Minimal Edge Domination Number of Graphs

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Abstract— The graph theory properties of χ'[J(G)] and its exact values for some standard graphs. The relation between minimal edge domination with other parameter is also discussed. We also present some results or middle edge domination graph M_ed (J(G)) for some graphs.

Key words: Minimal, Domination Set, Path, Cycle

I. Introduction

Over the past decay, the domination number of graph is one of the new concepts in graph theory which has attracted several researchers because of various applications such as linear algebra and optimization, communication networks, social sciences and in the existing literature [1]. The concept of edge domination was introduced by Mitchell and Hedetniemi [2] and it was explored by many researchers. Herein, we derived minimal edge domination graph.

In graph theory, a dominating set for a graph G = (V, E) is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number is the number of vertices in a smallest dominating set for G.

II. Mathematic Formulation

A. Definition: 2.1

A G_m is a minimal edge dominated graph, if every minimal edge dominating set of G_m has the same cardinality. The domination number of graph denoted by $\chi(J(G))$, is the minimum cardinality of a dominating set in $J(G)$.

B. Lemma: 2.2

If G_m is edge dominated graph and e_f is an edge of G_m , then there exists a minimum edge dominating set containing e_f and a minimum edge dominating set not containing e_f.

C. Definition: 2.3

A graph G_m is also called as class 1 or class 2 according to the equation

$$ds'(G_m) = \chi'(G_m) \text{ or } \chi'(G_m) + 1.$$

1) Proof:

To obtain an edge dominating set containing e_f Then D is minimal and since G_n is well-edge dominated, it is minimum. To obtain a minimum edge dominating set not containing e_f,

D. Theorem:2.4

If G_m is well-edge dominated, then G_m is of class 1

1) Proof:

In this every edge belongs to any one of the χ'-set. Therefore G_m is of class 1.

E. Theorem: 2.5

For cycle C_m with $m \geq 4$ vertices,

$$\chi'(C_m) = \frac{m}{3} \; for\ m \equiv 0(mod\ 3)$$
$$= \left\lceil \frac{m}{3} \right\rceil.$$

For path P_m with $m \geq 4$ vertices,

$$\chi'(P_m) = m, for\ t = 3m + 1, m = 1,2,3,4 \ldots$$
$$= \frac{m}{3} \; for\ m \equiv 0(mod\ 3)$$
$$= \left\lceil \frac{m}{3} \right\rceil otherwise.$$

F. Theorem: 2.6

For any path P_m, with $t \geq 5$, $\chi'[(P_m)] = 2$.
For any Cycle C_m, with $t \geq 5$, $\chi'[(C_m)] = 2$
For any Complete graph K_m with ≥ 5 , $\chi'[(K_m)] = 3$.

Theorem: 2.7

For cycle C_m,

$$\chi'(T(C_m)) \begin{cases} 2\left\lceil \frac{m-1}{3} \right\rceil & if\ m \equiv 0\ or\ 2\ (mod\ 3) \\ 2\left\lceil \frac{m+1}{3} \right\rceil & otherwise \end{cases}$$

1) Proof:

Consider two sets of C_m. Let $v_1, v_2, \ldots, v_m$ be the vertices of the first set of C_m and let $u_1, u_2, \ldots, u_m$ be the vertices of the second set of C_m. Let $e_1, e_2, \ldots, e_m$ be the edges of the first set of C_m, and $e_1', e_2', \ldots, e_m'$ be the edges of the second set of C_m.

Then $|V(T(C_m))| = 2m$ and $|E(T(C_m)) = 4m - 1|$. minimal edge dominating set of $D_2(C_m)$ as follows:

F =

$$\begin{cases} \{e_2, e_5, \ldots, e_{3i+2}, e_2', e_5', \ldots, e_{3i+2}'\} & if\ m \equiv 0\ or\ 2\ (mod\ 3) \\ \{e_2, e_5, \ldots, e_{3i+1}, e_2', e_5', \ldots, e_{3i+2}'\} \cup \{e_m, e_m'\}, & \end{cases}$$

where $0 \leq i \leq \lfloor (m-2)/3 \rfloor$ with
$|F| = 2\lceil (m-1)/3 \rceil$ for $m \equiv 0\ or\ 2\ (mod\ 3)$ and
$|F| = 2\lceil (m+1)/3 \rceil$ for $m \equiv 1 (mod\ 3)$.

