Channel Estimation using Discrete Fourier Transform for MIMO Systems

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Abstract— DFT-based channel estimation method for pilot-type OFDM systems which mainly improves the performance has been introduced. Channel estimation based on pilot-type arrangement is studied through Discrete Fourier Transform (DFT) algorithm. Channel State Information (CSI) is useful at receiver to remove effect of channel in received signal for efficient recovery of data and at transmitter which is useful to vary power allocation and modulation order according to channel. The performance and complexity comparison is made between the Minimum Mean Square Error (MMSE), Least Square (LS) with and without Discrete Fourier Transform (DFT)-based technique. From the simulation results, it is concluded that the performance of DFT-base technique for MMSE is better than the LS. Thus simulation results show the performance which reduced the computation complexity.

Key words: Orthogonal Frequency Division Multiplexing (OFDM), Discrete Fourier Transform (DFT), Channel State Information (CSI), Minimum Mean Square Error (MMSE), Least Square (LS), Channel Estimation

I. INTRODUCTION

Multipath fading and bandwidth efficiency are two challenges in the wireless communication system. For mitigation of impairments caused by the fading channels, channel estimation is imperative [3]. Channel estimation means to estimate some characteristics and it is useful to minimizing transmission error and increase possibility in accuracy for data recovery at receiver [4].

The transmitted signal can be recovered by estimating the channel response for each subcarrier. The channel state information [6] derived from channel estimation is important for receivers to improve the performance, computational complexity. The two basic channel estimations are block-type and comb-type pilot channel estimation [8], where the frequency direction and time direction includes pilots respectively. Channel estimation can be used for data signal as well as training signal, or both. Channel estimation technique is chosen for OFDM system including different aspects of implementations, such as performance, computational complexity and time-variation of the channel. This paper elaborates the channel estimation based on pilot signals in each individual OFDM block.

Firstly, the OFDM system basics are introduced, the modulation is included the way based on DFT based channel estimation. The performance of LS or MMSE [1] is achieved by eliminating the noise outside the maximum channel delay. In this dissertation, we analyze the LS, MMSE and the DFT based LS estimation. DFT-based algorithms are largely researched due to its low computation complexity and good estimation performance [10].

In this paper, two improved channel estimation methods has been proposed. The performance comparison between the LS and MMSE using with and without DFT technique. The paper is arranged as follows: Section II shows the overview of the DFT-based channel estimation. Section III shows pilot-type estimation strategy includes LS with DFT, MMSE with DFT, MMSE without DFT, LS without DFT and theory MMSE and its modified version. Simulation results with the improved performance is explained in section IV and section V describes the conclusions.

II. SYSTEM OVERVIEW

The block diagram of DFT-based channel estimation, given the LS channel estimation, is as shown in the figure 1.

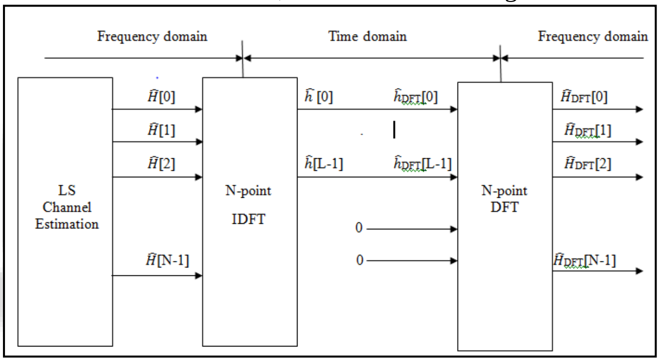

Fig. 1: DFT based channel estimation

Let $\hat{H}[k]$ denote the estimate of channel gain at the kth subcarrier, which gives the IDFT of the channel estimate $\{\widehat{H}[k]\}$ from k=0 to N-1.

$$\mathrm{IDFT}\{\widehat{H}[k]\}=h[n]+z[n]\cong \hat{h}[n], \text{ for } n=0,1,\ldots.N-1 \quad (1)$$

Where z[n] denotes the noise component in the time domain. The coefficients for the maximum channel delay L as depicted in equation 2:

$$\hat{h}_{DFT}[n]=\begin{cases} h[n]+z[n], & n=0,1,2..L-1 \\ 0 & otherwise \end{cases} \quad (2)$$

and transform the remaining L elements back to the frequency domain as follows:

