Supra Soft - GPR Separation Axioms in Soft Supra Topological Spaces

S. Vinnarasi[1] K. Mohana[2]
[1]Research Scholar [2]Assistant Professor
[1,2]Department of Mathematics
[1,2]Nirmala College for Women, Coimbatore - 641018, Tamil Nadu, India

Abstract— The aim of this paper is to study supra soft GPR-separation axioms and obtain some of their characterizations.
Key words: Supra Soft gpr-Ti, (i = 0, 1, 2, 3, 4) Spaces, Supra Soft gpr-Regular Spaces, Supra Soft GPR-Normal Spaces

I. INTRODUCTION

The concept of soft sets was first introduced by Molodtsov[10] in 1999 as a general mathematical tool for dealing with uncertain objects. In [9,10], Molodtsov successfully applied the soft theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, theory of measurement, and so on. After presentation of the operations of soft sets [11], the properties and applications of soft set theory have been studied increasingly [9, 7, 12]. To develop soft set theory, the operations of the soft sets are redefined and a uni-int decision making method was constructed by using these new operations. Recently, in 2011, Shabir and Naz [13] initiated the study of soft topological spaces. They defined basic notions of soft topological spaces such as open soft and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces, soft normal spaces and established their several properties. Min in [10] investigates some properties of these soft separation axioms mentioned in [11]. Banu and Halis in [4] studied some properties of soft Hausdorff space. Shabir et. al [7] and D. N. Georgiou et. al [6], defined and studied some soft separation axioms, soft θ-continuity and soft connectedness in soft spaces using (ordinary) points of a topological space X. The notion of supra soft topological spaces was initiated for the first time by El-sheikh and Abd El-latif [5]. Recently, Kandil et al. [8] introduced the concept of soft supra g-closed soft sets in supra soft topological spaces. A. M. Abd El-latif [2] investigate strongly generalized closed set in supra soft topological space.

The main purpose of this paper is to introduce, the notion of supra soft gpr separation axioms. Also, we study the properties of the supra soft gpr - regular spaces, supra soft gpr-normal spaces and supra soft gpr - irresolute open functions.

II. PRELIMINARIES

Let X be an initial universe set and E be a collection of all possible parameters with respect to X, where parameters are the characteristics or properties of objects in X. Let P (X) denote the power set of X, and let A $\subseteq$ E.

A. *Definition [2.1][4]*

A pair (F, A) is called a soft set over X, where F is a mapping given by F: A $\rightarrow$ P (X). In other words, a soft set over X is a parameterized family of subsets of the universe X. For a particular e $\in$ A. F (e) may be considered the set of e-approximate elements of the soft set (F, A).

B. *Definition [2.2][4]*

Two soft sets (F, A) and (G, B) over a common universe X is said to be soft equal if (F, A) is a soft subset (G, B) and (G, B) is a soft subset of (F, A).

C. *Definition [2.3][10]*

For two soft sets (F, A) and (G, B) over a common universe X, union of two soft sets of (F, A) and (G, B) is the soft set (H, C), where C = A $\cup$ B, and $\forall$e $\in$ C,

$$H(e) = \begin{cases} F(e) & , if\ e \in A - B \\ G(e) & , if\ e \in B - A \\ F(e) \cup G(e) & , if\ e \in A \cap B \end{cases}$$

We write (F, A) $\cup$ (G, B) = (H, C)

D. *Definition [2.4][13]*

Let μ be the collection of soft sets over X, and then μ is called a soft topology on X if μ satisfies the following axioms:

- φ , X belong to μ.
- The union of any number of soft sets in μ belongs to μ.
- The intersection of any two soft sets in μ belongs to μ.

The triplet (X, μ, E) is called a soft topological space over μ. Let (X, μ, E) be a soft space over X, then the members of μ are said to be soft open sets in X.

E. *Definition [2.5][13]*

The intersection (H, C) of two soft sets (F, A) and (G, B) over a common universe X denoted (F, A) $\cap$ (G,B) is defined as C = A $\cap$ B and H(e) = F(e) $\cap$ G(e) for all e$\in$ C.

F. *Definition [2.6][13]*

Let μ be the collection of supra soft sets over a universe X with a fixed set of parameters E, then μ is said to be a supra soft topology on X if

- $\varphi, X \in$ μ
- The union of any number of supra soft sets in μ belongs to μ.

