,

$$\sigma_3 = \mu * (\sigma_1 + \sigma_2) = \frac{2\mu}{\sqrt{2\pi r}}\left[k_I * \cos\frac{\theta}{2} - \sin\frac{\theta}{2}\right] \qquad (2.6)$$

In the case of plane stress, σ_3=0

Crack tip opening displacement (CTOD- **δ**) the crack tip opening displacement for the mixed mode fracture (MODE – I + II) is given by

$$\delta_{ctod} = \frac{K+1}{G} * \sqrt{\frac{x_c}{2\pi} * (K_I^2 + K_{II}^2)} \qquad (2.7)$$

Stress triaxiality factor (M) it is the ratio of the hydrostatic stress (σ_h) or means stress to the equivalent von Misses stress (σ_{eq}). Mathematically, it can be represented as

$$M = \frac{\sigma_h}{\sigma_{eq}} = \frac{\frac{(\sigma_1+\sigma_2)+\mu(\sigma_1+\sigma_2)}{3}}{\frac{\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\mu(\sigma_1+\sigma_2))^2+(\mu(\sigma_1+\sigma_2)-\sigma_1)^2+6\tau^2}}{\sqrt{2}}} \qquad (2.8)$$

Where, σ_1, σ_2 and σ_3 are the principal stresses.

Considering $\sigma = \frac{\sigma_1}{\sigma_2}$

$$M = \frac{\frac{(\sigma_1+\sigma_2)+\mu(\sigma_1+\sigma_2)}{3}}{\frac{\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\mu(\sigma_1+\sigma_2))^2+(\mu(\sigma_1+\sigma_2)-\sigma_1)^2}}{\sqrt{2}}} \qquad (2.9)$$

Squaring on both sides

$$M^2[(1+\mu^2-\mu)(\sigma_1^2+\sigma_2^2) - M^2\sigma_1\sigma_2(1+2\mu^2-2\mu)] = \left[(1+\mu^2-\mu)\left(\frac{\sigma_1^2}{9}+\frac{\sigma_2^2}{9}+\frac{2\sigma_1\sigma_2}{9}\right)\right]$$

By simplifying the equation we get

$$\sigma_1^2\left[\left(M^2-\frac{1}{9}\right)+\mu^2\left(M^2-\frac{1}{9}\right)-\mu\left(M^2+\frac{2}{9}\right)\right]+\sigma_2^2\left[\left(M^2-\frac{1}{9}\right)+\mu^2\left(M^2-\frac{1}{9}\right)-\mu\left(M^2+\frac{2}{9}\right)\right] - \sigma_1\sigma_2\left[\left(M^2+\frac{2}{9}\right)+\mu^2\left(2M^2+\frac{2}{9}\right)-\mu\left(2M^2-\frac{4}{9}\right)\right]=0$$

Dividing with σ_2^2 on both sides

$$\sigma^2\left[\left(M^2-\frac{1}{9}\right)+\mu^2\left(M^2-\frac{1}{9}\right)-\mu\left(M^2+\frac{2}{9}\right)\right]+\left[\left(M^2-\frac{1}{9}\right)+\mu^2\left(M^2-\frac{1}{9}\right)-\mu\left(M^2+\frac{2}{9}\right)\right]-\sigma\left[\left(M^2+\frac{2}{9}\right)+\mu^2\left(2M^2+\frac{2}{9}\right)-\mu\left(2M^2-\frac{4}{9}\right)\right]=0 \qquad (2.10)$$

The above equation is used to find the stress triaxiality ratio.

From Stress Triaxiality factor

$$TF = \frac{\sigma h}{\sigma eq} = \frac{\left(\frac{\sigma_{11}+\sigma_{22}+\sigma_{33}}{3}\right)}{\frac{\sqrt{(\sigma_{11}-\sigma_{22})^2+(\sigma_{22}-\sigma_{33})^2+(\sigma_{33}-\sigma_{11})^2+6\tau^2}}{\sqrt{2}}}$$

$$\mathrm{TF} = \frac{\frac{2}{3}\frac{(1+\mu)}{\sqrt{2\pi r}}\left(K_I\cos\frac{\theta}{2} - K_{II}\sin\frac{\theta}{2}\right)}{\frac{1}{\sqrt{2}\sqrt{2\pi r}}\sqrt{\left[\begin{array}{c}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\sin^2\theta + (2(1-2\mu)^2K_I^2 + 6K_{II}^2)\cos^2\frac{\theta}{2} + 8(1+\mu^2-\mu)K_{II}^2\sin^2\frac{\theta}{2} \\ +K_IK_{II}(3\sin 2\theta - 2(1-2\mu)^2\sin\theta)\end{array}\right]}}$$

$$\mathrm{TF} = \frac{2\sqrt{2}}{3}\frac{(1+\mu)\left(K_I\cos\frac{\theta}{2} - K_{II}\sin\frac{\theta}{2}\right)}{\sqrt{\left[\begin{array}{c}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\sin^2\theta + (2(1-2\mu)^2K_I^2 + 6K_{II}^2)\cos^2\frac{\theta}{2} + 8(1+\mu^2-\mu)K_{II}^2\sin^2\frac{\theta}{2} \\ +K_IK_{II}(3\sin 2\theta - 2(1-2\mu)^2\sin\theta)\end{array}\right]}} \quad (2.11)$$

IV. Finite Element Analysis

In the case of an inclined crack, the model was not symmetric. Thus, full model of the single edge cracked plate with dimensions 60 mm x 180 mm was analyzed using ANSYS 15.0 environment. The crack was inclined at different inclination angle varied from 0° to 50° with an incremental step of 10°. The crack length to width of the plate is varied from 0.1–0.4 with an incremental step of 0.1 is represented diagrammatically with local co-ordinate system.