Noting that the maximum channel delay L must be known in advance , which is given by:

$$\hat{H}_{DFT}[k]=\mathrm{DFT}\ \{\hat{h}_{DFT}(n)\}$$

III. CHANNEL ESTIMATION

The different types of channel estimators are explained in the further section. The block-type and comb-type pilot channel estimation are the basic types of estimation. The block-type pilot arrangement is used in this section, where OFDM symbols with pilots at all subcarriers are transmitted periodically for channel estimation. A typical block-type pilot arrangement is depicted in the figure 2. Each pilot symbol is transmitted for S_t Number of OFDM symbols.

The estimations for block-type pilot arrangement can be based on LS with and without DFT technique [10].

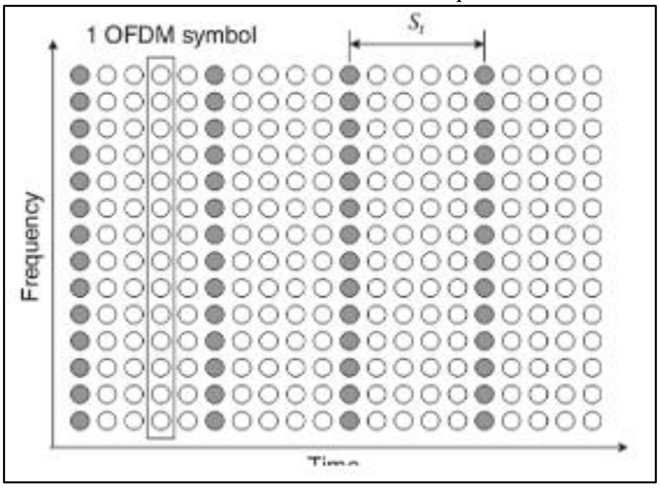

Fig. 2: Block type pilot arrangement [8]

A. Least Square Channel estimation

The training symbols for the received signal is represented as :

$$Y[k]= X[k]H[k]+Z[k]$$

Where Y(k),X(k),H(k),Z(k) are output of the samples, pilot tone at k^{th} subcarrier , channel gain, noise vector respectively.

The channel estimation for LS finds a method with cost function which is minimized:

$$J(\widehat{H})=\| Y-X\widehat{H} \|^2$$
$$=(Y-X\widehat{H})^H (Y-X\widehat{H})$$
$$=Y^HY-Y^HX\widehat{H}-\widehat{H}^HX^HY+\widehat{H}^HX^HX\widehat{H} \quad (3)$$

LS channel estimation solution is given by

$$\widehat{H}_{LS}=(X^HX)^{-1}X^HY=X^{-1}Y \quad (4)$$

Since X is assumed to be diagonal due to the Inter Carrier Interference -free condition, LS channel estimate for each subcarrier is given as:

The MSE of this LS channel is given by:

$$MSE_{LS} = \{(H-\widehat{H}_{LS})\hat{}H(H-H_{LS}) \quad (6)$$
$$= E\{(H-X^{-1}Y)^H(H-X^{-1}Y)\}$$
$$= E\{(X^{-1}Z)^H(X^{-1}Z)\}$$
$$=E\{Z^H(XX^H)^{-1}Z\}$$
$$=\frac{\sigma_z^2}{\sigma_x^2}$$

Hence equation 6 implies that it is inversely proportional to the SNR which is subjected to the noise enhancement. Thus LS is preferred for some cases.

