Performance Analysis of MIMO Receivers in Wireless Communication Systems

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Abstract— MIMO systems has been proved to be one of the most promising and emerging wireless technologies that can efficiently increase the data transmission rate, improves system coverage, and enhances the link reliability. By employing multiple antennas both at transmitter and receiver side, MIMO techniques enables a new dimension—the spatial dimension—that can be utilized in different ways to combat the impairments of wireless channels. In MIMO system technology, there is inter symbol interference present between the symbols. This paper is focussed on Equalization techniques for Rayleigh Flat fading and Rician Flat fading. It is a well-known technique for combating inter symbol interference. Furthermore, in this paper, we have discussed sphere decoder for MIMO wireless communication system. We analyze the performance of Sphere Decoder with other equalization techniques like Zero forcing(ZF), minimum mean squared error equalizer(MMSE) and maximum likelihood decoder by comparing the Bit error rate(BER). Simulation has been done on MATLAB and result shows that SD provides us the better result in comparison to other techniques for a 2 x 2 MIMO using BPSK in Rayleigh flat fading channel as well as in Rician flat fading.

Key words: Equalization; Zero Forcing (ZF); MMSE; Sphere Decoder; MIMO (Multiple Input Multiple Output); BER; Rayleigh & Rician Flat Fading

I. INTRODUCTION

Wireless system engineers are encountering a number of challenges. These include the limited availability of radio frequency spectrum, Power, and a time varying wireless environment. In addition, there is an increasing demand for higher data rates, better quality of service, and higher network capacity. Over the past decade, Multiple-Input Multiple-Output (MIMO) systems have evolved as a most promising technology in these measures.

MIMO systems use multiple antennas at both transmitter and receiver end. It is a key powerful performance-enhancing radio technology. It uses spatial dimension which provide the additional degree of freedom to deal with the wireless channel impaired predominantly by multipath-fading. Multi-path is the arrival of the transmitted signal at an intended receiver through differing angles and/or differing time delays and/or differing frequency (i.e., Doppler) shifts due to the scattering of electromagnetic waves in the environment. Consequently, the received signal power fluctuates in space (due to angle spread) and/or frequency (due to delay spread) and/or time (due to Doppler spread) through the random superposition of the impinging multipath components. This random fluctuation in signal level, known as fading, can severely affect the quality and reliability of wireless communication.

A. Functions of MIMO

MIMO functions can be divided into three categories.

1) Precoding

- It is multi-stream Beamforming. A spatial processing technique used at transmitter side. It maximizes signal power at receiver input. It helps to reduce multipath fading effect.

2) Spatial multiplexing

- It is a powerful technique which transmit several parallel streams of data without additional cost of frequency, power, and time. Thereby, increase the capacity of wireless channel.

3) Diversity

- In diversity methods, same signal is transmitted from each of the antenna using orthogonal space-time coding. Each transmitted signal encounters independent fading, diversity can be exploited to mitigate deep fading event.

II. MIMO SYSTEM MODEL

We consider MIMO systems with Nt transmit antennas in the transmitter side and Nr receiver antennas in the receiver side. The block diagram is shown in figure 1. The transmitted matrix is a Nt x1 column matrix x, where x is the jth component transmitted from the antenna i. We assume a Gaussian channel such that the elements of x are considered to be independent identically distributed (i.i.d) Gaussian variables.

Fig. 1: MIMO System Model

The channel matrix H is a Nr xNt complex matrix. The component hi of the matrix is the fading coefficient from the jth transmit antenna to the ith receive antenna. We assume that the received power for each of the receive antennas is equal to the total transmitted power Es. We assume that the channel Matrix is known at the receiver but unknown at the transmitter.

