A Case Study on Perlustrating the Circuits in Lamps using Ordinary Differential Equation

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Abstract— The purpose of this report is to verify how an ordinary differentiable equation is applicable in first order circuits. In this, the concept of Kirchoff's Current Law is used, by constructing a basic and simple circuit containing resistors, capacitors and inductors. The purpose of the report also includes measuring the current through each resistors and the voltage at each nodes in the circuit. In this study, we have derived some results using Ordinary Differential Equation (ODE) in first order circuits with different voltages and different types of circuits like RL and RC both in series and parallel. Using this we can examine the resistors or capacitors with respect to time and thereby checking the efficiency of circuits.

Key words: Ordinary Differential Equation (ODE)

I. INTRODUCTION

Ordinary Differential Equation is about differential equation which contains one or more functions of independent variable and its derivatives. Ordinary Differential Equation is shortly known as ODE .The term "ordinary" is used in distinction with the term partial differential equation with respect to more than one independent variable.

Linear differential equations, which have solutions that can be added and multiplied by coefficients, and have exact closed-form of solutions are obtained. By contrast, ODEs that lack additive solutions are nonlinear, and solving them is complicated, as one can rarely represent them by elementary functions in closed form. Instead, exact and analytic solutions of ODEs are also in series or integral form. Graphical and numerical methods, applied by hand or by computer, may approximate solutions of ODEs and possibly yield more useful information, often standing in the absence of exact, analytic solutions.

ODE is the study of linear equations and in pure mathematics the differential equations are studied from different viewpoint. The main concerned is about their solutions, functions and equations. The simplest differential equations are solvable by explicit formulas and other than that their properties of the solutions are finding without their exact form.

A. KIRCHOFF'S Current Law:

This fundamental law results from the conservation of charge. It applies to a junction or node in a circuit. A point in a circuit where charge has several possible paths to travel.

The algebraic sum of currents in a network of conductors meeting at a point is zero.

$$\sum_{k=1}^{n} I_k = 0$$

II. METHODOLOGY

A. First Order Circuits

First order circuits are circuits that contain only one energy storage element (capacitor or inductor), and that can, therefore, be described using only a first order differential equation. The two possible types of first-order circuits are:

1) RC
2) RL

RL and RC circuits is a term we will be using to describe a circuit that has either a) resistors and inductors (RL), or b) resistors and capacitors (RC).

B. RL Circuits parallel

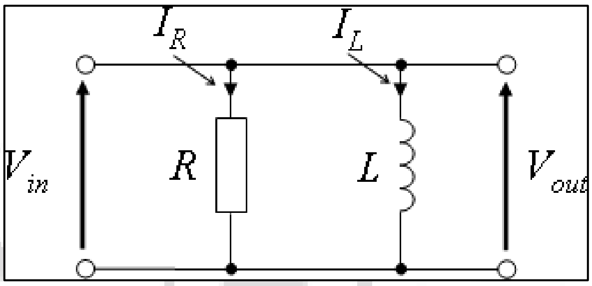


Fig. 1: An RL parallel circuit

An RL Circuit has at least one resistor (R) and one inductor (L). These can be arranged in parallel, or in series. Inductors are best solved by considering the current flowing through the inductor. Therefore, we will combine the resistive element and the source into a Norton Source Circuit. The Inductor then, will be the external load to the circuit. We remember the equation for the inductor:

If we apply KCL on the node that forms the positive terminal of the voltage source, we can solve to get the following differential equation:

$$v(t) = L(di/dt)$$

The transition period during which the current adjusts from its initial value of zero to the final value $i=\frac{V}{R}(1- e^{\frac{Rt}{T}})$ which is the steady state.

C. RC Circuits Parallel

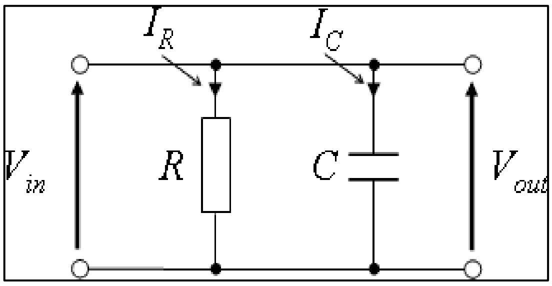


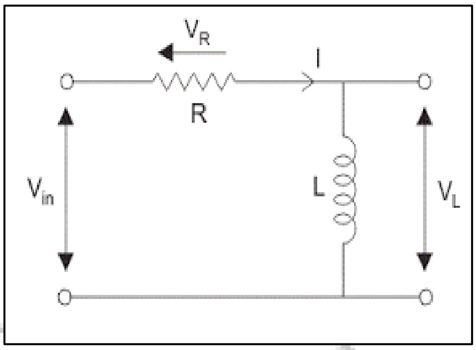
Fig. 2: A parallel RC Circuit

An RC circuit is a circuit that has both a resistor (R) and a capacitor (C). Like the RL Circuit, we will combine the resistor and the source on one side of the circuit, and combine them into a the venin source.

