

# Free Vibration Analysis of a Laminate Structure with Different Material Configuration FEM Analysis

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**Abstract**— This work is concerned with vibration (Modal) analysis of a Laminate structure with different material configuration. The finite Element method has been used computational by means of ANSYS 15.0 a main reason for adopting ANSYS 15.0 is that there is no analytical model has been develop for Composite structure in presence of singularities i.e. free i.e. natural vibration and harmonic response i.e. forced vibration. Following analysis have been carried out for SSSS (i.e. simply supported shear strength of laminated composite) boundary condition.

- Free vibration natural frequency
- Forced vibration harmonic response for different composite material
- And comparison of harmonic response of glass fiber, honeycomb, Carbone unidirectional. The obtained FEM results are validated with experimental and analytical result of available literature and it result shows good agreement.

**Key words:** Free & Forced Vibration, Natural Frequency, Shear Deformation and Stress, Laminated Composite Plate, Finite Element Method (FEM)

## I. INTRODUCTION

### A. What is a composite Material?

A broad definition of composite is: Two or more chemically distinct materials that when combined has improved properties over individual materials. The composites may be either natural or synthetic.

### B. Application

The glass-reinforced polyester has all the properties that make it ideal for this purpose and has become the standard material. Laminated plywood panels with thin layer of reinforced plastic are also widely used in truck trolleys. Various methods are used to produce them and the low cost, and the strength offered by the plywood that is attractive. They may not be as light as desired, but are found to be considerably more durable than conventional constructions, particularly under heavy duty conditions. There is also a growing market for products such as vans and trailers that are made from glass-reinforced plastic at competitive prices. Vehicles destined for consumer markets tend to be attractively designed, their appearance is different and the tools are easy. Mold reinforced plastic of the full size model of the piece are taken. The pattern can be made of wood, clay or plaster. The automotive sheet body is molded into molding molds paired with glass fiber or wick as reinforcements. Large and complex pieces can be produced and their reproduction can be made equally fast. Parts made using this technique generally have a uniform thickness, because the additional reinforcing fibers cannot be loaded easily or accurately. Composite molding materials are used when

small rigid parts have to be made and are usually prepared by casting. The bulk molding compound is composed of polyester resin, glass fibers and filler. The strength is less than that obtained by other molding processes. The molding compounds may be prepared as continuous sheets. Due to the longer fibers and a high glass content, its strength is better than those made in the bulk molding method. Injection molding are used to some extent to produce materials that can be applied to structural materials. After the separate manufacture of parts by molding or other methods, the assembly of the vehicle body has to be made. Tools and accessories similar to those used in metal parts are used for drilling and drilling operations. The assembly of several parts is usually accomplished by adhesion, using resins that are catalyzed to cure at room temperature in a short time. A good bonding strength is achieved even without pressure, although fastening devices are required to locate the assembled parts correctly in relation to each other. High-build primers are used to give a high-gloss finish so that surface imperfections do not become obvious. Commercial aircraft applications are the most important uses of the compounds. Aircraft, unlike other vehicles, need to place greater emphasis on safety and weight. They are achieved using materials with high specific properties. A modern civil aircraft must be designed to meet the numerous criteria of power and safety. Glass reinforced materials are the most desired materials as a result of advanced technology that has gone beyond the design and application. In cases where high moduli of elasticity values are less important, fiberglass is the natural option because of the low cost of material. The matrix material used with fiber glass, however, limits its use to low temperatures, usually below 121°C, although it is not a debilitating limitation for the fiber, as its properties can still be used and maintained at temperatures beyond 426 to 482°C. Fiber epoxy composites have been used in aircraft engine to enhance the performance of the system. The pilot's cabin door of aircrafts has also been made with fiber glass resin composites and these are now used in other transport systems. The boron-graphite materials were initially designed for fighter aircraft components and their use in commercial aircraft has been very less. There are a few instances of applications of these composites in wide use currently although experimental applications are several. They are presently limited to secondary structures which can be used in commercial aircraft with considerable safety. The data from such experimentation on the long term effects of loads and stress on the structure provides input for design. Both dynamic and static conditions are combined in the turbojet engine and research has always been directed towards this. These applications involve lightweight materials and this combination offers advantages. The weight of the rotors, compressors and bearings are reduced. Initially, turbojet engines were used in fighter aircraft and later in commercial planes. The need of a commercial plane is long