The channel matrix can be estimated at the receiver by transmitting a training sequence. If we require the transmitter to know this channel, then we need to communicate this information to the transmitter via a feedback channel. The elements of H can be deterministic or random. The noise at the receiver is another column matrix of size Nr x1.
\( x \times 1 \), denoted by \( n \). Each of the \( N_R \) receive branches has identical noise power of \( N_0 \). The receiver operates on the maximum likelihood detection principle over \( N_R \) receive antennas. The received signals constitute a \( N_R \times 1 \) column matrix denoted by \( y \), where each complex component refers to a receive antenna. Since we assumed that the total received power per antenna is equal to the total transmitted power, the SNR can be written as
\[
y = \frac{E_s}{N_0}
\]
Hence, the received vector can be written as,
\[
y = Hx + n
\]
(2)

### III. DECODING TECHNIQUES IN MIMO SYSTEM

Several MIMO receiver algorithms are proposed in the literature. In this section, we discuss about the background information of detectors used in MIMO receiver. Here we minimized or nullified interference signal from other transmit antenna for detecting desired signal from target transmit antenna. Many signal detectors are used in MIMO system.

#### A. Zero Forcing (ZF)

Zero forcing is one of the linear detection techniques which linearly filter the received signal using liner filter matrices and independently decode them, which applies the invers of the frequency response of the channel. It applies the inverse of the channel frequency response of received signal, to restore the signal after the channel. The name Zero Forcing corresponds to bring down the inter symbol interference (ISI) to zero in a noise free case. This will be useful when ISI is significant compared to noise. \( x \) is the transmitted data over \( H \) channel now we get \( y \) received data
\[
y = Hx + n
\]

Using zero forcing detector in the MIMO receiver part we get the estimated transmitted data is
\[
\hat{X} = H^\dagger y
\]
\[
= H^\dagger (Hx + n)
\]
\[
= x + H^\dagger n
\]

\( H^\dagger \) is the pseudo inverse of the channel matrix. With the addition of the noise vector, ZF estimate, \( \hat{X} \) consists of the decoded vector plus a combination of the inverted channel matrix and the unknown noise vector.

In reality ZF does not work in most of the application for the following reason:

1) If the channel has finite length, the impulse response of ZF needs to be infinity long. At some frequency they received signal may be weak. To compensate, the magnitude of the ZF grow very large. If any noise is added after the channel growled a large factor and then it destroys the overall SNR.

2) If channel have zeros in its frequency response that cannot inverted at all.

3) The ZF removes all ISI and it is ideal when the channel is noiseless. However, when the channel is noisy, the ZF will amplify the noise greatly at frequencies \( f \) where the channel response \( H(2\pi f) \) has a small magnitude(near zero of the channel) in the attempt to invert the channel completely, the MMSE detector was proposed, where the noise variance is considered in the construction of the filtering matrix.

#### B. Minimum Mean Square Error (MMSE)

Minimum Mean Square error does not usually eliminate ISI completely but instead minimizes the total power of the noise and ISI component in the output. MMSE is the small modification in the ZF denominator of the channel frequency. It is used to reduce error signal. A MMSE estimator is a method in which it minimizes the mean square error (MSE), which is a universal measure of estimator quality [3]. Minimum Mean Square Error (MMSE) approach alleviates the noise enhancement problem by taking into consideration the noise power when constructing the filtering matrix. The vector estimates produced by an MMSE filtering matrix becomes
\[
\hat{X}_{\text{MMSE}} = \left[ (H^\dagger H + \sigma^2 \mathbf{I})^{-1} \right] H^\dagger y
\]
(3)
Where \( \sigma \) is the noise variance. The added term \((1/\text{SNR} = \sigma^2\), in case of unit transmit power) offers a trade-off between the residual interference and the noise enhancement.

#### C. Maximum Likelihood (ML)

ML receiver, which is known as an optimal receiver [6], this receiver achieves optimal performance in the sense of maximizing the probability of correct data detection. The maximum likelihood detector for a MIMO receiver operates by comparing the received signal vector with all possible noiseless received signals in the receiver side searches across all possible combinations, and tries to solve the inter channel interference (ICI) caused by transmitting from all antennas simultaneously, on the same frequency [4]. Maximum-likelihood (ML) detection for high order MIMO systems face a major challenge in computational complexity that grows exponentially with the number of transmitted and received antennas and depends only on the spectral efficiency [4]. This limits the practicality of these systems from an implementation point of view, because it’s impossible to implement for large array sizes and high order digital modulation schemes particularly for mobile battery-operated devices [1][4]. The ML detection calculates the Euclidean distance between the received signal vector and the product of all possible transmitted signal vectors with the given channel \( H \), and finds the one with the minimum distance [2]. If \( C \) and \( N_T \) denote a set of signal constellation symbol points and a number of transmit antennas, respectively. Then, ML detection determines the estimate of the transmitted signal vector \( x \) as
\[
\hat{X}_{\text{ML}} = \arg \min_{x \in \mathbb{C}^{N_T}} \| y - Hx \|^2
\]
(4)
The ML decoding technique is the optimal decoding technique achieving the best performance in terms of Symbol Error Rate (SER). ML decoder uses an exhaustive search algorithm where all possible codeword are checked, and the one with minimum distance is selected as the final codeword.