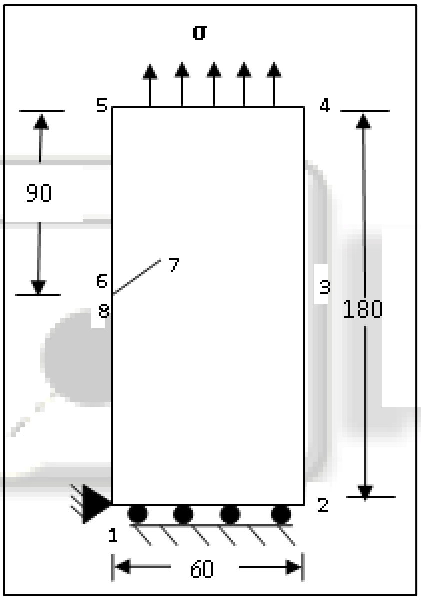

Fig. 1.1: Plate with dimensions

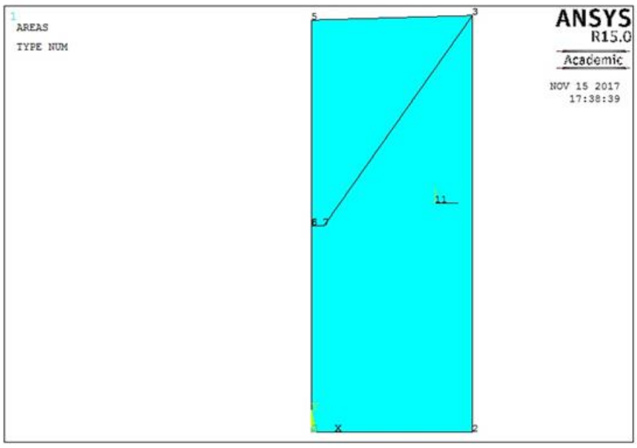

Fig. 1.2: 2D view of the plate

The problem is modelled as 2-D and PLANE 183 element with plane strain behaviour is considered for the generation of mesh. It is a 2-D higher order element with 2 degree of freedom at each node (u_x, u_y - translation in x and y) and exhibits quadratic (8 node) and triangular (6 node) behaviour. This element is well suited for the modeling of irregular meshes. The plate is free meshed with triangles and increase in size surrounding the crack tip. 3573 nodes generated, 1703 elements generated and the radius around the crack tip is 0.005 m.

Mixed mode loading is a combination of tension and shear loads. The crack is modelled as an inclined crack with the uniformly distributed load (pressure) acting perpendicular to the width of the crack. When the load is resolved into components, the component along the length of the crack gives the shearing effect and the perpendicular component gives the tensile loading or tension effect. The load applied is equal to the yield strength of the material. Deflection at the base of the plate is arrested in y-direction and the deflection at key point -1 is arrested in x-direction also, the results obtained from finite element analysis are tabulated and presented graphically as shown below. Distribution of First Principal stress, Second Principal stress and Von Mises stress for crack length 12mm crack inclination angle varying from 0^0 to 40^0.

At crack length a=12 (a/w) ratio=0.2, and inclination β=0^o in mixed mode cracked plate at crack opening the stress are shown in fig 1.3, fig1.4, & fig1.5

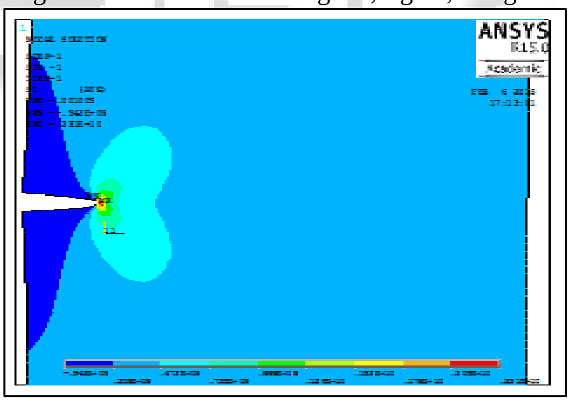

Fig. 1.3: first principle stress

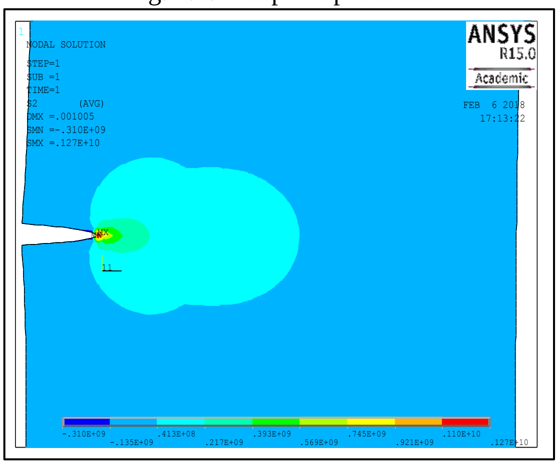

Fig. 1.4: second principle stress

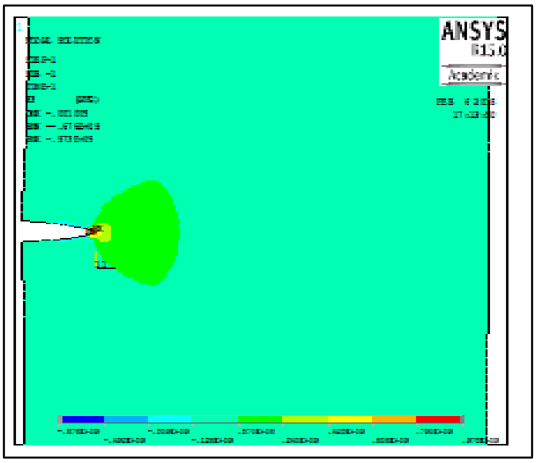

Fig. 1.5: Von Mises stress

At crack length a=12 (a/w) ratio=0.2, and inclination β=10° in mixed mode cracked plate at crack opening the stress are shown in fig 1.6, fig 1.7, & fig 1.8

