

Regular Equitable Domination Number in Fuzzy Graph

S. Revathi¹ C. V. R. HariNarayanan² R. Muthuraj³

^{1,2,3}Department of Mathematics

¹Kings College of Engineering, Thanjavur ²Govt.Arts College, Paramakudi ³H. H. The Rajah's College, Pudukkottai

Abstract— In this paper the new kind of parameter regular equitable domination number in a fuzzy graph is defined and established the parametric conditions. The properties of regular equitable domination number are discussed.

Key words: Dominating Set, Equitable Dominating Set, Regular Equitable Dominating Set

I. INTRODUCTION

Fuzzy graphs were introduced by Rosenfeld [6]. Rosenfeld has described the fuzzy analogue of several graph theoretic concepts like paths, cycles, trees and connectedness and established some of their properties. Nagoorgani A and K. Radha [5] introduced the concept of regular fuzzy graphs. Ravi Narayanan .S, et all.[8] discussed the regular domination in fuzzy graphs. Revathi .S, et all [2] introduced the concept of equitable domination in fuzzy graphs. In this paper we discussed the regular equitable domination in fuzzy graphs and establish the relationship with parameter which is also investigated.

II. PRELIMINARIES

A fuzzy graph $G=(\sigma,\mu)$ is a pair of functions $t:V \rightarrow [0,1]$ is a fuzzy subset $\mu: V \times V \rightarrow [0,1]$ fuzzy relation on the fuzzy subset σ such that $\mu(u,v) \leq t(u) \wedge t(v)$ for all $u,v \in V$.

A fuzzy graph $G=(t,\mu)$ with the underlying set V , the order of G is defined and denoted by $O(G) = \sum_{u \in V} t(u)$

and size of G is define and denoted by $S(G) = \sum_{u,v \in V} \mu(u,v)$.

Let $G=(t,\mu)$ be a fuzzy graph. The degree of a node is defined as $d(u) = \sum_{v \neq u, v \in V} \mu(u,v)$

Let $G=(t,\mu)$ be a fuzzy graph .Let $u, v \in V$.We say that u dominates v in G if (u,v) is a strong arc or strong edge. A subset S of V is called a dominating set of G if for each vertex v is not in S , there exists $u \in S$ such that u dominates v .

A dominating set S of a fuzzy graph G is said to be a minimal dominating set, if for each vertex v in S , $S-\{v\}$ is not a dominating set of G .

The minimum fuzzy cardinality of a minimal dominating set of G is called the domination number of a fuzzy graph G . It is denoted by $\gamma_f(G)$.

A dominating set S is a equitable dominating set of a fuzzy graph G if for every vertex $v \in V - S$ there exists a vertex $u \in S$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$.

The minimum fuzzy cardinality taken over all minimal equitable dominating set if for each vertex u in S , $S-\{u\}$ is not a equitable dominating set of G .

The minimum fuzzy cardinality taken over all minimal equitable dominating set of G is called the equitable

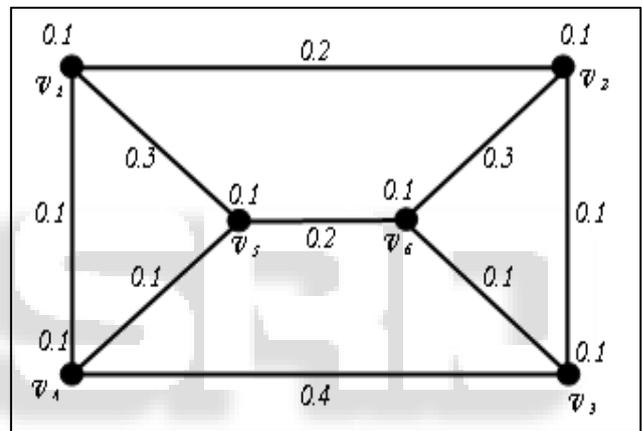
domination number of a fuzzy graph G . It is denoted by $\gamma_{fe}(G)$.

The dominating set S of a fuzzy graph $G=(t,\mu)$ is said to be regular dominating set if every vertex in S is of same degree.

A dominating set S of a fuzzy graph G is said to be minimal regular dominating set, if for each vertex v in S , $S-\{v\}$ is not a regular dominating set of G .

The minimum fuzzy cardinality taken over all minimal regular dominating set of G is called the regular dominating set of G is called the regular domination number of a fuzzy graph G .It is denoted by $\gamma_{fr}(G)$.