The triplet (X, μ, E) is called a supra soft topological space.

Every member of μ is called a supra soft open set. A supra soft set (F, E) is called supra soft closed in X if (F, E) $^c \in \mu$.

G. *Definition [2.7][15]*

Let (X, μ, E) be a supra soft topological space and (F, E) be a supra soft set over X.

- The supra soft closure of (F, E) is the supra soft set (F, E)$^{-s}$ = $\cap$ {(G, E): (G, E) is supra soft closed and (F, E) $\subseteq$ (G, E).
- The supra soft interior of (F, E) is the supra soft set (F, E)^{OS} = $\cup$ {(H, E): (H, E) is supra soft open and (H, E) $\subseteq$ (F, E).

Clearly, $(F, E)^{-s}$ is the smallest supra soft closed set over X which contains (F, E) and $(F, E)^{OS}$ is the largest supra soft open set over X which is contained in (F, E).

H. Definition [2.8] [15]

Let (X, μ, E) be a supra soft topological space. A supra soft set (F, E) is called supra soft pre-open set in X if $(F, E) \subseteq ((F, E)^{-s})^{OS}$. The relative complement of a supra soft pre-open set is called a supra soft pre -closed set.

I. Definition [2.9][15]

Let (X, μ, E) be a supra soft topological space over X. A supra soft set (F, E) is called a supra soft generalized pre-regular closed set (supra soft gpr-closed) in X if $(F, E)^{-sp} \subseteq (G, E)$ whenever $(F, E) \subseteq (G, E)$ and (G, E) is supra soft –regular-open in X.

J. Remark [2.10] [2]

Let S(X) and S(Y) be families of soft sets. For the soft function f_{pu}: S(X) → S(Y), the soft function f_{pu} is called surjective if p and u are surjective, also it is said to be injective if p and u are injective. Then for each F, F_1, $F_2 \in S(X)$ and for each G, G_1, $G_2 \in S(Y)$ the following statements hold,

- $f_{pu}^{-1}(G^c) = (f_{pu}^{-1}G)^c$.
- $f_{pu}(f_{pu}^{-1}(G)) \subseteq G$. If f_{pu} is surjective, then the equality holds.
- $F \subseteq f_{pu}^{-1}(f_{pu}(F))$. If f_{pu} is injective, then the equality holds.
- $f_{pu}(X) \subseteq Y$. If f_{pu} is surjective, then the equality holds.
- $f_{pu}^{-1}(Y) = X$ and $f_{pu}(\varphi) = \varphi$.
- If $F_1 \subseteq F_2$, then $f_{pu}(F_1) \subseteq f_{pu}(F_2)$.
- If $G_1 \subseteq G_2$, then $f_{pu}^{-1}(G_1) \subseteq f_{pu}^{-1}(G_2)$.
- $f_{pu}^{-1}(G_1 \cup G_2) = f_{pu}^{-1}(G_1) \cup f_{pu}^{-1}(G_2)$ and $f_{pu}^{-1}(G_1 \cap G_2) = f_{pu}^{-1}(G_1) \cap f_{pu}^{-1}(G_2)$.
- $f_{pu}^{-1}(F_1 \cup F_2) = f_{pu}^{-1}(F_1) \cup f_{pu}^{-1}(F_2)$ and $f_{pu}^{-1}(F_1 \cup F_2) = f_{pu}^{-1}(F_1) \cup f_{pu}^{-1}(F_2)$.
  If f_{pu} is injective, then the equality holds.

III. SUPRA SOFT GPR- SEPARATION AXIOMS

In this section, we introduce and investigate some weak supra soft separation axioms by using the notion of supra soft gpr-open sets, which is a generalization of the supra soft separation axioms.

A. Definition 3.1

Let (X, μ, E) be a supra soft topological space and μ be an associated supra soft topology with X. Let x,y ∈ X such that x ≠ y. Then, (X, μ, E) is called

- Supra soft gpr-T_0-space, if there exists a supra soft gpr -open set (F, E) containing one of the points x, y but not the other.
- Supra soft gpr-T_1-space, if there exist supra soft gpr -open sets (F,E) and (G,E) such that x ∈ (F, E), y ∉ (F,E) and y ∈ (G,E) , x∉ (G,E).
- Supra soft gpr-Hausdorff space (Supra soft- gpr-T_2) if there exist a supra soft gpr -open sets (F,E) and (G,E) such that x ∈ (F,E) y ∈ (G,E) and (F,E) ∩ (G,E) = Ø.

