On Soft Slightly $\pi g * b *$ - Continuous Functions in Soft Topological Spaces

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Abstract— The aim of this paper is to introduce two new classes of functions, namely soft slightly $\pi g * b *$ -continuous functions and totally $\pi g * b *$ -continuous functions and study its properties.

Key words: soft set, soft topology, soft $\pi g * b *$ - closed set, soft $\pi g * b *$ -open set, soft slightly $\pi g * b *$ - continuous functions

I. Introduction

Molodstov [1] initiated the concept of soft set theory as a new mathematical tool and presented the fundamental results of the soft sets. Muhammed Shabir and Munazza naz[2] introduced soft topological spaces and the notions of soft open sets, soft closed sets, soft closure, soft interior points, soft neighborhood of a point and soft separation axioms. In 1997, Slightly continuity was introduced by Jain[3] and has been applied for semi-open and pre open sets by Nour[5] and Baker[4] respectively. We shall introduce a new functions and investigated their properties in terms of composition and restriction. Also we establish the relationship between soft slightly $\pi g * b *$ -continuous functions with other functions.

II. Preliminaries

Throughout this paper (X, τ, E), (Y, σ, E) and (Z, η ,E) (or X,Y and Z) represent non-empty soft stopological spaces on which no separation axioms are assumed unless explicitly stated. For a subset A of a space (X, τ), soft $\pi g * b * Cl$(A) and soft $\pi g * b * Int$(A) denote the soft $\pi g * b *$ -closure and the soft $\pi g * b *$ -interior of A respectively

A. Definition:2.1[1]

Let U be an initial universe set and E be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. Let P(U) denote the power set of U, and let A ⊆ E. Let $\tilde{X}$,be an initial universal set and E be the set of parameters. Let P($\tilde{X}$) denote the power set of $\tilde{X}$, and A $\tilde{\subseteq}$ E. The pair (F, A) is called a soft set over $\tilde{X}$, where F is a mapping given by F : A→ P($\tilde{X}$).

B. Definition:2.2 [6]

A soft set (F,E) over $\tilde{X}$ is said to be

1) A null soft set, denoted by φ , if ∀ e∈ E , F(e) = φ .
2) ii) An absolute soft set, denoted by $\tilde{X}$, if e ∈ E , F(e) = $\tilde{X}$.

C. Definition:2.3 [6]

Let τ be the collection of soft sets over $\tilde{X}$, then τ is said to be a soft

Topology on $\tilde{X}$ if

1) $\tilde{X}$ are belongs to τ .
2) The union of any number of soft sets in τ belongs to τ.
3) The intersection of any two soft sets in τ belongs to τ.

The triplet ($\tilde{X}$, τ, E) is called a soft topological space over $\tilde{X}$ and any member of τ is known as soft open set in $\tilde{X}$. The complement of a soft open set is called soft closed set over $\tilde{X}$.

D. Definition:2.4 [1]

A pair (F,A) is called a soft set over U, where F is a mapping given by F : A ⟶ P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U. For a particular e ∈ A. F(e) may be considered the set of e-approximate elements of the soft set (F,A).

E. Definition:2.5 [8]

A subset A of a topological space (X,τ,E) is called a soft $\pi g * b *$ -closed set if X\A is soft $\pi g * b *$ -open. The collection of all soft $\pi g * b *$ -closed sets in (X,τ,E) is denoted by soft $\pi g * b * Cl$(X,τ,E).

F. Definition:2.6 [8]

A topological space (X,τ,E) is said to be soft $\pi g * b *$ -$T_{1/2}$ space if every soft $\pi g * b *$ -open set of X is open in X.

G. Definition:2.7 [9]

A topological space (X,τ,E) is said to be soft $\pi g * b *$ -locally indiscrete space if every soft $\pi g * b *$ -open set of X is closed in X.

H. Definition:2.8

A function f : $(X, \tau, E) \to (Y, \sigma, E)$ is said to be soft contra $\pi g * b *$ -continuous if the inverse image of every soft open set in Y is soft $\pi g * b *$ -closed in X.