service life and durability. Some of the turbofan engines are designed to meet the manifold requirements of transport sector. The engine can be improved by improving the efficiency of propulsion or reducing the weight. The notable stiffness and strengths of composites permit reduction in the number of compressor stages by higher blade loading. The use of composites in rotors, compressors and engine parts are estimated to lead to weight savings. Aeronautical engineering comprises of various distinct areas that produces vehicles capable of performing distinct flight programmes. Initially importance was given to weight, speed and power, but other parameters that influence market acceptance of the aircraft should also be considered during design. These conditions call for selection of materials that give less than optimum efficiency in terms of structure and systems. Hence, it is important to consider performance needs as well as service properties. Airframe design starts with evaluation of flight conditions which the aircraft will encounter. In recent designs, wind tunnel tests and analysis are being done to determine the lift and drag forces. Once determined, they are used to develop various related factors of structural engineering. The selection of material, it follows, enters naturally into the picture at the early stage of design itself. The high strength of composites allows designing of higher aspect ratio wings in aero foil sections. Nowadays, composites are used in peripheral structures of aerodromes. Conventional constructions of composites ought to cost much less in future and will not be a constraint. Automation along with high standard for filament and matrix materials will also decrease fabrication costs, as the rejection on grounds of quality will be less.

## II. LITERATURE SURVEY

Kenneth et. al 1985 Present a finite element formulation for examining the large amplitude, steady-state, forced vibration response of arbitrarily laminated composite rectangular thin plates. [1] To analysis nonlinear forced vibration laminated composite rectangular plate, the nonlinear stiffness and harmonic force matrices has been developed and parametric variation in form of laminate angles and number of plies, in-plane boundary conditions has also been presented. The FE results are compared with available approximate continuum solutions.

Sivakumaran 1987 applied R-R energy approach for estimating natural frequencies of laminated rectangular plates having completely free edges. The obtained result is utilized in determination of location and extent of defects in laminated plates from measurements of natural frequencies of damaged plates. The plate under investigation satisfy Kirchhoff's hypothesis as they are thin enough to undergo small deflection. [2] However parametric analysis has also been carried out with varied aspect ratio of fiber orientation. The acquired result are compared with experimental and FEA.

Bicos and Springer 1989 derive an equation which illustrates the free damped vibrations of plates and shells which are furnished by laminated fiber-reinforced, organic-matrix composites. [4] A FEM has been implemented to solve the governing equation which is utilized to compute natural frequencies, mode shapes, and damping factors of rectangular plates, cylinders, and cylindrical panels with free, clamped, or simply supported edges, and with or without circular cutouts for both isotropic and composite plate.

Mottram demonstrates that high-order modelling predicts more precisely the effect of shear deformation than the FEM. [7] A specific high-order laminated plate with central patch load has been developed and found that by providing  $S$  is greater than 35, then the maximum contribution to deflections and stresses from shear deformation for multi-layered plates is 10% in comparison with CPT

Lee and Ng 1994 on the bases of Rayleigh-Ritz method free vibrations of symmetrically laminated rectangular plates with stiffeners is investigated. The effect of orientation and locations of stiffness and their bending and torsional stiffness has been analyzed in there corresponding natural frequency and mode shape. The material properties and stacking effect of the laminate are also examined.

In 1994 Mohamad adopted novel Ritz method which is coupled with non-orthogonal algebraic polynomials to calculate natural frequencies for cantilevered laminated composite angle-ply triangular and trapezoidal plates. Through detailed study has been carried out and found that the adopted method is more superior and accuracy of obtained results are in acceptable limit and shows good agreement with the available literature. [9]

Taylor and Nayfeh 1994 extend analyses and numerical calculations for free vibration of Simply Supported, thick, layered composite plate strips composed of off-axis lamina. It is presupposed that each layer of the composite plate has a arbitrary thickness and possesses up to monoclinic symmetry. [8] The solution of the characteristics for the total system advances by utilizing the matrix transformation scheme, which relies on gratifying appropriate interfacial conditions across the constituents, and results in a third-order eigen problem. The order of the free vibration eigen problem of the layered plate strip has been rendered independent of the number of layers in the composite plate. Numerical illustrations are specified in the form of normalized natural frequency vs non-dimensionalized mode number. The effect of the plate's microstructure on its vibration characteristics is investigated by examining alterations in natural frequencies for various laminate lay-ups and material combinations.