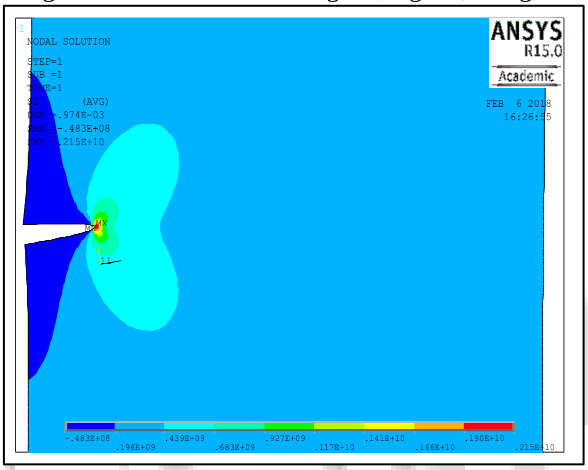

Fig. 1.6: first principle stress

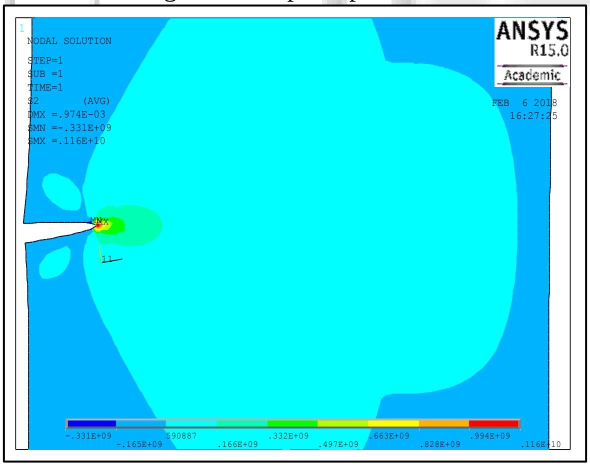

Fig. 1.7: second principle stress

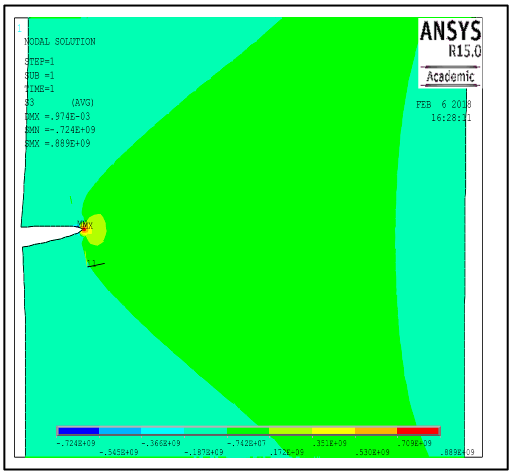

Fig. 1.8: Von Mises stress

At crack length a=12 (a/w) ratio=0.2, and inclination β=20° in mixed mode cracked plate at crack opening the stress are shown in fig 1.9, fig 1.10, & fig 1.11

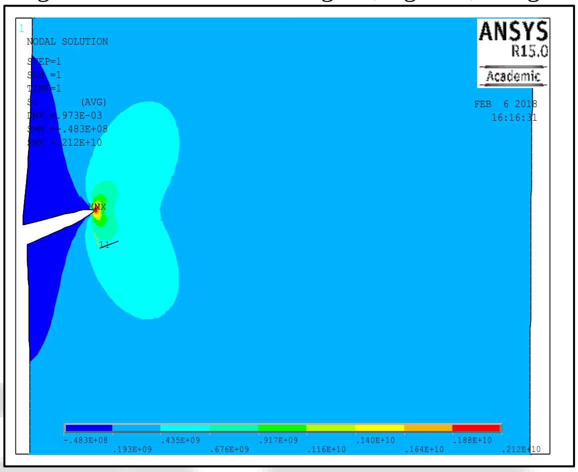

Fig. 1.9: first principle stress

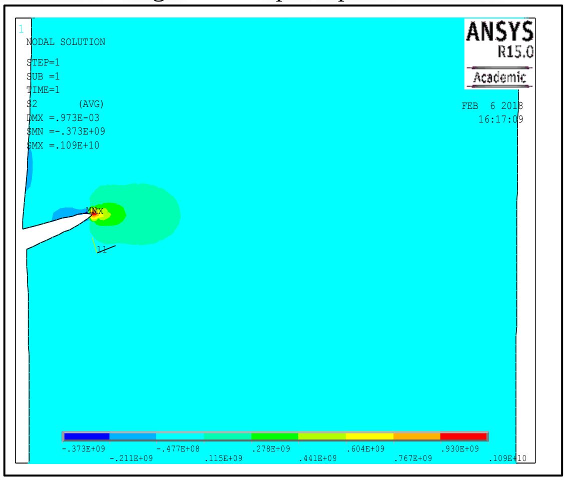

Fig. 1.10: second principle stress

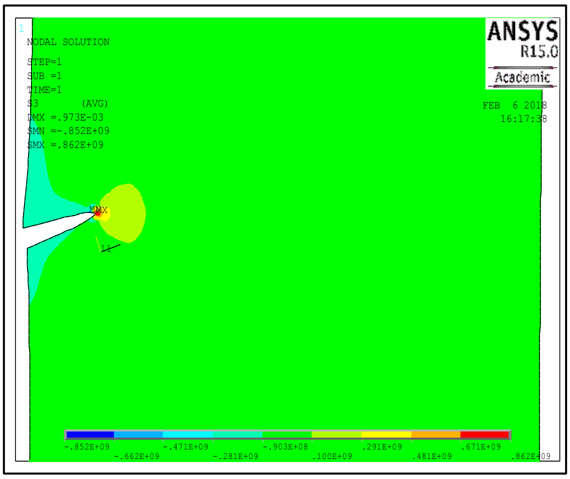

Fig. 1.11: Von Mises stress

At crack length a=12 (a/w) ratio=0.2, and inclination $\beta=30^{o}$ in mixed mode cracked plate at crack opening the stress are shown in fig 1.12, fig 1.13, & fig 1.14

