A Study on Bernoulli's Equation & Its Application in Fluid Mechanics

K. Nirmaladevi[1] M. Mythilee[2] M. Navein[3] K. Manikandaprabhu[4]
[1]Assistant Professor [2,3,4]UG Scholar
[1,2,3,4]Department of Mathematics
[1,2,3,4]Sri Krishna Arts & Science College, Coimbatore, India

Abstract— The main aim of the paper is to use differential equation in real life to solve world problems. Under differential equation, Bernoulli's equation is used to measure the pressure held in CNC machine which is applied in fluid mechanics. We have applied Bernoulli's equation to find the pressure to be held in pressure control valves during the production of hydraulic pumps in HI-TECH ENGINEERING WORKS and determined best results.

Key words: Bernoulli, Steady Incompressible Flow, Pressure Intensities, Rate of Flow

I. Introduction

Differential equation has vast application in our day to day life both directly and indirectly. It has been used in medicine, engineering, chemistry, mechanics and also in sports. In Bernoulli's equation, using pressure control valves, the pressure in compressing the hydraulic pumps can be determined and controlled. It can be concluded from Bernoulli's equation that the static pressure rises in the direction of flow in case of nozzle due to conservation of velocity pressure into static presssure and vice versa.

II. Bernoulli's Equation

The Bernoulli Equation is a different way of the conservation of energy principle, applied to flowing fluids. It relates the pressure, the kinetics energy and the gravitational potential energy of a fluid in a container or flowing in a tube.

Pressure + ½ density * square of the velocity + density * gravity acceleration* height = constant

The equation is written as,

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

That says the whole formula holds along the system, each term can change but the sum is the same.

We have:

- P: Static Pressure head
- v: velocity of the fluid
- ρ: Density of the fluid
- h: height of the container or the pipe here the fluid is flowing
- g:gravitational acceleration
- $\frac{1}{2}\rho v^2$: velocity head
- ρgh : Hydrostatic pressure head

III. Derivation of Bernoulli's Equation

Consider the motion of a fluid particle in a flow field in steady flow. Applying Newton's second law in the s-direction on a particle moving along a streamline gives,

$$\sum F_s = m a_s \qquad (1)$$

In regions of flow where net frictional forces are negligible, there is no pump or turbine, and no heat transfer along the streamline, the significant forces acting in the s-direction are the pressure (acting on both sides) and the component of the weight of the particle in the s-direction. Therefore equation (1) becomes

P dA - (P+dP) dA - W sinθu 223 mV d s

Where θ is the angle between the normal of the stream line and the vertical z-axis at the point, $m = \rho v = \rho dAds$ is the mass, W=mg = ρg dA ds is the weight of the fluid particle, and $\sin\theta = dz/ds$. Substituting,

$$-dP\,dA - \rho g\,dA\,ds\frac{dz}{ds} = \rho\,dA\,ds\,V$$

Cancelling dA from each term and simplifying,

$$-dP - \rho g\,dz = \rho\,V\,dV$$

$$\text{Noting that } V\,dV = \frac{1}{2}\,d(V^2) + g\,dz = 0$$

The last two terms are exact differentials. In the case of incompressible flow, the first term also becomes an exact differentials and integration gives ,

$$\text{Steady incompressible flow } \frac{p}{\rho} + \frac{v^2}{2} + gz = constant$$

A. Problem 1

In a machine there is a separate oil tank from which the oil gets sump, from the figure the oil is required to pump 60 litres/sec of oil through a 0.1 m diameter pipe from the sumo to a point 10 m above to determine the power required. Also determine pressure intensities at L and M.

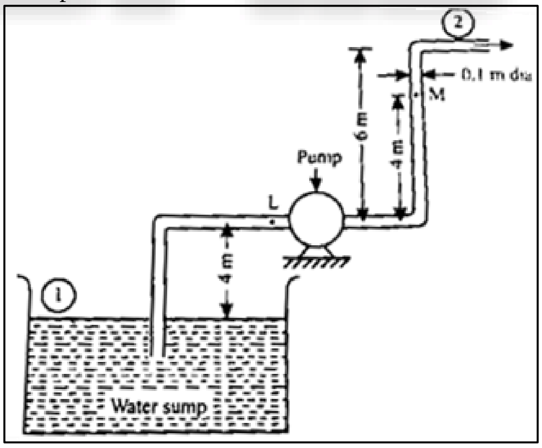

Fig. 1:

1) Solution

Quatity of oil to be pumped Q = 60

Diameter of a pipe d = 0.1m

Area of the pipe A = π/4 * 0.1^2 = 0.00785 m^2

Overall efficiency η_0 = 70%

Power required P is

Q = AV

V = 0.06/0.00785

= 7.64 m/s

Applying Bernoulli's equaiton,

$$\frac{p_1}{w}+\frac{V^2{}_1}{2g}+z_1+H_p=\frac{p_2}{w}+\frac{V^2{}_2}{2g}+z_2$$

H_p = 12.97m of oil

Applying Bernoulli's equation at 1 and L, we get

$$\frac{p_1}{w}+\frac{V^2{}_1}{2g}+z_1=\frac{p_L}{w}+\frac{V^2{}_L}{2g}+z_{L_2}$$

P_L = -68.4 kN/m ^{2}

Applying Bernoulli's equation at 1 and M, we get

$$\frac{p_1}{w}+\frac{V^2{}_1}{2g}+z_1+H_p=\frac{p_M}{w}+\frac{V^2{}_M}{2g}+z_M$$

P_M = 19.62 kN/m ^{2}

Pressure Intensities At L P_L is -68.4 kN/m ^{2} and pressure intensities at M

P_M is 19.62 kN/m ^{2}

B. Problem 2

The diameter of the pipeline in a inclined venturimeter is 400mm. Inclination of the pipeline with the horizontal is 30^0 and thorat diameter is 200mm. The distance between the mouth and throat off the meter is 600mm. Specific gravity of oil flowing through the pipeline is 0.7 and specific gravity of heavy liquid (U – Tube) is 13.6. Reading of the differential manometer is 50mm. The co-efficient of the meter is 0.98. Determine the rate of flow in the pipeline.

