A Study on Torque Capacity of Clutch using Ordinary Differential Equation

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Abstract— The purpose of this report is to verify how an ordinary differentiable equation is applicable in clutch. The project deals with the problems on estimating the torque capacity of a clutch by considering the force, friction coefficient, inner and outer radius under the concept of ordinary differential equations. Here the main objective is to find the torque capacity of different types of clutches.

Key words: Clutch, Ordinary Differential Equation

I. INTRODUCTION

Ordinary Differential Equation is about differential equation which contains one or more functions of independent variable and its derivatives. Ordinary Differential Equation is shortly known as ODE .The term "ordinary" is used in distinction with the term partial differential equation with respect to more than one independent variable.

Linear differential equations, which have solutions that can be added and multiplied by coefficients, and have exact closed-form of solutions are obtained. By contrast, ODEs that lack additive solutions are nonlinear, and solving them is complicated, as one can rarely represent them by elementary functions in closed form. Instead, exact and analytic solutions of ODEs are also in series or integral form. Graphical and numerical methods, applied by hand or by computer, may approximate solutions of ODEs and possibly yield more useful information, often standing in the absence of exact, analytic solutions.

ODE is the study of linear equations and in pure mathematics the differential equations are studied from different viewpoint. The main concerned is about their solutions, functions and equations. The simplest differential equations are solvable by explicit formulas and other than that their properties of the solutions are finding without their exact form.

It introduces key concepts and techniques in the field and shows how they are used in current mathematical research and modelling. It deals specifically with initial value problems, which play a fundamental role in a wide range of scientific disciplines, including mathematics, physics, computer science, statistics and biology.

II. METHODOLOGY

A. *How to Calculate the Torque Capacity of a Clutch:*

Modern automotive power train and drive train systems have at least one clutch as a component. An AWD or 4WD vehicle can have several clutches, depending on the architecture and type of the power train and drive train.

A clutch can be a stand-alone component, used to connect the internal combustion engine (ICE) to the transmission, or can be the sub-component or another main component: torque converter, automatic transmission, transfer case, limited-slip differential (LSD), etc.

Depending on the number of friction plates, a clutch can be:

– Single plate clutch
– Multi-disc clutch

Depending on the type of friction, we can have:

– Dry clutches
– Wet (oil) clutches

Usually, single plate clutches (except torque converter lock-up clutches) have dry friction and multi-disc clutches are with wet friction.

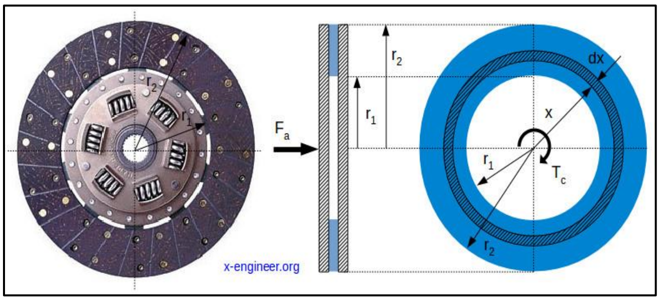
In operation, a clutch can have 4 operating states:

1) Open, zero torque is transmitted between the input and the output shafts.
2) Slipping, some amount of torque is transmitted between input and output shafts; the speed difference between input and output shaft is significant (e.g.500rpm).
3) Micro slipping, almost all of the input torque is transmitted through the clutch; the speed difference between input and output shaft is very small, around5-10rpm.
4) Closed (clamped, locked up), there is no slip between input and output shaft, all the input torque is transmitted through the clutch. Regardless of the type, every clutch has a torque capacity. The torque capacity of the clutch is the amount of torque that can be transmitted by the clutch when it slipping or when it's fully closed. The torque capacity of a clutch depends on a series of factors:

Total area of the friction surface

– Friction coefficient
– Normal force acting on the clutch
– Number of friction elements

To calculate the torque capacity of the clutch we have a look at the geometry of the clutch (friction) disc. Within the area of the friction surface we are going to represent an elementary area dx, at the distance x from the centre.



Where,
Fa [N]–the normal force pressing the clutch plate,
Tc [Nm]–the torque capacity of the clutch,
r1 [m]–the inner radius of the friction surface,
r2 [m]–the outer radius of the friction surface.

The pressure p [Pa] acting on the clutch is equal with the ratio between the normal force Fa and the area of the friction surface S[m^2]:

$$p = \frac{Fa}{S} \quad (1)$$

Assuming that the area of the rivets is negligible, the area of the friction surface is calculated as:

$S=S_2-S_1$

$S=\pi r_2^2-\pi r_1^2$

$S=\pi(r_2^2-r_1^2)$ (2)

Replacing (2) in (1), we get the expression for the clutch pressure:

$$p = \frac{F_a}{(r_2^2 - r_1^2)} \quad (3)$$

The elementary area dA is calculated as

$dA=2\pi xdx$ (4)

The elementary normal force dN, acting on the elementary area is calculated as:

$dN=pdA$ (5)

Replacing (3) and (4) in (5), we get

$$dN = \frac{2xdxF_a}{r_2^2 - r_1^2} \quad (6)$$

The elementary friction force dF is calculated as:

$dF=\mu dN$ (7)

Where μ is the friction coefficient of the clutch disc.