Gim 1997 developed a plate finite element on the bases on a lamination theory which includes the effect of transverse shear deformation. [10] By the means of a single layer of plate elements un-delaminated region is modeled while the delaminated region was modeled by two layers of plate elements whose boundary contains the de-lamination. A multipoint constraint algorithm has been developed in order to verify the compatibility of deformation and equilibrium of resultant forces and moments at the delamination crack tip. Moreover the strain energy release rate could be computed by the optimized crack closure technique using plate elements.

Zamanov 1999 calculate natural frequency of thin rectangular plate by applying periodically bent structures in a thin rectangular plate made of a composite material. The effect bending parameters are analyzed by using variation principle by FEM. [12]

Yucoğlu and ozerciyes 2000 conduct free bending vibrations of an orthotropic, composite base plate

or panel reinforced by a central stiffening plate strip. [13] The orthotropic base plate and the dissimilar, orthotropic, central stiffening plate strip are linked together with an exceptionally thin adhesive layer. The effects of the 'hard' and the 'soft' adhesive layers on the mode shapes and the natural frequencies of the composite system has been investigated. It has found that on Natural frequency increase at first for the increasing values of the 'bending cross stiffness ratio' of the base plate and the stiffening plate strip.

Zak et. al 2000 develops a theoretical models and carried out an extensive experimental investigation to establish changes in the first three bending natural frequencies due to de-laminations by the means of Vibration-Monitoring techniques as Non-Destructive methods for detection of De-Laminations in layered composite plate or Beam.[14]

Jaehong [15] presents free vibration analysis of a laminated beam with de-laminations using a layer wise theory Numerical results are compared with other theories addressing the effects of the lamination angle, location, size and number of de-lamination on vibration frequencies of delaminated beams. It has also revealed that a layer wise approach is adequate for vibration analysis of delaminated composites.

Singh and Yadav 2001 consider material properties in random variables for accurate prediction of the system behavior in accounts with rotatory inertia effect which correspond to higher order shear theory.[16] The effects of side to thickness ratio and variation in standard deviation of the material properties has been investigated for cross-ply symmetric and anti-symmetric laminates by employing first order perturbation technique. The higher order shear deformation theory answers has been validated with Monte Carlo simulation results and compared with the results based on classical laminate and first order shear deformation theories.

Lee and Han 2005 investigated Natural frequency for both isotropic and composite laminates. Natural strain method has been employed in order to membrane and shear locking phenomena. The developed laminated shell element can applicable for forced and free vibration analysis. The presented result show good precision with 3D elastic and analytical solution. Moreover the effect of damping has also been investigated for forced vibration analysis of laminated composite shells and plates.[20]

Chen et. al 2006 studied significance of nonlinear contact upon natural frequency of the stiffened composite plate with pre-damages using FEM using FSDT Rayleigh and energy method has been applied and explore the effect of delaminating and debonding in natural frequency of the plate.[21]

Rastgaar et. al 2006 exploits order shear deformation theory of plates (TSDT) for evaluating natural frequencies of square laminated composite plates for different supports at edges. [22] A novel set of linear equations of motion for square multi-layered composite plates has been derived. The FEM result of natural frequency for different combination of layers and supports are postulated and the compared result shows good agreement with equivalent single layer theories.

Ferreira develop an innovative numerical scheme, collocation by radial basis functions and pseudo-spectral methods are combined to produce highly accurate results for symmetric composite plates [23].

Yongshenga and Shuangshuang 2007 numerically investigate the Large Amplitude Flexural Vibration of Shape Memory Alloy Fibers embed in composite plate. The obtained result shows the significance of temperature on forced reaction behavior during phase alteration from Martensite to Austenite.[24]

Thai and Kim 2010 present two variable refined plate theory for Free vibration of laminated composite plates. [26] The theory comprises of r parabolic distribution of the transverse shear strains through the plate thickness, and satisfies the zero traction boundary conditions on the surfaces of the plate without using shear correction factors. Hamilton's principle is used to derive equation of motion and the closed-form solutions of antisymmetric cross-ply and angle-ply laminates are determined by using Navier technique. It can be revealed the proposed theory is not much accurate but the numerical results are compared with three-dimensional elasticity solutions which are computed by first-order and the other higher-order theories.

