Modeling and Analysis of Queueing Theory in CDCC Bank

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Abstract— The purpose of this paper is to attain optimum solution with the data collected from Coimbatore District Central Co-operative bank. This work consists of two problems, one is about choosing the exact server (single or multiple) and the other one is about whether a teller can hold both withdrawal and deposits. This paper has some intent i) To show the best server ii) To conclude whether the two channels can handle both withdrawals and deposit and to choose the advantage for the customers.

Key words: Queueing Theory, CDCC Bank, Queueing Model

I. INTRODUCTION

Waiting for service is a part of our daily life. Queueing theory deals with the problems that associate waiting (or queueing). It is quite natural that phenomenon of queue occur in our daily life. We wait at hotels, we form a queue at the railway reservation counter and this waiting circumstance is not only for human beings alone. Jobs wait to get refined on a machine. Aeroplane's coil in air before the permission is given to land at an airport. Car's waits at the traffic signals. We cannot avoid wait without contracting expense or delay. The study of queues deals with quantifying the circumstances of waiting lines adopting representative measures of performances, such as average queue length, average waiting time in queue and average facility utilization. [1] Explains the mathematical support in real life problems and develop probabilistic models which can be used in several areas of science and engineering by Balaji.G. The basic concepts of queueing theory are explained in [2] by Kanthi Swarup, Gupta P.K, Man Mohan. [3] explains the best approach for allocating resources that have been developed which may include inventory and production resources by Sundaresan.V, Ganapathy Subramanian K.S, Ganesan K. [4] enables us to make more effective decisions and more production system on given data and careful prediction of outcomes.

II. PRELIMINARIES & BASIC DEFINITIONS

A. Queueing System

The customer appears at the service counter (single or in a group) and attended by one or more servers. A served customer leaves the system after obtaining the service. In general, a queueing system consists of two components, the queue and the service facility. The queue is where the customers wait to get served. The service facility is where the customer is served and the individual servicestation.

A queueing system can be completely illustrated by:

1) Theinput(arrival pattern),
2) Theservicemechanism(servicepattern),
3) Thequeuediscipline,
4) Customer's behaviour.

B. The Input (Arrival Pattern)

The input illustrates the way in which the customers arrive and join the system. Generally, customers appear in more or less random manner which is not possible for prediction. Hence the arrival pattern can be illuminated in terms of probabilities and there upon the probability distribution for inter-arrival times (the time between two successive arrivals) must be defined. We deal with those Queueing system in which the customers arrive in Poisson process. The mean arrival rate is denoted λ.

C. The Service Mechanism:

The service mechanism means the arrangement of service facility to serve customers. If there is infinite number of servers, then all the customers are served spontaneously or arrival and there will be no queue. If the number of server is finite then the customers are served according to a specific order with a service time a constant or a random variable. Distribution of service time follows "EXPONENTIAL DISTRIBUTION" defined by $f(t) = \lambda e^{-\lambda t}$. The mean service rate is μ.

D. Queueing Discipline:

Queueing Discipline is a rule according to which the customers are chosen for service when a queue is formed. The most natural disciplines are

1) First In First Out-(FIFO) or(FCFS)
2) 2. Last In First Out– (LIFO)
3) 3. Service In Random Order– (SIRO)
4) 4. Priority Schemes - i. Preemptive ii. Non-Preemptive

E. Customer Behaviours:

Customer behavior is classified into four types namely: Bulk Arrival, Jockeying, Balking, and Reneging.

F. Single Server Queue:

The [M/M/1]: [infinity/infinity/FIFO] system is one which the arrivals and desertion area Poisson distribution with a single server, infinite queue length, calling infinite population and the queue discipline is First In First Out. This is the easiest queue system that can be studied mathematically. This queue system is also called or simply referred to as the M/M/1queue.

$$P_0 = P\begin{bmatrix} system\ is \\ empty(idle) \end{bmatrix} = 1 - \frac{\lambda}{\mu}$$

$$L_q = \text{Average number in the queue} = \frac{\lambda^2}{\mu(\mu-\lambda)}$$

$$L_S = \text{Average number in the system} = \frac{\lambda}{(\mu-\lambda)}$$

$$W_q = \text{Average waiting time in queue} = \frac{\lambda}{\mu(\mu-\lambda)}$$

$$W_s = \text{Average waiting time in system} = \frac{1}{(\mu-\lambda)}$$

Where

λ is the arrival rate.

μ is the service rate.

G. Multiple Server Queue:

The [M/M/S]: [infinity/infinity/FIFO] system is one in which the arrivals and desertion area Poisson distribution with a multiple server, infinite queue length, calling infinite population and the queue discipline is First In First Out.