B. Minimum Mean Square Error Channel Estimation.

The MMSE channel estimation method finds a better i.e, linear estimate in terms of weight matrix W in such a way that the MSE in the below equation 7 is minimized

$$J(\widehat{H})=E\{\|e\|^2\}=E\{\|H-\widehat{H}\|^2\} \quad (7)$$

According to orthogonality principle,

$$e=H-\widehat{H} \text{ is orthogonal to } \widetilde{H},$$
$$E\{e\widetilde{\boldsymbol{H}}^H\}=E\{(H-\widehat{\boldsymbol{H}})\widetilde{\boldsymbol{H}}^H\}$$
$$=E\{(H-W\widetilde{\boldsymbol{H}})\,\widetilde{\boldsymbol{H}}^H\}$$
$$=E\{H\widetilde{\boldsymbol{H}}^H\}-WE\{\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^H\}$$
$$= R_{H\tilde{H}}-WR_{\tilde{H}\tilde{H}}=0 \quad (8)$$

where R_{AB} is the cross-correlation matrix of NxN matrices A and B and $\widetilde{\boldsymbol{H}}$ is the LS channel estimate given as

$$\widetilde{\boldsymbol{H}} = \boldsymbol{X}^{-1}Y=H+X^{-1}Z$$

The solution is given by

$$W=R_H\, R^{-1}$$

Where $R\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}$ is the autocorrelation matrix of $\widetilde{\boldsymbol{H}}$ given as

$$\boldsymbol{R}_{\widehat{H}\widetilde{H}}=E\{\widetilde{\boldsymbol{H}}\widetilde{\boldsymbol{H}}^H\}$$
$$=E\{X^{-1}Y(X^{-1}Y)^H\}$$
$$=E\{(H+X^{-1}Z)(H+X^{-1}Z)^H\}$$
$$=E\{HH^H+X^{-1}ZH^H+HZ^H(X^{-1})^H+X^1ZZ^H(X^1)^H\}$$
$$=E\{HH^H\}+E\{X^{-1}ZZ^H(X^{-1})^H\}$$
$$=E\{HH^H+\frac{\sigma_z^2}{\sigma_x^2}\boldsymbol{I}\} \quad (9)$$

And $R_{H}\widetilde{\boldsymbol{H}}$ is the cross-correlation matrix between the true channel vector and temporary channel estimate vector in the frequency domain.

The MMSE channel estimate follows as

$$\widehat{\boldsymbol{H}} = \boldsymbol{W}\widetilde{\boldsymbol{H}} = \boldsymbol{R}_{H\widetilde{H}}\boldsymbol{R}^{-1}_{\widetilde{H}\widetilde{H}}\widetilde{\boldsymbol{H}}^{-1}H\tilde{} \quad (10)$$

IV. SIMULATION RESULTS

The table shows the simulated parameters used channel estimation system, which is as shown below:

Total number of sub channels	256
Number of pilots	32
Total no. of data sub channels	224
Cyclic prefix length	8
FFT size	64
Guard Interval length	64
Fading	Rayleigh , Rician
Modulation Technique	4-QAM, 8-QAM

Table 1: Simulation Parameters

The above figure 3 and 4 shows the simulated results. In the figure 3, the performance between MMSE, LS channel estimation with different channel estimations has been illustrated and shown and the figure 4 shows the performance improvement with DFT-channel estimators.

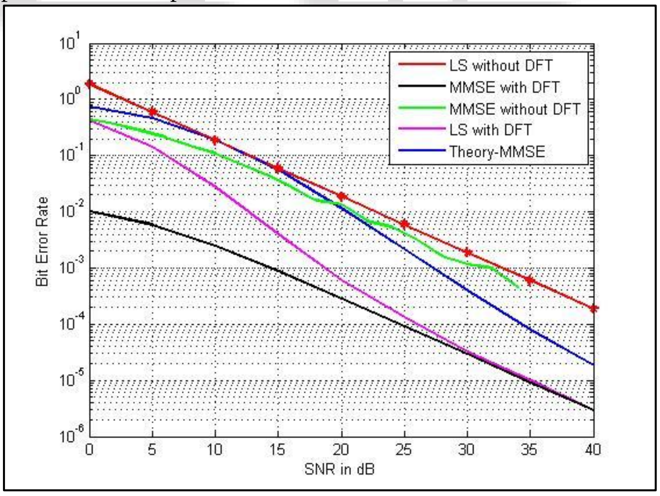

Fig. 3: Performance of comparison of different channel estimation techniques

V. CONCLUSIONS

Thus the improvement in the performance of DFT channel estimation technique has been achieved .hence it is concluded that for high SNRs the LS estimator is both simple and adequate. On comparison, the DFT based estimation improves the performance; therefore it clearly states that MMSE shows better performance than the LS with DFT.