A differential equation is linear if the unknown function and its derivatives have degree 1 (products of the unknown function and its derivatives are not allowed) and nonlinear otherwise. The characteristic property of linear equations is that their solutions form an affine subspace of an appropriate function space, which results in much more developed theory of linear differential equations.

The current stops flowing as the capacitor becomes fully charged.

D. *RL Circuit series*



Let us consider the RL (resistor R and inductor L) circuit shown above. At t = 0 the switch is closed and current passes through the circuit. Electricity laws state that the voltage across a resistor of resistance R is equal to R i and the voltage across an inductor L is given by L di/dt (i is the current). Another law gives an equation relating all voltages in the above circuit as follows: L di/dt + Ri = E

Where E is a constant voltage. Let us solve the above differential equation which may be written as follows L [di / dt] / [E - R i] = 1 which may be written as - (L / R) [- R d i] / [E - Ri] = dt Integrate both sides - (L / R) ln(E - R i) = t + c , c constant of integration. Find constant c by setting i = 0 at t = 0 (when switch is closed) which gives c = (-L / R) ln(E) Substitute c in the solution - (L / R) ln(E - R i) = t + (-L/R) ln (E) which may be written (L/R) ln (E)- (L / R) ln(E - R i) = t ln[E/(E - Ri)] = t(R/L) Change into exponential form [E/(E - Ri)] = $e^{t(R/L)}$ Solve for i to obtain i = (E/R) (1-$e^{-Rt/L}$)

The starting model for the circuit is a differential equation which when solved, gives an expression of the current in the circuit as a function of time.

Since the voltage drop across the resistor, V_R is equal to IxR (Ohms Law), it will have the same exponential growth and shape as the current. However, the voltage drop across the inductor, V_L will have a value equal to: $Ve^{(-Rt/L)}$. Then the voltage across the inductor, V_L will have an initial value equal to the battery voltage at time t = 0 or when the switch is first closed and then decays exponentially to zero as represented in the above curves.

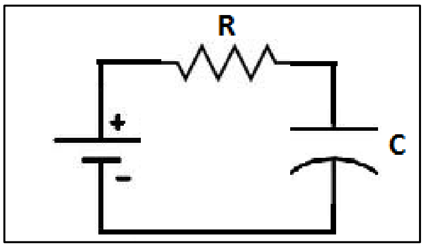
III. APPLICATION PROBLEMS

A. *Modelling 1*

In Electronic Signals and Controls (ESICO) is a lamp production company, manufactures reflection typed LED lamp, in series circuit. A lamp costs Rs.100. Every year 6000 reflection typed LED lamps are manufactured and sold by ESICO. This lamp is 12v, whose resistance=1000Ω, capacitance=0.085F.

Derive the equation of current at t → ∞. And also verify the condition of $v_R = v_c$ to check its strength. Thus the manufacturers came to know about the strength of the circuit.

IV. ANALYSIS



where, V=12v
R=1000Ω c=0.085F
The equation is,

$$\mathrm{Ri} + \frac{1}{c}\int i\,dt = \mathrm{V}$$

$$\mathrm{R}\frac{dq}{dt} + \frac{q}{c} = \mathrm{V}$$

$$1000\frac{dq}{dt} + \frac{q}{0.085} = 12$$

Dividing both sides by 1000,

$$\frac{dq}{dt} + 0.01q = 0.012$$

Integrating factor= $e^{\int p(x)dx} = e^{\int 0.01dt} = e^{0.01t}$

Multiplying I.F and integrating on both sides,

$(e^{0.01t}.150q + e^{0.01t}.\frac{dq}{dt}) = 0.012\int e^{0.01t}$ dt

$qe^{0.01t} = 0.012\int e^{0.01t}dt + k = 1.2(e^{0.01t}) + k$

When i=0, t=0, we get K= -1.2

Therefore the required equation is,

$$q = q\boldsymbol{e}^{\mathbf{0.01}\boldsymbol{t}} = 1.2(1-\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}})$$