I. Definition:2.9 [7]

A function f: (X,τ,E) ⟶ (Y,σ,E) is called a soft contra-continuous if f^{-1}(U) is closed in (X,τ,E) for every soft open subset U of (Y,σ,E).

J. Definition:2.10 [8]

A mapping f : $(X,\tau,E) \to (Y,\sigma,E)$ is said to be soft $\pi g*b*$-continuous if the inverse image of every soft open set in Y is soft $\pi g*b*$-open in X.

K. Definition:2.11 [9]

A map f : $(X,\tau,E) \to (Y,\sigma,E)$ is said to be soft $\pi g*b*$-irresolute[7] if the inverse image of every soft $\pi g*b*$-open set in Y is soft $\pi g*b*$-open in X.

L. Definition:2.12 [9]

A mapping f : $(X,\tau,E) \to (Y,\sigma,E)$ is said to be soft slightly $\pi g*b*$-continuous if the inverse image of every soft $\pi g*b*$-open set in Y is soft open in X.

M. Definition:2.13 [3]

A function f : $(X,\tau,E) \to (Y,\sigma,E)$ is said to be soft slightly-continuous if the inverse image of every soft $\pi g*b*$-open set in Y is soft open and soft closed in X.

III. SOFT SLIGHTLY $\pi g*b*$ - CONTINUOUS FUNCTION

A. Definition: 3.1

A function f : $(X,\tau,E) \to (Y,\sigma,E)$ is said to be soft slightly $\pi g*b*$-continuous at a point $x \in X$ if for each subset V of Y containing f(x), there exists a soft $\pi g*b*$-open subset U in X containing x such that f(U)$\subseteq$ V. The function f said to be soft slightly $\pi g*b*$ continuous at each of its points.

B. Definition: 3.2

A function f : $(X,\tau,E) \to (Y,\sigma,E)$ is said to be soft slightly $\pi g*b*$-continuous if the inverse image of every soft clopen set in Y is soft $\pi g*b*$-open in X.

1) Proposition:

3.3 Let f : $(X,\tau,E) \to (Y,\sigma,E)$ be a function then the following are equivalent

2) Proof:

Suppose the definition 3.1 holds. Let V be a soft clopen set in Y and let x∈f^{-1}(V) . Then f(x)∈V and thus there exists a soft $\pi g*b*$-open set U_x such that x∈ U_x and f(U_x)⊆V. Now x∈U_x⊆f^{-1}(V). And $f^{-1}(V) = \bigcup_{x\in f^{-1}(V)} U_x$. Since arbitrary union of soft $\pi g*b*$-open sets is soft $\pi g*b*$-open, f^{-1}(V) is soft $\pi g*b*$-open in X and therefore f is soft slightly $\pi g*b*$-continuous. suppose the definition 3.2 holds. Let f(x) ∈V where V is a clopen set in Y. Since f is soft slightly $\pi g*b*$-continuous, x∈f^{-1}(V) where f^{-1}(V) is soft $\pi g*b*$-open in X. Let U=f^{-1}(V) . Then U is soft $\pi g*b*$-open in X, x$\in$ U and f(U)⊆V.

3) Theorem: 3.4

Let f : $(X,\tau,E) \to (Y,\sigma,E)$ be a function then the following are equivalent.

1) f is soft slightly $\pi g*b*$-continuous.
2) The inverse image of every soft clopen set V of Y is soft $\pi g*b*$-open in X.
3) The inverse image of every soft clopen set V of Y is soft $\pi g*b*$-closed in X.
4) The inverse image of every soft clopen set V of Y is soft $\pi g*b*$-clopen in X.

Proof: (1)⟹(2): By Theorem 3.4 the proof is obvious.
(2)⟹(3): Let V be a soft clopen set in Y which implies V^c is soft clopen in Y. By (2), $f^{-1}(V^c)$ = $(f^{-1}(V))^c$ is soft $\pi g*b*$-open in X. Therefore f^{-1}(V) is soft $\pi g*b*$-closed in X.
(3)⟹(4): By (2) and (3) f^{-1}(V) is soft $\pi g*b*$-clopen in X.
(4)⟹(1): Let V be a clopen subset of Y containing f(x) . By (4) f^{-1}(V) is soft $\pi g*b*$- clopen in X. Put U=f^{-1}(V) then f(U)⊆V. Hence f is soft slightly $\pi g*b*$-continuous.