#### D. Sphere Decoder (SD)

Sphere Decoding is a new type of decoding technique which aims to reduce the computational complexity of the decoding technique [2]. ML decoder uses an exhaustive search algorithm where all possible code words are checked, and the one minimum distance is selected as the final codeword [1]. The ML decoding technique is certainly an optimal decoding technique but with the increase in the number of transmit and receive antennas along with a complex constellation adds to
the computational complexity of the decode mechanism. In the case of sphere decoder, the search can easily be restricted by drawing a circle around the received signal [3]. So the search allows only those codewords to be checked that happen to fall within the sphere. All the remaining code words outside the sphere are not taken into consideration for decoding [2]. In case of sphere decoder, the received signal is compared to the closest lattice point, since each codeword is represented by a lattice point. The number of lattice points scanned in a sphere decoder depends on the initial radius of the sphere [7]. The radius must be chosen in such a way that the value should cover the lattice. The initial radius selected plays a critical role in identifying the correct point in the lattice [7]. The main idea of the sphere decoder is to reduce the computational complexity of the maximum likelihood detector by only searching over only the noiseless received signals that lie within a sphere of radius R around the received signal [2].

\[ \hat{X}_{\text{ML}} = \arg \min_{x \in \mathbb{C}^{N_T \times 1}} \| y - Hx \|^2 \leq R^2 \]  

(5)

Figure 1 shows the idea behind the sphere decoding. Further in SD here are different algorithms used to achieve optimal results. The calculation of the radius in the SD is the most difficult task, and it is to be done at the pre-processing level [7]. If the radius is too large, average processing cycle becomes extremely high, making real time operation impossible. On the other hand, if the radius is too small, even the ML solution cannot satisfy the sphere constraint shown in figure 2. Thus setting the appropriate radius is very critical to successful implement the SD.

![Fig. 2: Diagram of Sphere Decoder](image)

It increases the radius when there exists no vector within a sphere, and decreases the radius when there exist multiple vectors within the sphere. It is way better than all other techniques as it helps in improving the bit error rate and spatial complexity. The original sphere decoder, after the computation of the first point in the lattice, reduces the radius of the sphere to the value of the distance of this new point to the received point [7].

**IV. SIMULATION RESULTS & DISCUSSION**

In this section we showed sphere decoder is good when compared with other decoder such as zero forcing, minimum mean square error receivers. The simulation results are obtained for Zero-Forcing (ZF) Receiver, Minimum-Mean-Square-Error (MMSE) Receiver, Maximum Likelihood (ML) Receiver and Sphere Decoding (SD) Receiver. Performance in the form of BER is plotted against different values of Eb/N0.
Figure 5 show the BER performance of the Maximum Likelihood Receiver, from the result we clearly understand that, Using Maximum Likelihood receiver, the SNR is above 20 we get BER $10^{-5}$.

![Figure 5: BER curves of SD and ML Receiver](image)

**V. CONCLUSION**

Till now SD is the best technique among all the decoding techniques. ZF and MMSE also provide the optimal results but the problem of the bit error rate remains as it is. By using some of the algorithms of these techniques we can achieve a little bit better results in terms of BER.

From the MATLAB results we can see that ML has got a better performance in terms of BER. But the complexity increases as the number of transmitters increase. The Sphere Decoding algorithm is a solution for this problem. This reduces the complexity of ML and give the better and optimal results. Which are required in a wireless communication system is also achieved.

**REFERENCES**