(i) $\boldsymbol{v_R}$ & $\boldsymbol{v_C}$

$$\boldsymbol{v_R} = \mathrm{Ri} = \frac{\boldsymbol{dq}}{\boldsymbol{dt}}\mathrm{R}$$

$$\boldsymbol{v_R} = \frac{\boldsymbol{d}}{\boldsymbol{dt}}(1.2(1-\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}}))(1000) = 12\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}}$$

$$\boldsymbol{v_C} = \frac{\mathbf{1}}{\boldsymbol{c}}\int \boldsymbol{idt} = \frac{\boldsymbol{q}}{\boldsymbol{c}} = \frac{\mathbf{1.2(1-e^{-0.01t})}}{\mathbf{0.085}}$$

$$\boldsymbol{v_C} = 16(1-\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}})$$

(ii) $\boldsymbol{v_c} = \boldsymbol{v_R}$

$$12\,\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}} = 16(1-\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}})$$

$$\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}} = 0.6$$

Taking log on both sides

$$0.01t = \log\ 0.6$$

$$t = 50s$$

since, $\boldsymbol{v_R} = \boldsymbol{v_c} = 12\,\boldsymbol{e}^{-\mathbf{0.01}\boldsymbol{t}} = 12\,\boldsymbol{e}^{-\mathbf{0.01(70)}} =$

$$\boldsymbol{v_R} = \boldsymbol{v_c} = 7\mathrm{V}$$

Therefore, voltage across resistor and voltage across capacitance crosses at voltage of 7 in 50s

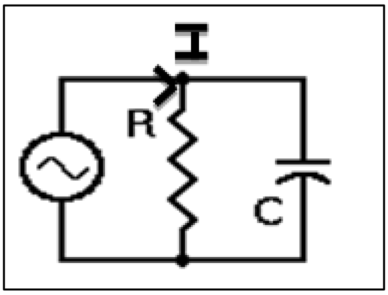
1) *Deduction:*

By this we can conclude that, the stability of voltage in reflection typed LED lamp is below medium level, since the average level is form 11.3v to 15v . Hence it can function with the period of nearly 2 years. Therefore ESICO can earn Rs.6,00,000/year.

B. Modelling 2

Electronic signals and controls (ESICO), lamp production company manufactures multi-tier LED lamp, in parallel circuit. Every year 6000 multi-tier LED lamps are manufactured and sold by ESICO where each lamp costs Rs.1000 .This lamp is 6v, resistance=56Ω, capacitance=0.02F. Prove that current is reduced to certain amperes when the capacitor is fully charged and also detect at which second the flow of current is constant.

1) Analysis:



Where , R=56Ω, V=6v, C=0.02F

By kirchoff's current law,

$$I_s = i_R + i_c t$$

$$I_s = \frac{V_R}{R} + C\frac{dv_R}{dt}$$

$$\frac{I_s}{C} = \frac{dv}{dt} + \frac{V}{RC}$$

Integrating factor = $e^{\int dt/RC} = e^{\int t/RC}$

$$\int \frac{d}{dt}[e^{t/RC} V] = \int \frac{I_s}{C} e^{t/RC} + K$$

$$[e^{t/RC} V] = I_s R e^{t/RC} + K$$

Divide by $e^{t/RC}$,

$$V = I_s R + k\, e^{-t/RC}$$

When, v(0)=0=$I_s R$+k

$$K = -I_s R$$

Therefore, v(t) = $I_s R(1 - e^{-t/RC})$, is the required general equation.

Substitute the values to obtain t, at any time,

6= $I_s 56(1 - e^{-t/1.1})$, is the required equation

By this equation we can get the value of voltage at any time and also resistance & current.

When substituting 't' in the current equation,

t=1sec, I_s =0.2

t=2sec, I_s =0.15

t=3sec, I_s =0.13

t=4sec, I_s =0.123

t=5sec, I_s =0.121

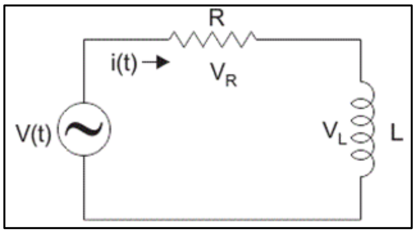
1) Deduction:

Hence it is proven that the current reduces when capacitor is fully charged and the current is maintained at constant level of 0.12A from 4th second. Normally current reduces from 1.3A-0.7A when capacitor is fully charged in a 6v lamp. Therefore the stability of multi-tier LED lamp is at medium level and can function 4 years efficiently. Therefore the company can earn Rs.60,00,000/year.