C. Theorem: 3.5

Every soft slightly continuous function is soft slightly $\pi g*b*$-continuous.

Proof: Let f : $(X,\tau,E) \to (Y,\sigma,E)$ be soft slightly continuous. Let U be a soft clopen set in Y . Then f^{-1}(U) is soft open in X. since every soft open set is soft $\pi g*b*$-open, f^{-1}(U) is soft $\pi g*b*$-open. Hence f is soft slightly $\pi g*b*$-continuous.

1) Remark 3.6

The converse of the above theorem need not be true as can be seen from the following example

Example: 3.7 Let X ={a,b,c}=Y, E = {e_1,e_2}. Let F_1, F_2,…, F_7 are functions from E to P(X) and are defined as follows :

$F_1(e_1)$ = {a}, $F_1(e_2)$ = {a}, $F_2(e_1)$ = {b), $F_2(e_2)$ = {b}, $F_3(e_1)$ = {c}, $F_3(e_2)$ = { ϕ },
$F_4(e_1)$ = {a,b}, $F_4(e_2)$ = {a,b}, $F_5(e_1)$ = {a,c}, $F_5(e_2)$ = {a},
$F_6(e_1)$ = {b,c}, $F_6(e_2)$ = {b},
$F_7(e_1)$ = {X}, $F_7(e_2)$ ={a,b}.

Then τ = { ϕ , $\tilde{X}$, (F_1,E),(F_2,E),…,(F_7,E) is a soft topology and elements in τ are soft open sets. Let G_1,G_2 are functions from E to P(Y) and are defined as follows :
$G_1(e_1)$ = {a}, $G_1(e_2)$ = {a}, $G_2(e_1)$ = {b,c}, $G_2(e_2)$ = {b,c}
Then σ = { ϕ , $\tilde{Y}$,(G_1,E),(G_2,E) is a soft topology on Y.
Let f : (X,τ , A) → (Y,σ ,B) be a map by f(a)=b, f(b)=a, f(c)=c.

Here the inverse image f^{-1}{{a,c},{b}}={{b,c},{a}}of the soft $\pi g*b*$-open but not soft open in X. hence f is soft slightly $\pi g*b*$- continuous but not slightly continuous.

D. *Theorem: 3.8*

Every soft $\pi g*b*$-continuous function is soft slightly $\pi g*b*$-continuous.

1) Proof:

Let f : $(X,\tau,E)\to(Y,\sigma,E)$ be a $\pi g*b*$ -continuous function. Let U be a soft clopen set in Y .Then $f^{-1}(U)$ is soft $\pi g*b*$-open in X and soft $\pi g*b*$-closed in X. Hence f is soft slightly $\pi g*b*$-continuous.

E. *Theorem 3.12:*

If the function f: $(X,\tau,E)\to(Y,\sigma,E)$,σ is soft slightly $\pi g*b*$ -continuous and (Y,σ,E) is a soft locally indiscrete space then f is soft $\pi g*b*$-continuous.

1) Proof:

Let U be an open subset of Y. since Y is soft locally indiscrete, U is soft closed in Y. since f is soft slightly $\pi g*b*$-continuous, $f^{-1}(U)$ is soft $\pi g*b*$-open in X. Hence f is soft $\pi g*b*$-continuous.