B. Proposition 3.2

Let (X, μ, E) be a supra soft topological space and x, y ∈ X such that x ≠ y. If there exist supra soft gpr -open sets (F,E) and (G,E) such that either x∈ (F,E) and y∈ $(F,E)^c$ or y∈ (G,E) and x ∈ $(G,E)^c$. Then, (X, μ, E) is supra soft- gpr-T_0-space.

1) Proof

Let x, y ∈ X such that *x* ≠ *y*. Let (F,E*)* and (*G*,E) be supra soft gpr- open sets , such that either *x* ∈ (*F*,*E*) and y ∈ $(F,E)^c$ or y ∈ (G,E) and x ∈ $(G,E)^c$. If x∈ (F,E) and y ∈ $(F,E)^c$, then, y ∈ $(F(e))^c$, for each e ∈ E. This implies that, y ∉ F(e) for each e ∈ E. Therefore, y ∉ (F,E). Similarly, if y ∈ (G,E) and x ∈ $(G,E)^c$, then x∉ (G,E). Hence, (X, μ ,*E*) is supra soft gpr-T_0-space.

C. Proposition 3.3

Let (X, μ, E) be a supra soft topological space and x, y ∈ X such that x≠ y. If there exist supra soft gpr-open sets (F,E) and (G,E) such that x ∈ (F, E) and y ∈ $(F,E)^c$ and y ∈ (G, E) and x ∈ $(G,E)^c$. Then (*X*, μ, *E*) is supra soft gpr -T_1-space.

1) Proof

It is similar to the proof of Proposition 3.2.

D. Proposition 3.4

- Every supra soft gpr-T_1-space is a supra soft gpr- T_0-space.
- Every supra soft gpr-T_2-space is a supra soft gpr -T_1-space.

1) Proof

- Let (Y, μ, E) is a supra soft gpr- T_1-space, and x, y ∈ X, such that x ≠ y. So there exist supra soft- open sets (F,E) and (G,E) such that x∈ (F,E) and y ∉ (F,E) and y∈ (G,E) and x∉ (G,E). Obviously, then we have x∈ (G,E) and y ∉ (G,E) or y∈ (F,E) and x∉ (F,E). Hence (X, μ, E) is supra soft gpr- T_0 - space.
- If (Y, μ, E) is a supra soft gpr-T_2-space, then for x, y ∈ X, x ≠ y, there exist supra soft- open sets (F, E) and (G, E) such that x∈ (F, E), y∈ (G, E) and (F, E)∩ (G, E) = Ø. Since (F,E)∩ (G,E) = Ø, x∉ G, y ∈ F. Thus it follows that, (X, μ, E) is a supra soft gpr -T_1-space.

The converse of the above theorem is not true.

E. Example 3.5

Let X={a, b}, E={e_1,e_2}, and μ ={Ø, X, (F_1,E), (F_2,E), (F_3, E)}, where (F_1,E), (F_2,E), (F_3,E) are supra soft sets over X defined as follows:

$F_1(e_1)$ ={a}, $F_1(e_2)$={X}, $F_2(e_1)$ ={a}, $F_2(e_2)$={b}, $F_3(e_1)$ ={X}, $F_3(e_2)$={b}

- Let X = {a, b}, E = {e_1 ,e_2} and μ = {X,Ø,(F_1,E)}, where (F_1, E) is supra soft set over X defined as follows by $F_1(e_1)$ ={a}, $F_1(e_2)$={X}. Then, μ defines a soft supra topology on X. Also (X, μ, E) is supra soft gpr- T_0-space. But not a supra soft gpr-T_1-space, since a, b∈ X, $a \neq b$, but all the open supra soft sets which contain a, also contain b.
- Let X = {a, b}, E = {e_1, e_2} and μ = {Ø, X, (F_1,E)...... (F_3, E)} where(F_1,E), (F_2,E), (F_3,E) are supra soft sets over X defined as follows:

$F_1(e_1)$ ={a}, $F_1(e_2)$={X},$F_2(e_1)$ ={a} ,$F_2(e_2)$={b},$F_3(e_1)$ ={x}, $F_3(e_2)$={b}.

