Analysis of Beam using Timeshenko Method & Compare with Eulers Elementry Beam Theory

Akhilesh Sharma[1] Rashmi Sakalle[2] Nitin Tiwari[3]
[1]Mtech Scholar [2,3]Assistant Professor
[1,2,3]TIEIT, Bhopal, India

Abstract— This paper describes analysis of simply supported beam subjected to uniformly distributed load with different length to depth ratios by Timoshenko method and compared it with Euler's Bernoulli Theory. According to IS code 456-2000 a beam is considered as deep, if the span to depth ratio is 2 or less for simply supported beam and 2.5 or less for continuous beam. The parameters observed were flexural stress, shear stress and strain components. The effective span to depth ratios of beam considered were 8.0, 6.0, 4.0, 3.0 and 2.0.

Key words: Timeshenko Method, Eulers Elementry Beam Theory

I. Introduction

In recent years the use of RC deep beam has become more prevalent. In IS code-456 (2000) clause 29, a simply supported beam is defined as deep beam when its effective span to depth ratio is less than or equal to 2.0, and for continuous beam it will be less than or equal to 2.5, where the effective span is center to center distance between supports or 1.15 times the clear span whichever is less. The elementary theory of bending of beam based on Euler-Bernoulli hypothesis that the effects of the shear deformation and stress concentration. This theory is applicable for slender beams and is not suitable for thick or deep beams because it is based on the assumption that the sections plane before bending remains plane after bending, implying that the transverse shear strain is zero. It means Euler's elementary theory neglects the transverse shear deformation, it underestimates deflections in case of thick beams where shear deformation effects are significant.

The ACI Building code 318-14(Committee, Institute et al. 2014) defines deep beams as members loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the supports, and have Clear spans, is, equal to or less than four times the overall depth. IS 456-2000 code defines deep beam as a beam has ratio of effective span-to-overall depth (l/h) less than: 2.0, for simply supported beam; 2.5, for a continuous beam. In RC beams, Regions where the shear span is less than twice the depth of the beams are defined as D-regions, D refers to disturbed or deep and it is controlled by arch action.

- The following are the major differences between deep beam and shallow beams based on the design assumptions are as follows:
- Two dimensional action i.e. deep beam behaves as a plate subjected to heavy loads in its own plane.

In deep beam plane section before bending don't remains plane after bending and thus strain distribution is no longer linear.

II. Literature Review

Vaibhav B. Chavan, et al (2015) have observed that elementary theory of bending of beam based on Euler-Bernoulli hypothesis disregards the effects of the shear deformation and stress concentration. The theory is suitable for slender beams and is not suitable for thick or deep beams.

Rakesh Patel, S.K. Dubey, K.K Pathak (2014) Effect of depth span ratio on the behavior of beam. The analyzed simply supported beam for different depth to span ratio using the MIF, FEM and Bending theory.

Niranjan B.R, Patil S.S (2012) Conducted experimental research and determine the strength of deep beam designed by strut and tie model. The experiment was done in two faces and experimental results were compared with the results obtained theoretically by finite strip method.

Niranjan, and patil (2011) The analysed simply supported beams for different shear stress ,flexural strain ,flexural stress using the finite element method and eulers elementary beam theory

III. Methodology

A. Timoshenko Method

Timoshenko given general method for analyzing the beam with uniform load over it and supports are simply supported which can be used for analyzing the behavior of beam for different length/depth ratio and helping in finding of the position of neutral axis , flexural stress variation ,shear stress variation.

The general formula for uniformly loaded beam can help in finding of the lever arm and validation can be done using that the general equation for uniformly loaded beam simply supported beam can be done using the polynomial of fifth degree.

As we need to find the strain components u and v for calculating the stress at different fiber of the beam.

Let us consider a stress function in the form of a polynomial of the fifth degree.

$$\phi_5 = \frac{a_5}{5(4)}x^5 + \frac{b_5}{4(3)}x^4 y + \frac{c_5}{3(2)}x^3 y^2 + \frac{d_5}{3(2)}x^2 y^2 + \frac{e_5}{4(3)}xy^4 + \frac{f_5}{5(4)}y^5$$

The design bending moments are calculated as follows:

In a simply supported beam, the bending moment is calculated as in ordinary beams. For a total load w uniformly distributed on the beam

$$M_{\max} = \frac{wL^2}{8}$$

In a continuous beam the bending moment, according to American practice for a uniformly distributed load, w per unit length is as follows:

At mid span, $M_{max} = \frac{wL^2}{24}(1 - e^2)$ positive

At face of support $M_{max} = \frac{wL^2}{8}(1 - e)(2 - e)$ Negative

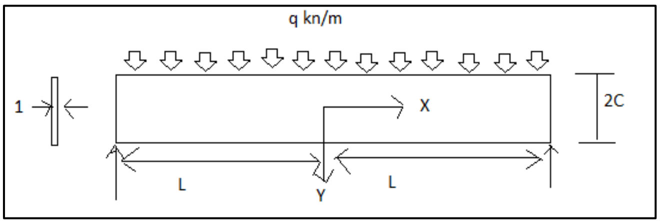
Where, e is the ratio of width of support to effective span.