Mohamad 2011 present a remarkable work for determining accurate natural frequencies for simply supported shallow shells on rectangular plan form subjected to 55 possible combinations of edge constraints. Thin shallow shell theory has been used to calculate natural frequencies of doubly curved shallow shells. Furthermore Natural frequencies for various shell curvatures including spherical, cylindrical and hyperbolic paraboloidal shells are also presented

Ngo-cong et. al 2011 present a new radial basis function (RBF) collocation technique for analyzing free vibration analysis of laminated composite plates. [27] First order shear deformation theory has been used. A Cartesian grid is employed for discretizing rectangular or non-rectangular plates. Instead of using conventional differentiated RBF networks, one-dimensional integrated RBF networks (1D-IRBFN) are employed on grid lines to approximate the field variables. A number of paradigms concerning various thickness-to-span ratios, material properties and boundary conditions are considered. Results obtained are compared with the exact solutions and numerical results by other techniques in the literature to investigate the performance of the proposed method.

Dey and Karmakar 2012 applied FEM to analyze free vibration characteristics with the effect of rotational speeds on delaminated twisted graphite-epoxy cross-ply composite conical shells.[28] The Mindlin's theory consider for the theoretical formulation where eight nodal iso-parametric plate bending element is there. A generalized dynamic equilibrium equation has been derived from Lagrange's equation of motion ignoring the Coriolis Effect for moderate rotational speeds. The multi-point constraint algorithm was utilized to promise the compatibility of deformation and equilibrium of resultant forces and moments at the de-lamination crack front. The QR iteration algorithm has been adopted for solution of standard eigen value problem. Finite element codes are extended to attain the numerical results concerning the combined effects of twist angle and rotational speed on the

natural frequencies of cross-ply composite shallow conical shells. The mode shapes for a classic laminate configuration are also depicted. Numerical results obtained for cross-ply laminates with de-lamination are the first known non-dimensional frequencies for the type of analyses carried out here.

Wen et. al 2013 consider in-plane free vibration of a rectangular plate with material properties varying periodically and rapidly in the plate plane. [29] state-space method has been used to analyzed homogenized model of the original heterogeneous plate which is established by the two-scale asymptotic expansion method. In their result they concluded that the plate can be designed to have particular frequencies through a proper selection of the material distribution law in each unit.

Zeki et. al 2013 experimentally and numerically examine the effect of interfacial root crack on the lateral buckling and free vibration responses of a sandwich composite beam[30]. Lateral buckling loads and natural frequencies in a thin sandwich composite cantilever beam with root crack are resolved. The crack with various lengths is opened between the face sheet and foam core, such as 50, 100, 150 and 200 mm. Lateral buckling and free vibration tests of these models are carried out. For the numerical analysis, ANSYS finite element software is used. Results obtained by numerical analyses and experiments are compared and it is seen that there is a good agreement between them.

Süleyman et. al 2014 investigates the nonlinear dynamic response of a hybrid laminated composite plate composed of composite material such as basalt, Kevlar/epoxy and Eglass/ epoxy under the blast load with damping effects. [32] von Kármán type of geometric nonlinearities are considered and parameters such as damping ratios, aspect ratios and different peak pressure values are analyzed

Bowyer and Kyrov experimentally investigate the indentation of power law profile for honeycomb sandwich panels. [33]It has found that for higher order acoustic black holes for flexural waves that can absorb a large fraction of the incident wave energy.

Sudip et. al 2015 presents a generic random sampling-high dimensional model representations (RS-HDMR) approach for free vibration analysis of angle-ply composite plates.[35] Meta-model has been develop for estimating stochastic natural frequencies. The parametric analysis in form of Elastic modulus, Mass density and Fibre-Orientation angle has been carried out to validated the proposed meta-model. On the basis of statistical analysis stochastic mode shapes are depicted for a typical laminate configuration for studying dynamic behavior of a system.

Sepahvand et. al 2015 examines the impact of elastic parameter uncertainty on the natural frequency and the radiated acoustic power of laminated composite plates. [36]By using generalized polynomial chaos expansions with arbitrary random basis a model is develop for laminate to elastic parameters, natural frequencies and acoustic power density. It has been concluded that the natural frequencies and as well as the radiated acoustic powers are strongly affected from uncertainty in input parameters of Composite laminate.

Xin and Hu 2015 applied new develop hybrid approach for analyzing free vibration of simply supported and multilayered magneto-electro-elastic plates. [37]The Approach include state space approach (SSA) and the discrete singular convolution (DSC) algorithm which is based on three-dimensional elasticity theory. The thickness direction of plate is chosen as the transfer direction in SSA, and the DSC is concerned to discretize the in-plane domains. Hence, the original partial differential equations are decoded into a state equation consisting of first-order ordinary differential equations. The relevance of DSC makes it possible to treat various boundary conditions, and shows admirable performance for high frequency vibration. The accuracy and convergence of SS-DSC is validated through numerical examples.