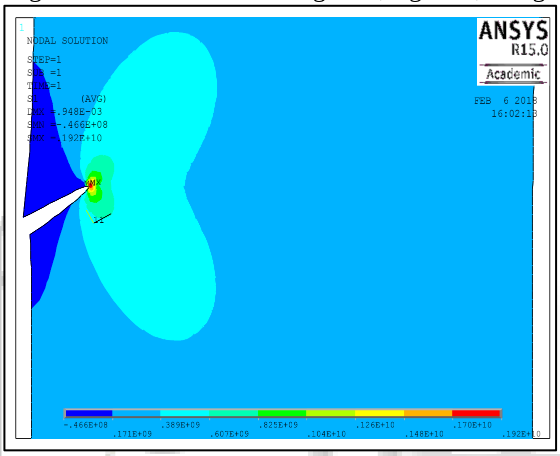

Fig. 1.12: first principle stress

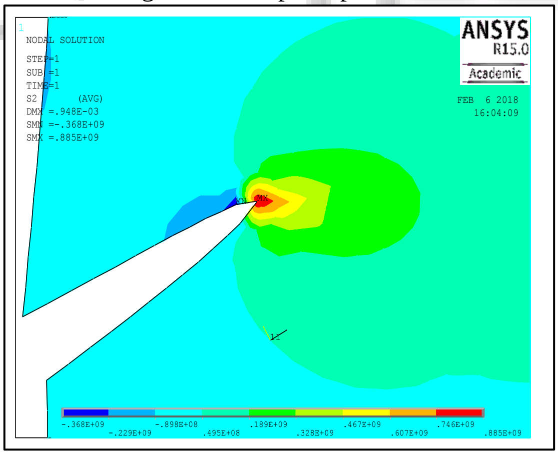

Fig 1.13: second principle stress

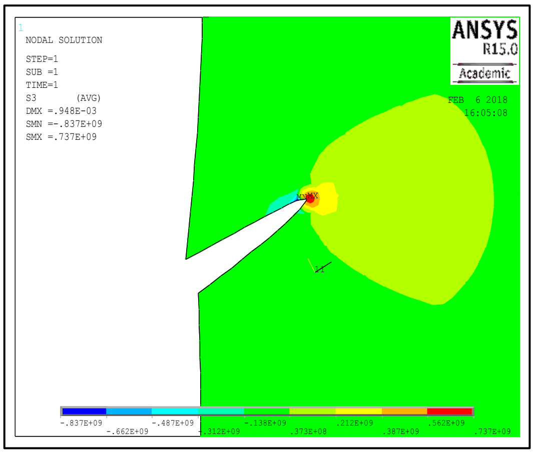

Fig 1.14: Von Mises stress

At crack length a=12 (a/w) ratio=0.2, and inclination $\beta=40^{o}$ in mixed mode cracked plate at crack opening the stress are shown in fig 1.15, fig 1.16, & fig 1.17