From this we can calculate bending moment value

IV. PROBLEM ANALYSIS

Following problem have been taken in this work:

Problem:



Simply supported beam

Length of the beam = 3000 mm

Over all depth of the beam =2C mm

Uniformly distributed load over the beam is = 30 N/mm

Width of the beam = 250 mm

S. NO	Depth (mm)	y/d	U(mm)	V(mm)	Normal Stress N/mm^2	Shear Stress N/mm^2	Bending Stress (Timoshenko theory) (N/mm^2)	Bending Stress (Bending theory) (N/mm^2)
1	0	0	1.290	1.096	0	0	-5.70	-5.76
2	37.5	0.1	1.032	1.096	0.69	0.259	-4.60	-4.61
3	75	0.2	0.776	1.096	2.94	0.460	-3.44	-3.45
4	112.5	0.3	0.516	1.097	6.33	0.600	-2.30	-2.30
5	150	0.4	0.258	1.097	10.41	0.690	-1.15	-1.16
6	187.5	0.5	0.003	1.098	14.85	0.719	0	0
7	225	0.6	-0.257	1.098	19.29	0.690	1.15	1.16
8	262.5	0.7	-0.514	1.098	23.37	0.600	2.30	2.30
9	300	0.8	-0.774	1.099	26.73	0.4600	3.44	3.45
10	337.5	0.9	-1.032	1.099	29.01	0.259	4.60	4.61
11	375	1	-1.310	1.100	30.00	0	5.70	5.76

Table 1: For l=3000 mm and depth d=375 mm (l/d= 8.0)

S. NO.	Depth (mm)	y/d	U(mm)	V(mm)	Normal Stress N/mm^2	Shear Stress N/mm^2	Bending Stress (Timoshenko theory) (N/mm2)	Bending Stress (Bending theory) (N/mm^2)
1	0	0	0.727	0.896	0	0	-3.180	-3.240
2	50	0.1	0.577	0.897	0.690	0.194	-2.561	-2.592
3	100	0.2	0.433	0.897	3.195	0.345	-1.931	-1.944
4	150	0.3	0.288	0.898	6.555	0.453	-1.295	-1.296
5	200	0.4	0.144	0.898	10.635	0.558	-0.647	-0.648
6	250	0.5	0.00033	0.899	15.075	0.595	0	0
7	300	0.6	-0.144	0.899	19.515	0.558	0.647	0.648
8	350	0.7	-0.288	0.899	23.595	0.453	1.295	1.296
9	300	0.8	-0.434	0.900	26.955	0.345	1.931	1.944
10	337.5	0.9	-0.58	0.901	29.010	0.194	2.561	2.592
11	375	1	-0.727	0.901	30.075	0	3.180	3.240

Table 2: for l=3000 mm and depth d=500 mm (l/d= 6)

S. NO.	Depth (mm)	y/d	U(mm)	V(mm)	Normal Stress N/mm^2	Shear Stress N/mm^2	Bending Stress (Timoshenko theory) (N/mm^2)	Bending Stress (Bending theory) (N/mm^2)
1	0	0	0.323	0.286	0.000	0	-1.380	-1.440
2	75	0.1	0.258	0.288	0.690	0.129	-1.121	-1.152
3	150	0.2	0.192	0.289	2.970	0.230	-0.851	-0.864
4	225	0.3	0.128	0.290	6.330	0.302	-0.572	-0.576
5	300	0.4	0.064	0.291	10.410	0.345	-0.287	-0.288
6	375	0.5	0.0003	0.293	14.850	0.360	0	0
7	450	0.6	-0.064	0.294	19.290	0.345	0.287	0.288
8	525	0.7	-0.128	0.294	23.370	0.302	0.572	0.576
9	600	0.8	-0.192	0.295	26.730	0.200	0.851	0.864
10	675	0.9	-0.258	0.297	29.010	0.129	1.121	1.152
11	750	1	-0.323	0.297	29.850	0	1.380	1.440