### III. METHODOLOGY

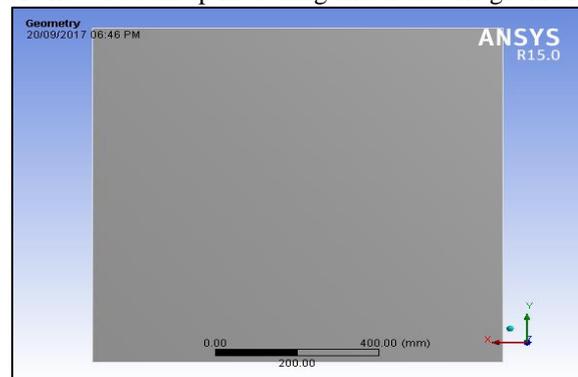
The equation of motion of Composite laminated plate with and without cutouts is solved using FEM tool (ANSYS) as the equation of motion for a laminate with cutouts is difficult to visualize therefore some FEM tool is the only solution method for analyzing vibration characteristics of laminate with cutouts. The ANSYS 15 finite element program was used for free vibration of the orthotropic (Composite) Laminated plate. For this purpose, the key points were first created and then line segments were formed. Numbers of layers are defined along with the Orientation and thickness. The lines were combined to create an area. Finally, this area was extruded a We modeled the laminated plate. A 20-node three-dimensional structural solid element was selected to model the beam. The Laminated plate was discretized into 10201 elements with 10000 nodes. Plate boundary conditions can also be modeled by constraining all degrees of freedoms of the nodes located on the left end of the beam. The subspace mode extraction method was used to calculate the natural frequencies of the Composite laminated plate. The subspace mode extraction method was used to calculate the natural frequencies of the blade. Following steps show the guidelines for carrying out Modal analysis.

#### A. Define Materials

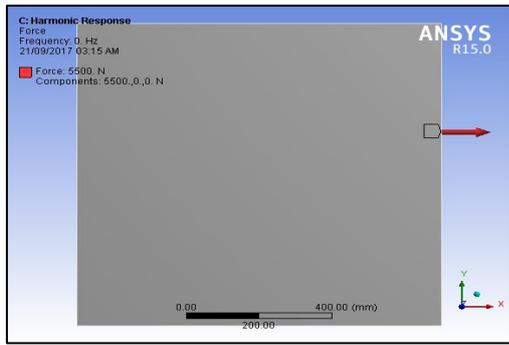
- Set preferences. (Structural)
- Define constant material properties. Properties of material are tabulated in table 1.

#### B. Model the Geometry

- Follow bottom up modelling and create the geometry



(a) SSSS boundary condition for free vibration



(b) SSSS boundary condition for force vibration

Fig. 1: (a) & (b) Modeled Geometry

- Define element type. i.e. shell 3D
- Mesh. i.e. Mapped mesh

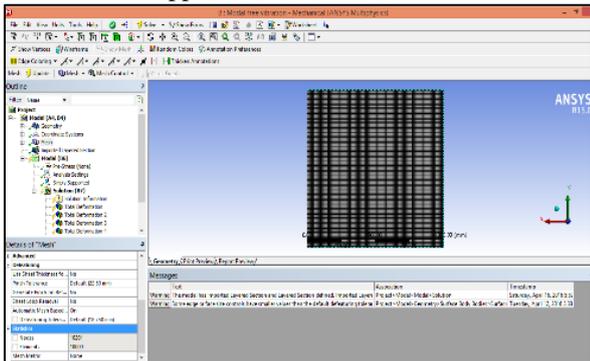


Fig. 2: Mesh model

C. Apply Boundary Conditions

- Apply constraints to the model. i.e. SSSS
- Convergence

D. Obtain Solution

- Specify analysis types and options.
- Solve.