C. Modelling 3

Electronic signals and controls (ESICO), lamp Production Company manufactures dome typed LED lamp, in series circuit. Every year 12000 dome typed LED lamps are manufactured and sold by ESICO where each lamp costs Rs.130. This dome typed lamp is manufactured at both voltages 24v and 36v, with same resistor and inductor where, resistance=1000Ω, inductance=2H. Compare the stability produced by both the voltages and conclude which voltage of dome typed led lamp yield profit to the company.

1) Analysis:



1) 24v lamp,

Where R=1000Ω, L=2H, V=24v

The equation is,

$$Ri + L\frac{di}{dt} = V$$

$$1000i + 2\frac{di}{dt} = 24V$$

Dividing both sides by 2,

$$500i + \frac{di}{dt} = 12$$

Integrating factor= $e^{\int p(x)dx} = e^{\int 500dt} = e^{500t}$

Multiplying I.F and integrating on both sides,

$$\int (e^{500t}.500i + e^{500t}.\frac{di}{dt}) = 12\int e^{500t}\, dt$$

$$= (0.024)\, e^{500t} + k$$

When i=0, t=0, we get k = - 0.024

Therefore the required equation is,

$$i = ie^{500t} = 0.024(1 - e^{-500t})$$

(i) v_R & v_L

$$v_R = Ri = 24(1 - e^{-500t})$$

$$v_L = L\frac{di}{dt} = 24e^{-500t}$$

(ii) $v_R = v_L$

$$24(1 - e^{-500t}) = 24e^{-500t}$$

$$e^{-500t} = 0.5$$

Taking log on both sides

500t= log 0.5, which gives t = 0.0014s

Since, $v_R = v_L = 24e^{-500(t)} = 24e^{-500(0.0014)}$

$$v_R = v_L = 12V$$

Therefore, voltage across resistor and voltage across inductor crosses at voltage of 12 in 0.0014s.

(ii) 36v lamp

Where R=1000Ω, L=2H, V=36v

The equation is,

$$Ri + L\frac{di}{dt} = V$$

$$1000i + 2\frac{di}{dt} = 36V$$

Dividing both sides by 2, $500i + \frac{di}{dt} = 13$

Integrating factor= $e^{\int p(x)dx} = e^{\int 500dt} = e^{500t}$

Multiplying I.F and integrating on both sides

$\int (e^{500t}.500i + e^{500t}.\frac{di}{dt}) = 13\int e^{500t}\, dt = (0.026)\, e^{500t}$+k

When i=0, t=0, we get k = - 0.026

Therefore the required equation is,

$$i = ie^{500t} = 0.026(1 - e^{-500t})$$

(i) v_R & v_L

$$v_R = Ri = 26(1 - e^{-500t}) \quad v_L = L\frac{di}{dt} = 26e^{-500t}$$

(ii)v_R = v_L

26(1- e^{-500t}) =26e^{-500t}gives e^{-500t} = 0.5

Taking log on both sides

500t= log 0.5 gives t=0.0014s

since, v_R = v_L= 26$e^{-500(t)}$ = 13V

Therefore, voltage across resistor and voltage across inductor crosses at voltage of 12 in 0.0014s.

2) Deduction:

By this we can conclude that 24v have a better stability than 36v of a dome typed LED lamp since 36v is progressed with only one volt when compared to 24v. The stability of this lamp ranges from 11.3v to 15v, where it can function efficiently with period of five years. So, the stability of 24 volt dome typed LED lamp is very quick. Hence the lamp can function with the period of 5 years efficiently with the earning of Rs.15,60,000/year.

V. CONCLUSION

Thus, in this paper we came to about how the circuits, resistance, voltage and the capacitors are working and also the strength and capability of the lamps under the voltage and current passing in it. By using the Ordinary Differential Equation, a manufacturer can understand the efficiency of the lamps which are used in the company. By knowing the potentials of the lamps we can avoid the fluctuations and damages of the lamps by passing the required voltages through them.

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