Then, μ defines a soft supra topology on X. Also, (X, μ, E) is supra soft gpr-T_1-space, but it is not a supra soft gpr-T_2-space, for a, b ∈ X and, $a \neq b$ but there is no supra soft gpr-open sets F and G such that a ∈ (F,E), b∈ (G,E) and $(F, E) \cap (G, E) = \phi$.

F. *Theorem 3.6*

Let (X, µ, E) be a supra soft topological space. If x_E is supra soft gpr - closed set in µ, for each x ∈ X, then (X, µ, E) is supra soft gpr -T_1-space.

1) Proof

Let x ∈ X and x_E be supra soft gpr-closed set in µ, then, x_E^c is supra soft gpr- open set in µ. Let x, y ∈ X such that x ≠ y. For x ∈ X and x_E^c is supra soft gpr -open set, such that x ∉ x_E^c and y ∈ x_E^c. Similarly y_E^c is supra soft gpr- open set in µ such that y ∉ y_E^c and x ∈ y_E^c. Thus, (X, µ, E) is supra soft gpr -T_1-space over X.

G. *Theorem 3.7*

If every point x of X is a supra soft gpr-closed, then X is a supra soft gpr -T_1-space.

1) Proof

Let x and y be two distinct points of X and by hypothesis x and y are supra soft gpr -closed. Then x^c and y^c are supra soft gpr-open set, such that y ∈ x^c, y ∉ x and x ∈ y^c, x ∉ y. Hence the supra soft topological space X is a supra soft gpr -T_1-space.

H. *Theorem 3.8*

Let (X, µ, E) be a supra soft topological space over X. If every supra soft point of a soft supra topological space (X, µ, E) is a supra soft gpr-closed set, then (X, µ, E) is a supra soft gpr-T_0-space.

1) Proof

Let x ∈ X be a supra soft point which is a supra soft gpr-closed set then x^c ∈ X is a supra soft gpr-open set. Then for distinct supra soft points x, y we have x^c, y^c are supra soft gpr-open sets such that y ∈ x^c and y ∉ x, x ∈ y^c and x ∉ y.

I. *Theorem 3.9*

Let (X, µ, E) be a supra soft topological space over X and Y ⊆ X. If (X µ, E) is a supra soft gpr -T_2-space, then (Y, µ, E) is a supra soft gpr-T_2-space.

1) Proof

Let x, y ∈ Y clearly x ≠ y. Then x, y ∈ X. Since (X, µ, E) is a supra soft gpr -T_2-space, therefore there exist, supra soft gpr-open sets (F,E), (G,E) such that x ∈ (F,E), y ∈ (G,E) and (F,E) ∩ (G,E) = Ø. Thus x ∈ Y ∩ (F,E)= (F_Y ,E), y ∈ X ∩ (G,E)= (G_Y, E) and (F_Y ,E) ∩ (G_Y, E) = Ø. This proves that (Y, µ, E) is a supra soft gpr-T_2-space.

J. *Theorem 3.10*

Let (X, µ, E) be a supra soft topological space over X. If (X, µ, E) is a supra soft gpr -T_2-space and for any x, y ∈ X such that x ≠ y, then there exist supra soft gpr -closed sets (F,E) and (G,E) such that x ∈ (F,E), y ∉ (F,E), and x ∉ (G,E) , y ∈ (G,E) and (F,E) ∪ (G,E) = X.

1) Proof

Since (X, µ, E) is a supra soft gpr-T_2-space and x, y ∈ X such that x ≠ y, there exist supra soft gpr-open sets (F_1,E) and (G_1,E) such that x ∈ (F_1,E), y ∈ (G_1,E) and (F_1, E) ∩ (G_1,E) = Ø. Clearly (F_1,E) ⊆ $(G_1,E)^c$ and (G_1,E) ⊆ $(F1,E)^c$. Hence x ∈ $(G_1,E)^c$. Put $(G_1,E)^c$ = (F,E). This gives x ∈ (F,E) and y ∉ (F,E). Also y ∈ $(F_1,E)^c$. Put $(F_1,E)^c$ = (G,E). Therefore x ∉ (G,E) and y ∈ (G,E). Moreover (F,E) ∪ (G,E) = $(G_1,E)^c$ ∪ $(F1,E)^c$ = X.