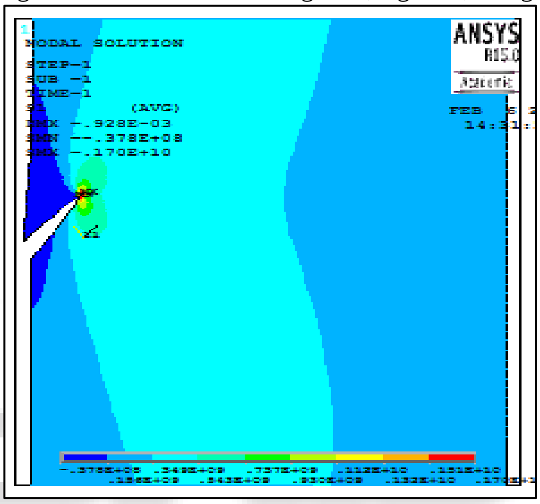

Fig. 1.15: first principle stress

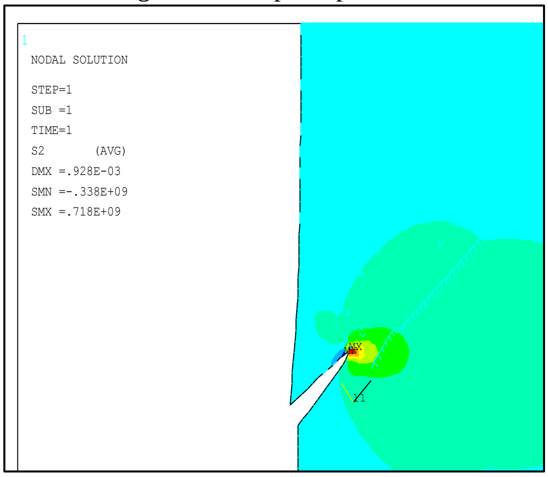

Fig. 1.16: second principle stress

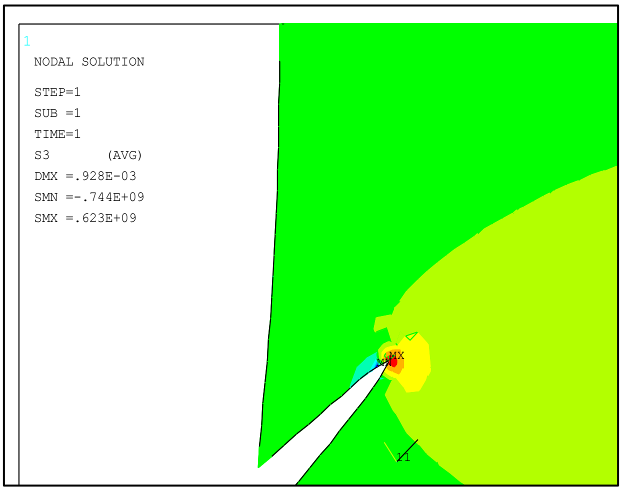

Fig. 1.17: Von Mises stress

V. IMPLEMENTING MODIFIED M-CRITERION

Triaxiality factor is considered as the parameter for the failure under mixed mode fracture and corresponding M – Criterion was used to find the maximum triaxiality factor. It states that the direction of to crack initiation coincides with the direction of maximum stress triaxiality factor along a constant radius around the crack tip. Mathematically M-criterion can be stated as:

$$\frac{\partial M}{\partial \theta} = 0$$

$$\frac{\partial^2 M}{\partial \theta^2} < 0$$

Where M – stress triaxiality factor (TF), θ – polar angle defining the direction of the point Now implementing the above equations to the plane stress condition, we get from the first condition

$$\frac{\partial TF}{\partial \theta} = 0$$

$$\frac{\frac{-1(1+\mu)}{2}\left(K_I\mathrm{Sin}\frac{\theta}{2} + K_{II}\mathrm{Cos}\frac{\theta}{2}\right)}{\sqrt{\left[\begin{array}{c}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\mathrm{Sin}^2\theta + (2(1-2\mu)^2K_I^2 + 6K_{II}^2)\mathrm{Cos}^2\frac{\theta}{2} + 8(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}^2\frac{\theta}{2} \\ +K_IK_{II}(3\mathrm{Sin}2\theta - 2(1-2\mu)^2\mathrm{Sin}\theta)\end{array}\right]}}$$

$$-\frac{1(1+\mu)}{2}\frac{\left(K_I\mathrm{Cos}\frac{\theta}{2} - K_{II}\mathrm{Sin}\frac{\theta}{2}\right)\left(\left[\begin{array}{c}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\mathrm{Sin}2\theta - (2(1-2\mu)^2)K_I^2 + 6K_{II}^2)\frac{\mathrm{Sin}\theta}{2} \\ +4(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}\theta + K_IK_{II}(6\mathrm{Cos}2\theta - 2(1-2\mu)^2\mathrm{Cos}\theta)\end{array}\right]\right)}{\left[\sqrt{\left[\begin{array}{c}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\mathrm{Sin}^2\theta + (2(1-2\mu)^2K_I^2 + 6K_{II}^2)\mathrm{Cos}^2\frac{\theta}{2} + 8(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}^2\frac{\theta}{2} \\ +K_IK_{II}(3\mathrm{Sin}2\theta - 2(1-2\mu)^2\mathrm{Sin}\theta)\end{array}\right]}\right]} = 0$$

$$\begin{array}{c}\left(K_I\mathrm{Sin}\frac{\theta}{2} + K_{II}\mathrm{Cos}\frac{\theta}{2}\right)\left[\begin{array}{c}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\mathrm{Sin}^2\theta + (2(1-2\mu)^2K_I^2 + 6K_{II}^2)\mathrm{Cos}^2\frac{\theta}{2} + 8(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}^2\frac{\theta}{2} \\ +K_IK_{II}(3\mathrm{Sin}2\theta - 2(1-2\mu)^2\mathrm{Sin}\theta)\end{array}\right] \\ +\left(K_I\mathrm{Cos}\frac{\theta}{2} - K_{II}\mathrm{Sin}\frac{\theta}{2}\right)\left[\begin{array}{c}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\mathrm{Sin}2\theta - (2(1-2\mu)^2)K_I^2 + 6K_{II}^2)\frac{\mathrm{Sin}\theta}{2} \\ +4(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}\theta + K_IK_{II}(6\mathrm{Cos}2\theta - 2(1-\mu)^2\mathrm{Cos}\theta)\end{array}\right] = 0\end{array}$$

Let

$$\frac{K_I}{K_{II}} = w$$

On simplification, we get

$$\begin{array}{c}\left[w\mathrm{Tan}\frac{\theta}{2} + 1\right]\left[\left(\frac{3}{2}w^2 - \frac{9}{2}\right)\mathrm{Sin}^2\theta + (2(1-2\mu)^2w^2 + 6)\mathrm{Cos}^2\frac{\theta}{2} + 8(1+\mu^2-\mu)\mathrm{Sin}^2\frac{\theta}{2} + w(3\mathrm{Sin}2\theta - 2(1-2\mu)^2\mathrm{Sin}\theta)\right] \\ +\left[K - \mathrm{Tan}\frac{\theta}{2}\right]\left[\begin{array}{c}\left(\frac{3}{2}w^2 - \frac{9}{2}\right)\mathrm{Sin}2\theta - (2(1-2\mu)^2w^2 + 6)\frac{\mathrm{Sin}\theta}{2} + 4(1+\mu^2-\mu)\mathrm{Sin}\theta \\ +w(6\mathrm{Cos}2\theta - 2(1-2\mu)^2\mathrm{Cos}\theta)\end{array}\right] = 0\end{array}$$

On further simplification, we get

$$\mathrm{Tan}^4\frac{\theta}{2} - 3w\mathrm{Tan}^3\frac{\theta}{2} - (1-2w^2)\mathrm{Tan}^2\frac{\theta}{2} + \frac{(1-w^2)w}{2}\mathrm{Tan}\frac{\theta}{2} - \frac{1+w^2}{2} = 0$$