F. *Theorem 3.14:*

Let f: $(X,\tau,E)\to(Y,\sigma,E)$ and g: (Y,σ,E) →(Z,η,E) be function

1) If f is soft $\pi g*b*$-irresolute and g is soft slightly $\pi g*b*$-continuous then $g\circ f$: (X,τ,E) →(Z,η,E) is soft slightly $\pi g*b*$-continuous.
2) If f is soft $\pi g*b*$-irresolute and g is soft $\pi g*b*$-continuous then $g\circ f$ is slightly soft $\pi g*b*$-continuous.
3) If f is soft $\pi g*b*$-irresolute and g is soft slightly continuous then $g\circ f$ is soft slightly $\pi g*b*$-continuous.
4) If f is soft $\pi g*b*$-continuous and g is soft slightly continuous then $g\circ f$ is soft slightly $\pi g*b*$-continuous.
5) If f is strongly soft $\pi g*b*$-continuous and g is soft slightly $\pi g*b*$-continuous then $g\circ f$ is soft slightly continuous.
6) If f is soft slightly $\pi g*b*$-continuous and g is soft perfectly $\pi g*b*$-continuous then $g\circ f$ is soft $\pi g*b*$-irresolute.
7) If f is soft slightly $\pi g*b*$-continuous and g is soft contra-continuous then $g\circ f$ is soft slightly $\pi g*b*$-continuous.
8) If f is soft $\pi g*b*$ -irresolute and g is soft contra $\pi g*b*$-continuous then $g\circ f$ is soft slightly $\pi g*b*$ -continuous.

G. *Proof:*

1) Let U be a soft clopen set in Z. since g is soft slightly $\pi g*b*$-continuous, g^{-1} (U) is soft -open in Y. since f is soft $\pi g*b*$-irresolute, $f^{-1}(g^{-1}(U))$ is soft $\pi g*b*$-open in X. since $(g\circ f)^{-1}(U)=f^{-1}(g^{-1}(U))$, $g\circ f$ is soft slightly $\pi g*b*$-continuous.
2) Let U be a soft clopen set in Z. since g is soft $\pi g*b*$-continuous, $g^{-1}(U)$ is soft $\pi g*b*$-open in Y. Also since f is soft $\pi g*b*$-irresolute, $f^{-1}(g^{-1}(U))$ is soft $\pi g*b*$-open in X. Hence $g\circ f$ is soft slightly $\pi g*b*$-continuous.
3) Let U be a soft clopen set in Z. Then $g^{-1}(U)$ is soft open in Y. Therefore $f^{-1}(g^{-1}(U))$ is soft $\pi g*b*$-open in X, since f is soft $\pi g*b*$-irresolute. Hence $g\circ f$ is soft slightly $\pi g*b*$-continuous.
4) Let U be a soft clopen set in Z. Then $g^{-1}(U)$ is soft open in Y, since g is slightly continuous. Also since f is soft $\pi g*b*$-continuous. Also since f is soft $\pi g*b*$-continuous, $f^{-1}(g^{-1}((U))$ is soft $\pi g*b*$-open in X. Hence $g\circ f$ is soft slightly $\pi g*b*$-continuous.
5) Let U be a soft clopen set in Z. Then $g^{-1}(U)$ is $\pi g*b*$-open in Y, since g is soft slightly $\pi g*b*$-continuous. Also $f^{-1}(g^{-1}(U))$ is open in X, since f is soft strongly $\pi g*b*$-continuous. Therefore $g\circ f$ is soft slightly continuous.
6) Let U be a soft $\pi g*b*$-open in Z. since g is perfectly soft $\pi g*b*$-continuous, g^{-1} (U) is soft open and soft closed in Y. Then $f^{-1}(g^{-1}(U))$ is soft $\pi g*b*$-open in X. since f is soft slightly $\pi g*b*$-continuous. Hence $g\circ f$ is soft $\pi g*b*$-irresolute.
7) Let U be a soft closed and soft open set in Z. since g is contra-continuous, $g^{-1}(U)$ is open and closed in Y. since f is soft slightly $\pi g*b*$-continuous, $f^{-1}(g^{-1}(U))$ is soft $\pi g*b*$-open in X. since $(g\circ f)^{-1}(U)=f^{-1}(g^{-1}(U))$, $g\circ f$ is soft slightly $\pi g*b*$-continuous.
8) Let U be a soft clopen set in Z. since g is soft contra $\pi g*b*$-continuous, $g^{-1}(U)$ is soft $\pi g*b*$-open and soft $\pi g*b*$-closed in Y. Therefore $f^{-1}(g^{-1}(U))$ is soft $\pi g*b*$-open and soft $\pi g*b*$-closed in X, since f is soft $\pi g*b*$-irresolute. Hence $g\circ f$ is soft slightly $\pi g*b*$-continuous.

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