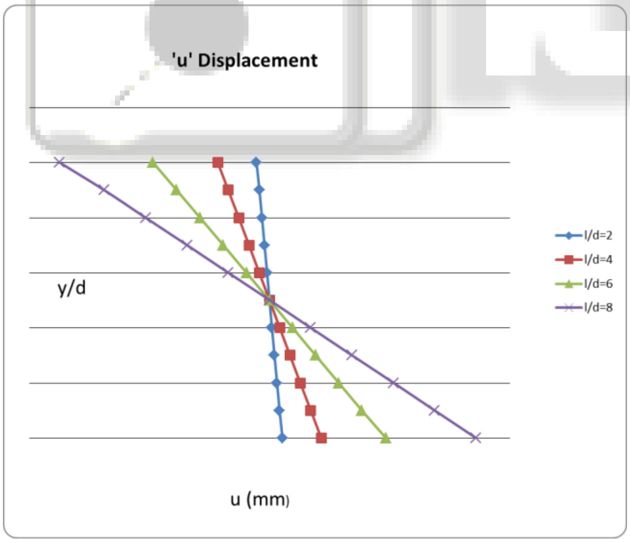
Table 3: for l=3000 mm and depth d=750 mm (l/d= 4)

S. NO.	Depth (mm)	y/d	U(mm)	V(mm)	Normal Stress N/mm^2	Shear Stress N/mm^2	Bending Stress (Timoshenko theory) (N/mm^2)	Bending Stress (Bending theory) (N/mm^2)
1	0.00	0.0	0.181	0.658	0.000	0.000	-3.252	-3.241
2	100	0.1	0.144	0.664	0.690	0.097	-2.583	-2.548
3	200	0.2	0.107	0.668	1.350	0.173	-1.930	-1.886
4	300	0.3	0.071	0.669	6.330	0.227	-1.281	-1.264
5	400	0.4	0.036	0.671	10.410	0.259	-0.670	-0.620
6	500	0.5	0	0.678	14.850	0.270	0.003	0
7	600	0.6	-0.036	0.679	19.290	0.259	0.632	0.620
8	700	0.7	-0.071	0.679	23.370	0.227	1.282	1.264
9	800	0.8	-0.107	0.68	26.730	0.170	1.926	1.886
10	900	0.9	-0.144	0.680	29.010	0.097	2.585	2.548
11	1000	1	-0.181	0.681	29.850	0	3.256	3.241

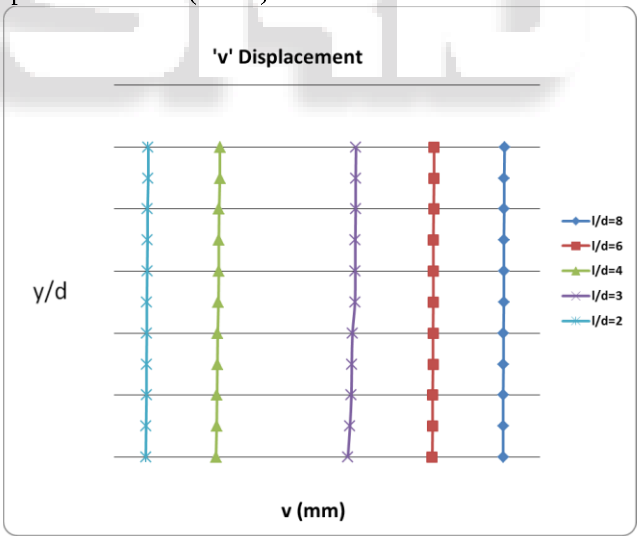
Table 4: for l=3000 mm and depth d=1000 mm (l/d= 3)

S. NO.	Depth (mm)	y/d	U(mm)	V(mm)	Normal Stress N/mm^2	Shear Stress N/mm^2	Bending Stress (Timoshenko theory) (N/mm^2)	Bending Stress (Bending theory) (N/mm^2)
1	0	0	0.081	0.088	0.000	0	-0.415	-0.360
2	150	0.1	0.063	0.089	0.022	0.065	-0.308	-0.288
3	300	0.2	0.047	0.09	2.970	0.115	-0.216	-0.216
4	450	0.3	0.031	0.09	6.330	0.152	-0.135	-0.144
5	600	0.4	0.015	0.091	10.410	0.173	-0.059	-0.072
6	750	0.5	0	0.091	14.850	0.180	0.015	0
7	900	0.6	-0.015	0.092	19.290	0.173	0.093	0.072
8	1050	0.7	-0.031	0.093	23.370	0.152	0.180	0.144
9	1200	0.8	-0.047	0.093	26.730	0.115	0.281	0.216
10	1350	0.9	-0.063	0.095	29.010	0.065	0.403	0.288
11	1500	1	-0.081	0.095	29.850	0	0.533	0.360