L_q = Length of the queue = $P_s \cdot \frac{\rho}{(1-\rho)(1-\rho)}$

L_s = length of the system = $\frac{\lambda}{\mu} + L_q$

W_q = waiting time in the queue = $\frac{L_q}{\lambda}$

W_s = waiting time in the system = $\frac{L_s}{\lambda}$

III. Analysis Of The Queueing Model:

1) In a bank there are 3 tellers who hold Savings account. Each teller can hold an average service mean rate of 4per hour and if the customer arrive at the rate of 6 per hour. i) Find the average number of a customer in queue. ii) Find the average number of a customer in system iii) Find the average waiting time for a customer in system iv) Find the average waiting time for a customer in queue. Verify which sever is the finest to save time for the given situation?

A. Analysis:

The mean arrival rate (λ) = 6per hour, The mean arrival rate (μ)= 4 per hour

$$\rho = \frac{\lambda}{s\mu}$$

$$P_0 = \left(\sum_{n=0}^{s-1}\left(\frac{1}{n!}\right)\left(\frac{\lambda}{\mu}\right)^n + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!(1-\rho)}\right)^{-1} = 0.228$$

1) *Average number of customers in queue (L_q):*

$$L_q = \left(\frac{1}{SS!}\frac{\left(\frac{\lambda}{\mu}\right)^{S+1}}{(1-\rho)^2}P_0\right) = 0.257$$

2) *The average number of a customer in system (L_S):*

$$L_S = L_q + \left(\frac{\lambda}{\mu}\right) = 1.757$$

3) *The average waiting time for a customer in system (W_S) :*

$$W_S = \frac{1}{\lambda} * L_S = 18 \text{ minutes}$$

4) *The average waiting time for a customer in queue (W_q):*

$$W_q = 1/\lambda * L_q = 3 \text{ minutes}$$

B. Conclusion:

No of server	L_s	L_q	W_s	W_q
Single	2.5	1.78	30 min	22 min
Multiple	1.757	0.257	18 min	3 min

From this we can bring down that the length of customers in queue and system as well as waiting time in queue and system of multiple server with infinity accommodation are very less when compared to single server. So we can select multiple server.

2) A bank has two tellers working on Recurring deposits. The first teller holds withdrawals only and the second teller holds deposits only. It has been found that the service time for both deposits and withdrawal are exponential with the mean service time of 3 minutes per customer. Deposits are found to arrive in Poisson fashion all through the day with mean arrival rate of 14 per hour. Withdrawals also arrive in Poisson fashion all through the day with mean arrival rate of 12 per hour. What would be the effect on the average waiting time for the customer if each teller could hold both withdrawal and deposits? Will this be an advantage for the customers?

A. Analysis

1) Given:

The model is (M/M/1): (∞/ FIFO)
The mean arrival rate (λ) =14 per hour
The mean arrival rate (μ) =3 minutes
Mean service rate (μ) = 1/3 per minute.
(i.e) μ = 20 per hour.

a) CHANNEL 1:

To find W_q:

$$W_s = \frac{1}{\mu-\lambda} = 10 \text{ min}$$

$$W_q = W_s - \frac{1}{\mu} = 7 \text{ min}$$

b) CHANNEL 2:

To find W_q:

$$W_s = \frac{1}{\mu-\lambda} = 8 \text{ min}$$

$$W_q = W_s - \frac{1}{\mu} = 5 \text{ min}$$

If both tellers do both the service, λ = 26 hours
Here the model is (M/M/2) : (∞/ FIFO)

$$\rho = \frac{\lambda}{s\mu} = \frac{13}{10}$$

$$P_0 = \left(\sum_{n=0}^{s-1}\left(\frac{1}{n!}\right)\left(\frac{\lambda}{\mu}\right)^n + \frac{\left(\frac{\lambda}{\mu}\right)^s}{s!(1-\rho)}\right)^{-1} = \frac{7}{33}$$

Average number of customers in a queue:

$$L_q = \left(\frac{1}{SS!}\frac{\left(\frac{\lambda}{\mu}\right)^{S+1}}{(1-\rho)^2}P_0\right) = 0.951$$

$$W_q = 1/\lambda * L_q = 2.194 \text{ minutes}$$

C. Conclusion:

From this it is concluded that the waiting time in teller 1 is 7 min and teller 2 is 5 min. If both the tellers do service, the average waiting time is 2 min. Hence this will be an advantage for customers.

IV. Conclusion

In this paper, we have studied applications of queueing theory in CDCC Bank along with queueing discipline and server system of the queue. There are two main systems – single server and multiple server. Several conditions are discussed for a queueing system. Some examples are illustrated and solutions are also obtained. Hence, single teller handles only one type which takes much more minutes than each teller handling both the types of recurring account. Therefore, Multiple server is the best to reduce the queue length.

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