Also, $\deg(e_f) = 6 = \deg(e_f') = \Delta'(T(C_m))$, $(1 \leq i \leq m)$ and each edge of $T(C_m)$ can dominate at most seven distinct edges of $T(C_m)$ including itself. But, at a time, each of at most $\lceil (m-2)/3 \rceil$ distinct edges of $T(C_m)$ can dominate seven distinct edges of $T(C_m)$ including itself and each of the remaining edges can dominate less than six distinct edges of $T(C_m)$. Therefore, any set containing the edges less than that of F cannot be an edge dominating set of $T(C_m)$. This implies that the above edge dominating set F is of minimum cardinality.

Hence, minimal edge domination with minimum cardinality among all minimal edge dominating sets is $T(C_m)$.

Thus,

$$\chi'(T(C_m)) = \begin{cases} 2\left\lceil \frac{m-1}{3} \right\rceil & if\ m \equiv 0\ or\ 2\ (mod\ 3) \\ 2\left\lceil \frac{m+1}{3} \right\rceil & otherwise \end{cases}$$

G. Theorem: 2.8

For cycle C_m.

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$$\chi'(T(C_m)) = \begin{cases} \left\lceil \frac{2m-1}{3} \right\rceil & if\ m \equiv 0\ or\ 1\ (mod\ 3) \\ \left\lceil \frac{2m+1}{3} \right\rceil & otherwise \end{cases}$$

1) Proof:

Let $v_1, v_2, \dots, v_m$ be the vertices of cycle C_m and let $u_1, u_2, \dots, u_{m-1}$ be the added vertices corresponding to the edges $e_1, e_2, \dots, e_m$ of C_m to obtain $T(C_m)$.

Thus, $|V(T(C_m)) = 2m|$ and

$|E(T(C_m)) = 4m|$.

Let the edges $f_1, f_2, \dots, f_m \in E(T(C_m))$

Where, $f_m = u_{\lceil m/2 \rceil} v_{\lceil m/2 \rceil}$ for odd m and

$f_m = u_{(m/2)} v_{(m/2)+1}$ for even m.

First, we construct the edge sets of $T(C_m)$ as follows:

$$F = \begin{cases} \{f_2, f_5, \dots, f_{3i+2}\} & if\ m \equiv 0\ or\ 1\ (mod\ 3) \\ \{f_2, f_5, f_{3i+2}\} \cup \{f_{2m}\} & otherwise, \end{cases}$$

for $0 \le i \le \lceil (2m-4)/3 \rceil$ with

$|F| = \lceil (2m-1)/3 \rceil$, if $m \equiv 0\ or\ 1\ (mod\ 3)$ and

$|F| = \lfloor (2m+1)/3 \rfloor$ if $m \equiv 2 (mod\ 3)$.

The above set F is an minimum edge dominating set of $T(C_m)$ because each edge in $E(T(C_m))$ is either in F or is adjacent to an edge in F. Since for any edge $e_f \in$ F, the set $F - \{e_f\}$ does not dominate the edges in $N(e_f)$ of $T(C_m)$, it follows that the above set F is an minimal edge dominating set of $T(C_m)$.

Now, $\deg(f_i) = 6 = \Delta'(T(C_m))$ for $1 \le i \le 2m$ and each edge of $T(C_m)$ can dominate at most seven distinct edges of $T(C_m)$ including itself. But, at a time each of at most $\lfloor m/2 \rfloor$ distinct edges of $T(C_m)$ can dominate the distinct edges of $T(C_m)$ and each of the remaining edges can dominate the edges of $T(C_m)$. Therefore, any set containing the edges less than that of F cannot be an edge dominating set of $T(C_m)$. This implies that the above edge dominating set F is of minimum cardinality.

Hence, the above set F is an minimal edge dominating set with minimum cardinality among all minimal edge dominating sets of $T(C_m)$.

Thus,

$$\chi'(T(C_m)) = \begin{cases} \left\lceil \frac{2m-1}{3} \right\rceil & if\ m \equiv 0\ or\ 1\ (mod\ 3) \\ \left\lceil \frac{2m+1}{3} \right\rceil & \end{cases}$$

III. CONCLUSION

The minimal domination number and the parameters of minimal domination number of jump graphs like C_m, p_m, k_m are also dicussed.

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