Material Properties Ref. [22]	
Orthotropic (Composite)	Value
Density, $\rho$	2200 kg/m <sup>3</sup>
Young's Modulus	
$E_x$	280GPa
$E_y$	7GPa
$E_z$	7GPa
Poisson's Ratio, $\nu_x=\nu_y=\nu_z$	
Shear Stress, $G_x$	4.2GPa
Shear Stress, $G_y$	3.5GPa
Shear Stress, $G_z$	4.2GPa

Table 1: Material Properties for glass fiber

IV. RESULTS AND DISCUSSIONS

A. Validation

The governing equations of the problem were solved, numerically, using a Element method, and finite element Analysis (FEA) used in order to calculate the Vibration characteristics of a Composite Laminated plate. As a result of a grid independence study, a grid size of 100x100 was found to model accurately the Natural Frequency of a Composite Laminated plate described in the corresponding results.

The accuracy of the computer model was verified by comparing results from the present study with those obtained

by Rastgaar [22], Ngo-Cong [27], Lam [5], Tkant [17] Experimental, Analytical and FEA results.

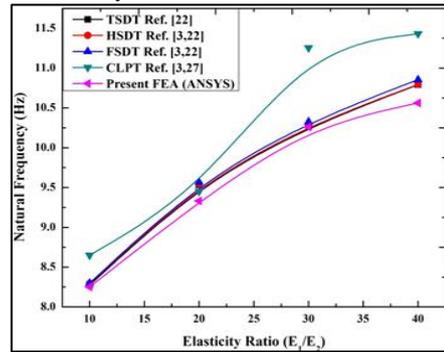


Fig. 3: Validation of Non Dimensional Natural Frequency with Varying Elastic Ratio

$E_1/E_2$	TSDT Ref. [22]	HSDT Ref. [3,22]	FSDT Ref. [3,22]	My Work Result
10	8.2741	8.294	8.2982	8.2512
20	9.5312	9.5439	9.5671	9.3309
30	10.2651	10.284	10.326	10.2591
40	10.7912	10.794	10.854	10.5619

Table 2: Validation of Non dimensional Natural Frequency with Varying Elastic Ratio

In table 2 and figure 3 shows the validation of FEM result obtained from the ANSYS tool. It has been seen that the obtained result of Laminated composite plate with different boundary condition shows good agreement with the analytical, Experimental and FEM of available literature.

The small variation in results is due to variation in grid sizing, operating condition, material properties, etc. but the obtained result shows the same trend so that the results are suitably verified.