Here $\theta = \theta_0$, crack inclination angle.
Second Condition is

$$\frac{\partial^2 TF}{\partial \theta^2} < 0$$

Final equation after simplification, we will get

$$(5+w^2)2\mathrm{Sin}\frac{\theta}{2} + (1+w^2)27\mathrm{Sin}\frac{3\theta}{2} + (5w^2-3)5\mathrm{Sin}\frac{5\theta}{2} - 2(5+w^2)\mathrm{Cos}\frac{\theta}{2} - 9w(1+w^2)\mathrm{Cos}\frac{3\theta}{2} - 5w(w^2-7)\mathrm{Cos}\frac{5\theta}{2} < 0$$

Finally, in M-criterion we get

$$\mathrm{Tan}^4\frac{\theta}{2} - 3w\mathrm{Tan}^3\frac{\theta}{2} - (1-2w^2)\mathrm{Tan}^2\frac{\theta}{2} + \frac{(1-w^2)w}{2}\mathrm{Tan}\frac{\theta}{2} - \frac{1+w^2}{2} = 0 \quad (2.12)$$

$$(5+w^2)2\mathrm{Sin}\frac{\theta}{2} + (1+w^2)27\mathrm{Sin}\frac{3\theta}{2} + (5w^2-3)5\mathrm{Sin}\frac{5\theta}{2} - 2(5+w^2)\mathrm{Cos}\frac{\theta}{2} - 9w(1+w^2)\mathrm{Cos}\frac{3\theta}{2} - 5w(5w^2-7)\mathrm{Cos}\frac{5\theta}{2} < 0 \quad (2.13)$$

A. Relation between CTOD and TF

Relation has been developed between local parameter, triaxiality factor (TF) and a global parameter, crack tip opening displacement (CTOD - **δ**). Using this equation, the effect of CTOD on triaxiality factor was studied.
Stress triaxiality factor,

$$M = \frac{2\sqrt{2}}{3}\frac{(1+\mu)\left(K_I\mathrm{Cos}\frac{\theta}{2} - K_{II}\mathrm{Sin}\frac{\theta}{2}\right)}{\sqrt{\begin{bmatrix}\left(\frac{3}{2}K_I^2 - \frac{9}{2}K_{II}^2\right)\mathrm{Sin}^2\theta + (2(1-2\mu)^2K_I^2 + 6K_{II}^2)\mathrm{Cos}^2\frac{\theta}{2} + 8(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}^2\frac{\theta}{2} \\ +K_IK_{II}(3\mathrm{Sin}2\theta - 2(1-2\mu)^2\mathrm{Sin}\theta)\end{bmatrix}}}$$

$$M = \frac{4}{3}\frac{(1+\mu)\left(K_I\mathrm{Cos}\frac{\theta}{2} - K_{II}\mathrm{Sin}\frac{\theta}{2}\right)}{\sqrt{\begin{bmatrix}(3K_I^2 - 9K_{II}^2)\mathrm{Sin}^2\theta + (4(1-2\mu)^2K_I^2 + 12K_{II}^2)\mathrm{Cos}^2\frac{\theta}{2} + 16(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}^2\frac{\theta}{2} \\ +K_IK_{II}(6\mathrm{Sin}2\theta - 4(1-2\mu)^2\mathrm{Sin}\theta) + 12(K_I^2 + K_{II}^2) + K_I^2 + K_{II}^2 - K_I^2 - K_{II}^2\end{bmatrix}}}$$

Crack tip opening displacement,

$$\delta = \frac{K+1}{G}\sqrt{\frac{xc}{2\pi}(K_I^2 + K_{II}^2)} \Rightarrow \delta^2 = \left(\frac{K+1}{G}\right)^2\frac{xc}{2\pi}(K_I^2 + K_{II}^2)$$

But $K = 3 - 4\mu$ Hence, K+1 = 4(1-μ)

$$(K_I^2 + K_{II}^2) = \frac{\delta^2G^2}{(4(1-\mu))^2}\frac{2\pi}{x_c} = \frac{\delta^2G^2}{(1-\mu)^2}\frac{\pi}{8x_c}$$

Substituting this expression in the above equation of triaxiality factor, we get

$$M_{ctod} = \frac{1.333(1+\mu)\left(K_I\mathrm{Cos}\frac{\theta}{2} - K_{II}\mathrm{Sin}\frac{\theta}{2}\right)}{\sqrt{\begin{bmatrix}(3K_I^2 - 9K_{II}^2)\mathrm{Sin}^2\theta + (4(1-2\mu)^2K_I^2 + 12K_{II}^2)\mathrm{Cos}^2\frac{\theta}{2} + 16(1+\mu^2-\mu)K_{II}^2\mathrm{Sin}^2\frac{\theta}{2} \\ +K_IK_{II}(6\mathrm{Sin}2\theta - 4(1-2\mu)^2\mathrm{Sin}\theta) + 12\left(\frac{\delta^2G^2}{(1-\mu)^2} * \frac{\pi}{8xc}\right) + K_I^2 + K_{II}^2 - K_I^2 - K_{II}^2\end{bmatrix}}}$$

This equation related the CTOD, TF, SIF's and the direction of the point from the crack tip. Relation between TF & CTOD will be helpful for studying the relationship between micro-damage mechanisms and the macro fracture criteria of materials.