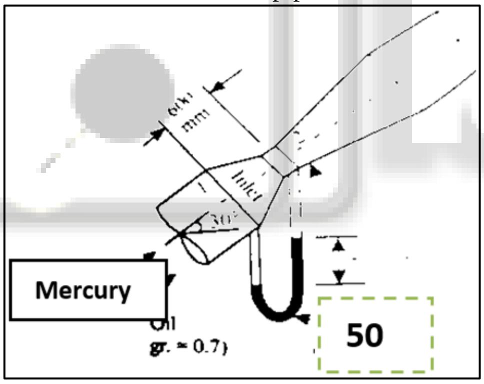

Fig. 2:

Diameter at inlet D_1 = 0.4m

Area of inlet A_1= 0.1257 m^2

Throat diameter D_2 = 0.2 m

Area of throat A_2 = 0.0314 m^2

y = 0.05 m

Difference is given by

$$h=y\left[\frac{S_{hl}}{S_p}-1\right]$$

Where S_{hl} = sp. Gravity of heavy liquid in U- Tube = 13.6

S_p =sp. Gravity of liquid flowing through the pipe = 0.7

$$h=0.05\left[\frac{13.6}{0.7}-1\right]=0.92\ m\ of\ oil$$

With Bernoulli's equaiton,

$$\frac{p_1}{w}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{w}+z_2+\frac{V_2^2}{2g} \quad (1)$$

$$\left[\frac{P_1}{w}+z_1\right]-\left[\frac{P_2}{w}+z_2\right]=h$$

$$\left[\frac{P_1}{w}-\frac{P_2}{w}\right]-[z_1-z_2]=h$$

$$h+\frac{V_1^2}{2g}-\frac{V_2^2}{2g}=0 \quad (2)$$

Applying continuity equation at sections '1' and '2' we get

$A_1V_1 = A_2V_2$

$$V_1=\frac{A_2V_2}{A_1}=\frac{\pi/4\times 0.2^2}{\pi/4\times 0.4^2}\times V_2=\frac{V_2}{4}$$

Substituting the value of V_1 and h in equation 2 , we get

$$0.92+\frac{V_2^2}{16\times 2g}-\frac{V_2^2}{2g}=0$$

$$\frac{V_2^2}{2g}\left(1-\frac{1}{16}\right)=0.92$$

$$V_2^2=\frac{0.92\times 2\times 9.81\times 16}{15}=19.25$$

$V_2 = 4.38\ m/s$

Rate of flow of oil, Q = A_2V_2=0.0314 $\times$ 4.38

$=0.1375\ m^3/s$

C. Problem 3

1) Velocity Measurement by a Pitot tube

A pyrometer and a Piton tube are tapped into a horizontal water pipe, as shown in Figure, to measure static and stagnation (static 224 dynamic) pressures. For the water lengths, find the velocity of the pipe.

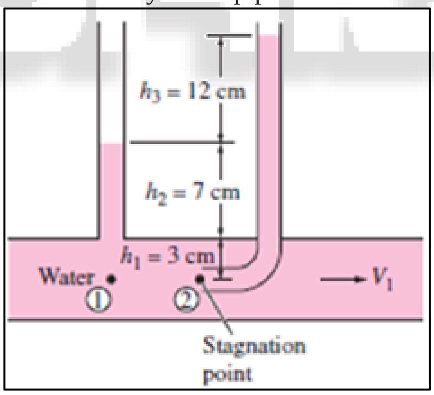

Fig. 3:

2) Solution

a) Assumptions

The flow is steady and incompressible.

b) Analysis

We take points 1 and 2 along the centerline of the pipe, with point 1 directly under the piezo meter and point 2 at the tip of the Pitot tube.1 and 2 can be expressed as

$P_1=\rho g(h_1+h_2)$

$P_2=\rho g(h_1+h_2+h_3)$

Noting that the point 2 is stagnating point and thus $V_2=0$ and $z_1=z_2$, the application of Bernoulli's equations between point 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad \frac{V_1^2}{2g}=\frac{P_2-P_1}{\rho g}$$

Substituting the P_1 and P_2 expressions gives,

$$\frac{V_1^2}{\rho g} = \frac{P_2 - P_1}{\rho g} = \frac{\rho g\,(h_1 + h_2 + h_3) - \rho g(h_1 + h_2)}{\rho g} = h_3$$

Solving for V_1 and substituting ,

$$V_1 = \sqrt{2gh_3} = \sqrt{2(9.81\ m/s^2)(0.12\ m)} = 1.53\ m/s$$

$$V_1 = 1.53\ m/s$$

IV. CONCLUSION

In this paper, the main focus is to present a study on Bernoulli's equation and its application on fluid mechanics, this concept is mainly based on the forms of energy. In this paper we gain some knowledge about heat treatment process and the working of CNC machine. Using Bernoulli's concept we obtained the solution for newly worked out real life applications problem.

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