Replacing (6) in (7), we get:

$$dN = \frac{2\mu xdxF_a}{r_2^2 - r_1^2} \quad (8)$$

The elementary friction torque dT is calculated as:

$dT=xdF$ (9)

Replacing (8) in (9), we get:

$$dN = \frac{2\mu x^2dxF_a}{r_2^2 - r_1^2} \quad (10)$$

Integrating equation (10) from r_1 to r_2, we get the mathematical expression of the torque capacity of the clutch:

$$Tc = \int_{r_1}^{r_1} dT$$

$$Tc = \frac{2\mu F_a}{r_2^2 - r_1^2}\int_{r_1}^{r_2} x^2 dx$$

$$Tc = \frac{2\mu F_a}{r_2^2 - r_1^2}\left[\frac{r_2^3 - r_1^3}{3}\right]$$

The resulting mathematical expression for the torque capacity of a single plate clutch is:

$$Tc = \frac{2}{3}\mu\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}F_a \quad (11)$$

For a multi-disc clutch expression (11) becomes:

$$Tc = z\frac{2}{3}\mu\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}F_a \quad (12)$$

Where z is the number of friction plates (discs).

Assuming a mean radius r_m[m] of the clutch calculated as:

$$r_m = \frac{r_1 + r_2}{2}$$

We can deduce a simplified expression, with an acceptable error, for the torque capacity of the clutch:

$$T_c = z\mu F_a r_m$$

III. Application Problems

A. Problems 1:

Calculate the torque capacity of single plate dry clutch, which has the normal force 250N, the outer radius 0.3m, the inner radius 0.2m and the friction coefficient 0.4.

GIVEN:

F_a=250N

r_2=0.3m

r_1=0.2m

μ=0.4

W.K.T,

$$Tc = \frac{2}{3}\mu\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}F_a$$

SOLUTION:

$$Tc = \frac{2}{3}\mu\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}F_a$$

$$Tc = \frac{2}{3}(0.4)\frac{(0.3^3 - 0.2^3)m}{(0.3^2 - 0.2^2)m}250N$$

$$Tc = \frac{2}{3}(0.4)\frac{(0.027 - 0.008)}{(0.09 - 0.04)}250 \quad Nm$$

$$Tc = \frac{2}{3}(0.4)\frac{0.019}{0.05}250 \quad Nm$$

$$Tc = \frac{0.8}{3} * \frac{0.019}{0.05}250 \quad Nm$$

$$Tc = \frac{3.8}{0.15} \quad Nm$$

T_c=25.333Nm.

Hence, the Torque capacity is 25.3Nm.

B. Problem 2:

Calculate the torque capacity of multi-disc wet clutch, which has: 5friction discs (plates), the normal force 250N, the outer radius 0.3m, the inner radius 0.2m and the friction coefficient 0.07.

GIVEN:

Z=5plates

F_a=250N

r_2=0.3m

r_1=0.2m

μ=0.7

W.K.T,

$$Tc = z\frac{2}{3}\mu\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}F_a$$

SOLUTION:

$$Tc = z\frac{2}{3}\mu\frac{r_2^3 - r_1^3}{r_2^2 - r_1^2}F_a$$

$$T_c = 5\frac{2}{3}(0.07)\frac{(0.3^3 - 0.2^3)}{(0.3^2 - 0.2^2)}250Nm$$

$$T_c = \frac{10}{3}(0.07)\frac{(0.027 - 0.008)}{(0.09 - 0.04)}250Nm$$

$$T_c = \frac{0.7}{3} * \frac{0.019}{0.05}250Nm$$

$$T_c = \frac{0.0133}{0.15}250Nm$$

$$T_c = \frac{3.325}{0.15}Nm$$

Hence, the Torque capacity is 22.2Nm.

C. Problem 3:

Calculate the normal force of single plate dry clutch, which has the Torque capacity 22.3Nm, the outer radius 0.4m, the inner radius 0.3m and the friction coefficient 0.6.

GIVEN:

T_c=22.3Nm

r_2=0.4m

r_1=0.3m

μ=0.6

W.K.T,

$$F_a = \frac{T_c * 3({r_2}^2 - {r_1}^2)}{2 * \mu({r_2}^3 - {r_1}^3)}$$

SOLUTION:

$$F_a = \frac{T_c * 3({r_2}^2 - {r_1}^2)}{2 * \mu({r_2}^3 - {r_1}^3)}$$

$$F_a = \frac{22.3 * 3(0.4^2 - 0.3^2)}{2 * 0.6(0.4^3 - 0.3^3)}$$

$$F_a = \frac{22.3 * 3(0.16 - 0.09)}{2 * 0.6(0.064 - 0.027)}$$

$$F_a = \frac{22.3 * 3(0.07)}{2 * 0.6(0.037)}$$

$$F_a = \frac{4.683}{0.0444}$$

$$F_a = 105N$$

Hence the Normal Force is 105N.

IV. CONCLUSION

Thus in this paper, we came to know how to calculate the torque capacity of a clutch so that we can analyse the efficiency of the corresponding clutch. By using the Ordinary Differential Equation, a manufacturer can understand the efficiency of the clutches which are used in the company. By knowing the torque capacity of the clutches, they can deliver the clutches with best quality to their customers.

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