B. Relation between ERR & TF

Relation has been developed between local parameter, Triaxiality factor (TF) and a global parameter, Energy release rate (ERR). Using this equation, the effect of ERR on Triaxiality factor was studied.

$$M = \frac{\frac{4}{3}(1+\mu)(K_I\cos\frac{\theta}{2} - K_{II}\sin\frac{\theta}{2})}{\sqrt{\begin{bmatrix}\left(3K_I^2 - 9K_{II}^2 + 16(\mu^2-\mu)K_{II}\right)\sin^2\frac{\theta}{2} + \left(4K_I^2 + 16(\mu^2-\mu)K_I\right)\cos^2\frac{\theta}{2} + 16K_IK_{II}\sin 2\theta \\ -4K_IK_{II}\sin\theta + 16(\mu^2-\mu)\sin\theta + 12(K_I^2 + K_{II}^2)\end{bmatrix}}}$$

It is the energy released per unit increase in area during crack growth. In the mixed mode fracture, energy release rate is given by

$$G_0 = \frac{\pi(K+1)}{8G}[K_I^2 + K_{II}^2]$$

In plane strain condition,

$$K = \frac{3-4\mu}{1+\mu}$$

$$K+1 = \frac{3-4\mu+1+\mu}{1+\mu} = \frac{4-3\mu}{1+\mu}$$

Also,

$$G = \frac{E}{2(1+\mu)}$$

On substituting the expressions for *G* and *K+1*, we get finally

$$[K_I^2 + K_{II}^2] = G\frac{E}{\pi}$$

$$M_{err} = \frac{\frac{4}{3}(1+\mu)(K_I\cos\frac{\theta}{2} - K_{II}\sin\frac{\theta}{2})}{\sqrt{\begin{array}{c}[(3K_I^2 - 9K_{II}^2)\sin^2\theta + (4(1-2\mu)^2K_I^2 + 12K_{II}^2)\cos^2\frac{\theta}{2} + 16(1+\mu^2-\mu)K_{II}^2\sin^2\frac{\theta}{2} \\ + K_IK_{II}(6\sin2\theta - 4(1-2\mu)^2\sin\theta) + 12(G\frac{E}{\pi}) + K_I^2 + K_{II}^2 - K_I^2 - K_{II}^2\end{array}}} \quad (2.15)$$

This equation related the ERR, TF, SIF's and the direction of the point from the crack tip. Relation between TF & ERR will be helpful for studying the energy release rate at the crack tip opening.

VI. Optimization

Constrained optimization is performed. Two different equations are optimized in this work. the stress intensity vs CTOD in plane strain condition is derived based on the criterion proposed by M-criterion. It is minimized subject to the constraints in equations 2.12 and 2.13.the stress intensity vs ERR in plane strain condition derived based on the M-criterion. It is minimized subject to the constraints in equations 2.12 and 2.13.

S. No	Crack inclination angle (deg)	ANSYS Results			GA Results			
		TF	CTOD (m)	ERR (kPa-m)	TF	CTOD (m)	ERR (kPa-m)	Crack initiation angle (deg)
1	0	1.4594	3.3e-5	0.0234	2.1662	9.58e-5	1.3826	-0.100
2	10	1.4194	3.2e-5	0.0226	2.1639	8.96e-5	1.4327	-11.818
3	20	1.3350	2.7e-5	0.0203	2.1346	7.43e-5	0.3757	-22.20
4	30	1.2125	2.2e-5	0.0169	2.0407	5.41e-5	0.159	-31.2
5	40	1.0918	1.6e-5	0.0130	1.8821	3.54e-5	0.101	-38.9
6	50	0.9802	2.1e-5	0.0090	1.695	4.02e-5	0.049	-45.18

Table 1: Comparison of Stress triaxiality from ANSYS & M-Criterion for SS 304 (considering effect of CTOD) (for a/w=0.1)

S. No	Crack inclination angle (deg)	ANSYS Results			GA Results			
		TF	CTOD(m)	ERR (kPa-m)	TF	CTOD(m)	ERR (kPa-m)	Crack initiation angle (deg)
1	0	1.4568	5.32e-5	0.0594	2.0476	15.1e-5	1.06678	-0.004
2	10	1.4259	5.06e-5	0.0573	2.0466	14.2e-5	0.410	-9.947
3	20	1.3311	4.38e-5	0.0515	2.0332	11.8e-5	0.206	-19.050
4	30	1.2148	3.47e-5	0.0430	1.987	8.89e-5	0.127	-27.000
5	40	1.0909	2.55e-5	0.0330	1.8987	5.95e-5	0.083	-34.695
6	50	0.9769	3.42e-5	0.0229	1.7796	7.35e-5	0.054	-39.57

Table 2: Comparison of Stress triaxiality from ANSYS & M-Criterion Al-6061 (considering effect of CTOD) (for a/w=0.1)

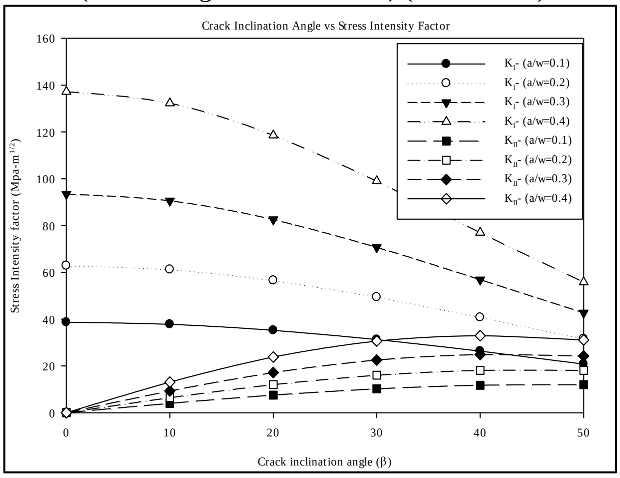

Fig. 1.18 Variation of Stress Intensity Factor vs Crack inclination angle (SS304)