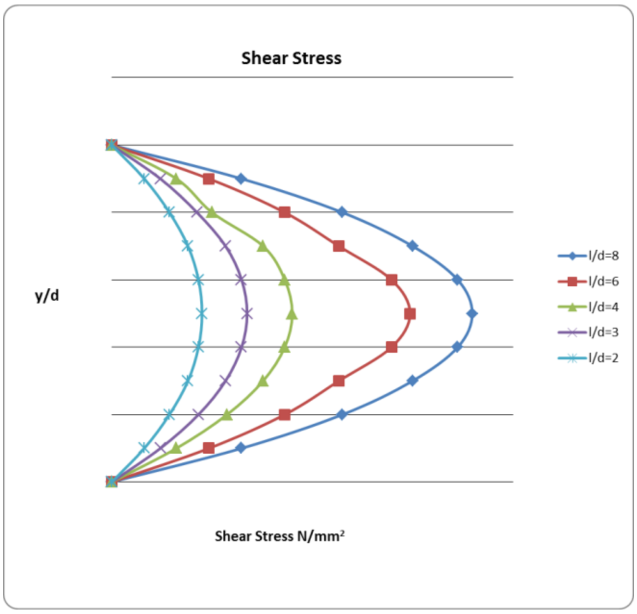
Table 5 : for l=3000 mm and depth d=1500 mm (l/d= 2)

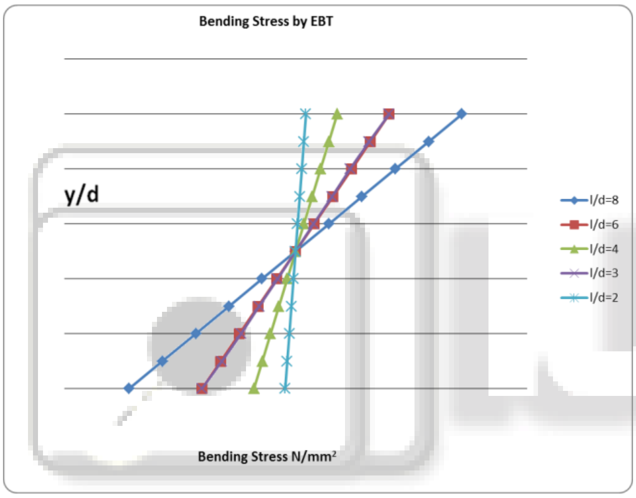


Variation of "Displacement u" for different span to span ratios. The displacement (u) is more at the top surface of the beam as compared to the bottom surface. Its value decreases with the increase in depth.

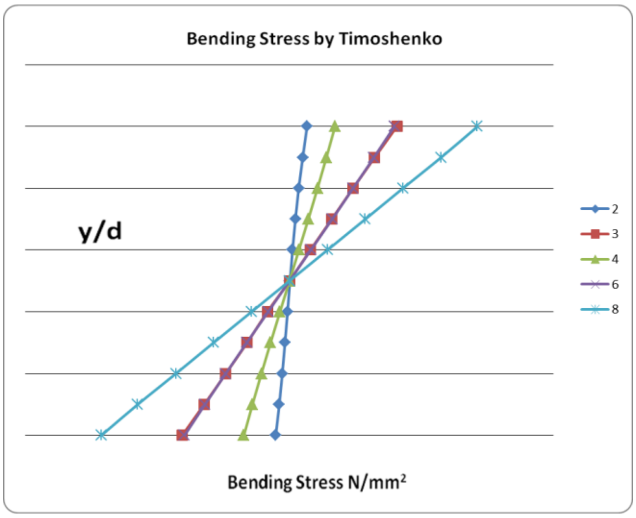


Variation of "Displacement v" for different span to depth ratios. The variation of displacement (v) is almost linear across the depth. It decreases with increase in depth of the beam section





Variation of "Bending stress σ x" by "Euler's Bending theory for different span depth ratios. It is observed that the distribution of bending stress across the depth of beam section is linear in all cases.



Variation of "Bending stress σ x" by "Timoshenko theory" for different span depth ratios. It is observed that the distribution of bending stress across the depth of beam section is linear in case of l/d = 8.0, 6.0 and 5.0, but stress distribution across the depth becomes nonlinear when l/d = 4.0, 3.0 and 2.0. The neutral layer of the beam is shifted from its original position and reaches the depth lower than the middle layer.

V. CONCLUSION

According to is code for deep beam the span to depth ratio for simply supported beams is equal to or less than 2 but a researcher observe that the behavior of deep beam is seen where the span to depth ratio of beam is 4

The variation of strain component is not linear when we decrease the span to depth ratio. I also observe that the neutal axis is shift towards the bottom (tensile phase) when i decrease span to depth ratio and it start when the span to depth ratio is 4

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