B. Mode shape in SSSS boundary conditions of free vibration

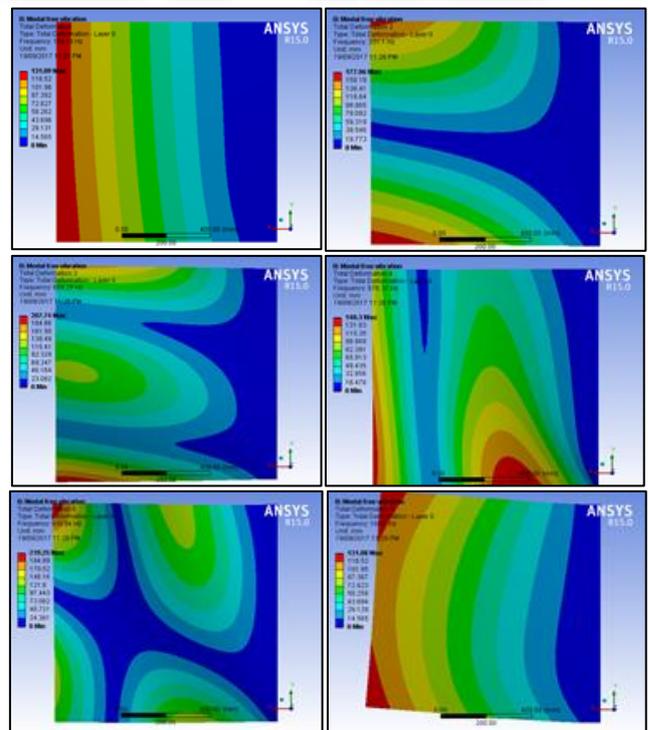


Fig. 4: Mode shapes in SSSS boundary condition of free vibration for glass fiber

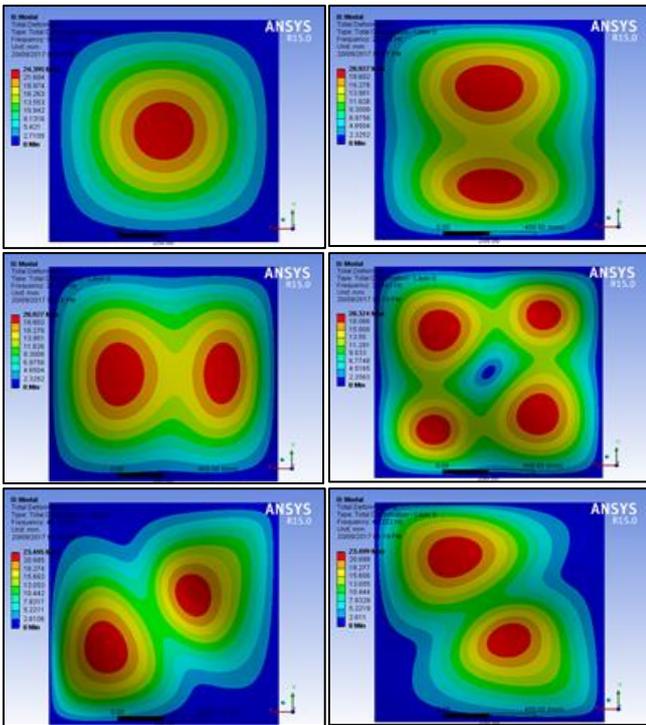


Fig. 5: Mode shapes in SSSS boundary condition of free vibration for honeycomb

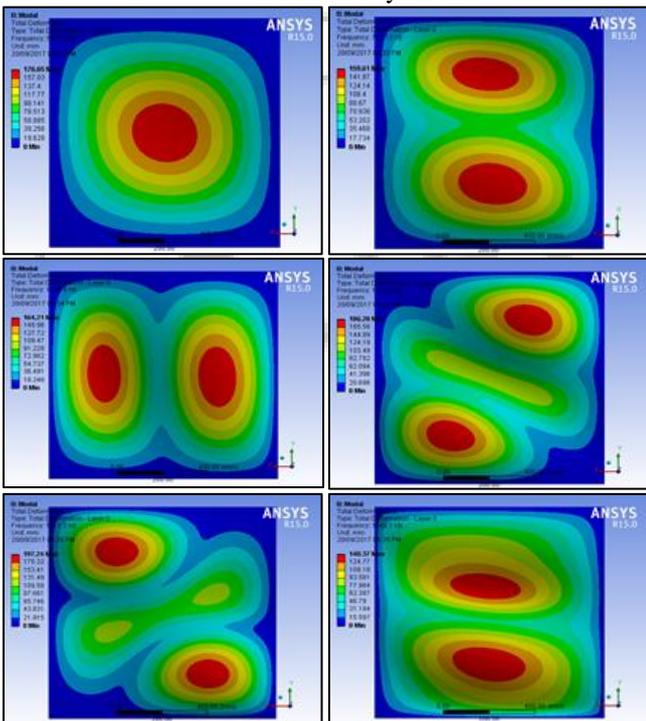


Fig. 6: Mode shapes in SSSS boundary condition of free vibration for carbon-ud

Mode	Glass fibre	Honey comb	Carbon-ud
	Frequency[Hz]	Frequency[Hz]	Frequency[Hz]
1.	378.57	14.918	547.74
2.	622.87	25.409	773.13
3.	808.68	25.409	1108.4
4.	1004.3	39.443	1274.6
5.	1132.7	49.216	1411.2
6.	1396.9	49.223	1466.1

Table 3: Natural frequency of free vibration for laminated plate with different composite material

From figure 4, 5 & figure 6 and in table 3 illustrates the natural frequencies of free vibration for different composite material like glass fiber, honeycomb and carbon-ud (carbon unidirectional) are to be shows that the maximum and minimum amount of total deformation. Here are the 6 modes of fracture are to be showed for their maximum frequency level. If the material meets the respective highest frequencies of these 6 modes the material will fail at that point. By comparing these three materials, carbon-ud will shows that the maximum value of natural frequency for all 6 modes.

### V. CONCLUSION

- The natural frequencies of free and force vibration for different composite material like glass fiber, honeycomb and carbon-ud (carbon unidirectional) are to be shows that the 6 modes of fracture are to be showed for their maximum frequency level. If the material meets the respective highest frequencies of these 6 modes the material will fail at that point. By comparing these three materials, carbon-ud will shows that the maximum value of natural frequency for all 6 modes.
- For laminated composite plate the natural frequency is minimum at center of the plate in simply supported condition.

In the view of parametric Vibration analysis of Laminated Composite plate structure (following conclusions have been drawn as below.

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