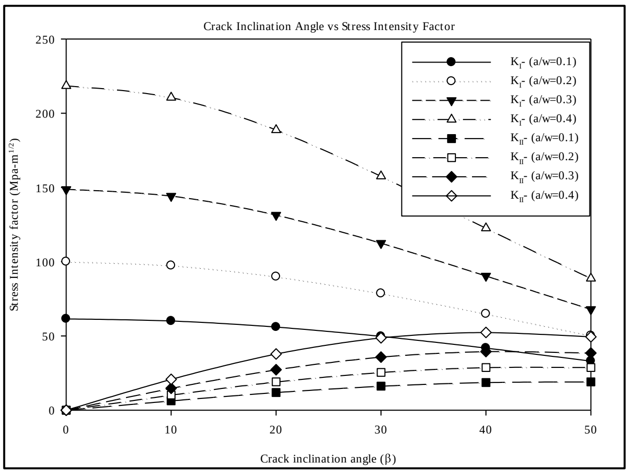

Fig 1.19 Variation of stress intensity Factor vs crack inclination angle (AL6061)

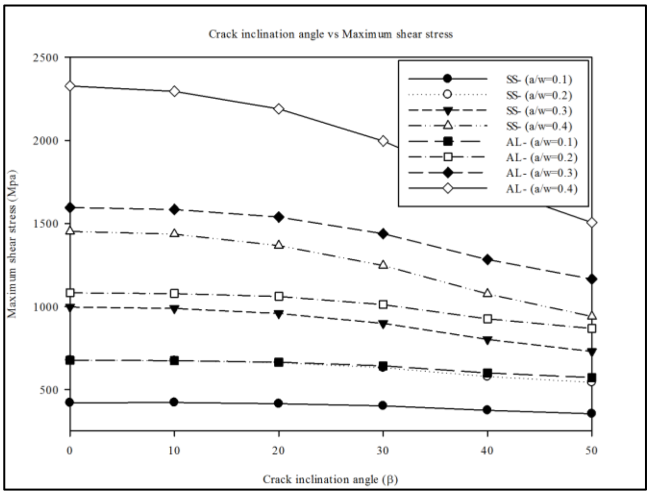

Fig. 1.20 Variation of Maximum Shear Stress vs Crack inclination angle

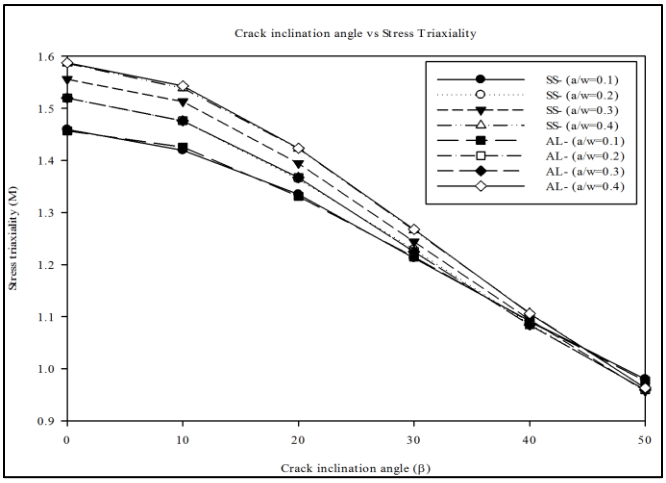

Fig 1.21 stress triaxiality vs crack inclination angle

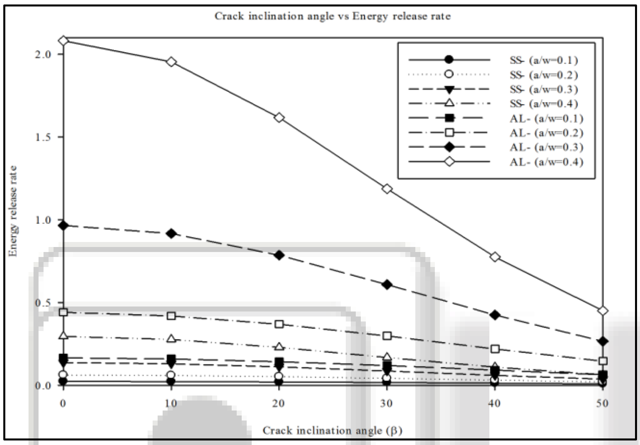

Fig 1.22 Energy release rate vs Crack inclination angle

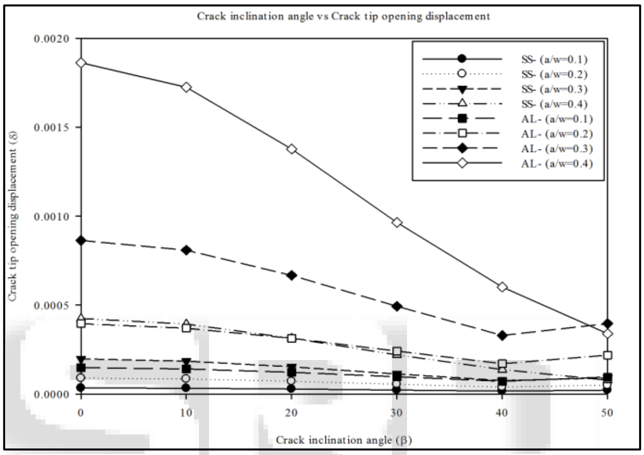

Fig 1.23 crack inclination angle vs crack tip opening displacement

VII. Conclusion

1) With an increase in crack inclination angle,
   a) Mode-I SIF is observed to be decreasing and mode-II SIF is increasing, which is due to the increase of domination of shear stress. It further results in decrease of stress triaxiality.
   b) Strain energy density stored in the member decreases which leads to reduction of rate of energy release for the propagation of crack. If energy released decreases the bonds will not break and it further results in decrease of CTOD.
2) With an iincrease in the a/w ratio, the mode-I SIF and stress triaxiality are decreasing and mode-II SIF and stress triaxiality are increasing. This is also due to decrease in the effective area resisting the applied load.
3) Finally we conclude that at the smaller angles the M-Criterion and modified M-Criterion give the same results but at higher angles Modified M-Criterion gives better results.

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