A. Example



$S=\{v_5,v_6\}$

Regular domination number $= .1+.1=.2$

$\deg(v_5)=.6$

$\deg(v_6)=.6$

Every vertex in S has same degree

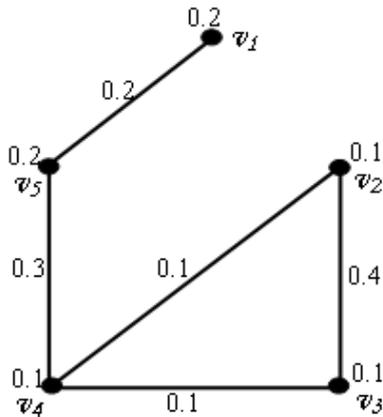
III. MAIN RESULTS

The equitable dominating set S of a fuzzy graph $G=(t,\mu)$ is said to be regular equitable dominating set if every vertex in S is of same degree.

A dominating set S of a fuzzy graph G is said to be minimal regular equitable dominating set ,if for each vertex u in S , $S-\{u\}$ is not a regular equitable dominating set of G .

The minimum fuzzy cardinality taken over all minimal regular equitable dominating set of G is called the regular equitable domination number of a fuzzy graph G . It is denoted by $\gamma_{rfe}(G)$.

A. Example



$S = \{v_3, v_5\}$
 Regular equitable domination number $\gamma_{rfe}(G) = 0.5 + 0.5 = 1.0$
 $\deg(v_3) = 0.5$
 $\deg(v_5) = 0.5$

B. Theorem 3.1

A regular equitable dominating set exists for any regular strong fuzzy graph G.

1) Proof

Let $G = (\sigma, \mu)$ be a regular strong fuzzy graph. It is clear that $d(u_i) = R$ for every $u_i \in G$. Suppose a strong fuzzy graph G has a equitable dominating set, obviously it contains the vertices with $d(u_i) = k$ for every $u \in S$. Therefore every regular strong fuzzy graph is a regular equitable Dominating set and it exists for strong fuzzy graph.

C. Theorem 3.2

Let G be a complete bipartite fuzzy graph with $\sigma(u_i) = R$ and $\sigma(v_i) = R$ for every $u_i \in V_1$ and $v_i \in V_2$ then $\gamma_{rfe}(G) = 2R$.

1) Proof

Let G be a complete fuzzy graph with $\sigma(u_i) = R$ and $\sigma(v_i) = R$ for every $u_i \in V_1$ and $v_i \in V_2$. By the definition of regular fuzzy equitable dominating set = $\{\min \sigma(u_i), \min \sigma(v_i) : u_i \in V_1, v_i \in V_2, \text{clearly } d(u) = d(v)\}$. Therefore regular equitable dominating set exists. That is $\gamma_{rfe}(G) = 2R$.

IV. PROPERTIES OF REGULAR EQUITABLE DOMINATING SET IN FUZZY GRAPH USING STRONG ARCS

In this section we introduce the concept of regular equitable dominating set in fuzzy graph using strong arcs, strong fuzzy graph, and also discuss some parameters.

A. Theorem 4.1

For a regular fuzzy graph G $\gamma_{fe}(G) \leq \gamma_{rfe}(G)$

1) Proof

It is clear that every regular equitable dominating set is a equitable dominating set. we get

$$\gamma_{fe}(G) \leq \gamma_{rfe}(G).$$

B. Theorem 4.2

For a fuzzy graph $G = (\sigma, \mu)$ if S_{rfe} is a regular fuzzy equitable dominating set then $V - S_{rfe}$ is a dominating set of a fuzzy graph G.

1) Proof

Let v be any vertex in S, S is a regular equitable set in G. Since G has no isolated vertex $v \in N(u)$. It is clearly every regular fuzzy equitable dominating set is a equitable dominating set such that $v \in V - S$.

Hence every vertex of S dominates some of the vertices in V-S.

Hence V-S is a dominating set of fuzzy graph G.

C. Theorem 4.3

Let G be a fuzzy path with $\sigma(u_i) = R$ for every $u_i \in V$ and having all edges (that is G be a strong fuzzy graph) then regular fuzzy equitable dominating set $\gamma_{rfe}(G)$ set exists.

1) Proof

Let G be a fuzzy path $\sigma(u_i) = R$ for every $u_i \in V$ and having all effective edges, the regular equitable dominating set $\{u_i : i \neq 1 \text{ or } n\}$ such that S is a regular fuzzy equitable dominating set. Therefore γ_{rfe} set exists.

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