K. *Definition 3.11*

Let (X, µ, E) be a supra soft topological space over X, (G,E) be a supra soft closed set in (X, µ, E) and x ∈ X such that x ∉ (G,E). If there exist supra soft open sets (F_1,E)and (F_2,E) such that x ∈ (F_1,E), (G,E) ⊆ (F_2,E) and (F_1,E) ∩ (F_2,E) = Ø, then (X, µ,E) is called a supra soft gpr- regular space.

L. *Theorem 3.12*

Let (X, µ, E) be a supra soft gpr- regular space over X. Then every supra soft subspace of (X,µ,E) is supra soft gpr- regular.

1) Proof

Let (Y, µ, E) be a supra soft subspace of a supra soft -regular space (X, µ, E). Suppose (F,E) is a supra soft closed set in (Y, µ, E) and x ∈ Y such that x ∉ (F,E). Then (F,E) = (G,E) ∩ (Y,E) where (G,E) is a supra soft closed set in (X, µ, E) . Then x ∈ (G,E). Since (X, µ, E) is supra soft gpr- regular, there exist supra soft disjoint open sets F_{1X}, and F_{2X} in (X, µ, E) such that x ∈ (F_1,E), (G,E) ⊆ (F_2,E). Clearly x ∈ (F_1,E) ∩ Y = F_Y and F=(F_2 ,E) ∩ Y = F_Y such that F_Y ∩ F_Y = Ø. This proves that (Y, µ, E) is a supra soft gpr regular subspace of (X, µ, E).

M. *Definition 3.13*

Let (X, µ, E) be a supra soft topological space over X. Then (X, µ, E) is said to be a supra soft gpr -T_3-space, if it is a supra soft gpr- regular and a supra soft gpr -T_1-space.

N. *Remark 3.14*

- A supra soft gpr -T_3-space may not be a supra soft gpr -T_2-space.
- If (X, µ, E) is a supra soft gpr-T_3-space, then(X,$µ_e$) may not be a supra soft gpr-T_3-space, for each parameter e ∈ E.

O. *Definition 3.15*

Let (X, µ, E) be a supra soft topological space. (F,E) and (G, E) be supra soft gpr- closed sets in X. such that (F,E) ∩ (G,E) = Ø. If there exist supra soft gpr- open sets (F_1,E) and (F_2, E) such that (F.E) ⊆ (F_1,E), (G,E) ⊆ (F_2,E) and $(F_1,E)\cap(F_2,E)=\varphi$, then (X, µ, E) is called a supra soft- gpr -normal space.

P. *Definition 3.16*

Let (X, µ, E) be a supra soft topological space over X. Then (X, µ, E) is said to be a supra soft gpr -T_4-space, if it is a supra soft gpr- normal and a supra soft gpr- T_1-space.

Q. *Theorem 3.17*

A supra soft gpr- closed subspace (X, µ,E) of a supra soft gpr-normal space (Y, µ, E) is supra soft-gpr- normal.

1) Proof

Let (G_1, E) and (G_2,E) be supra soft gpr - closed sets in X such that (G_1, E)$\cap(G_2,E)=\varphi$. Then (G_1,E) =Y$\cap(F_1,E)$ and $(G_2,E)=Y\cap(F_1,E)$ for some supra soft gpr- closed sets (F_1, E) and (F_2,E) in X. Since Y is a supra soft gpr- closed subset of X. Then (G_1,E) and (G_2,E) are supra soft gpr- closed sets in X such that (G_1,E)$\cap(G_2,E)$ $=\varphi$. Hence by supra soft gpr normality, there exist supra soft gpr- open sets (H_1, E) and (H_2, E) such that $(G_1,E)\subseteq(H_1,E)$, $(G_2,E)\subseteq(H_2,E)$ and $(H_1,E)\cap(H_2,E)=\varphi$. Since ($G_1$,E),($G_2$,E) ⊆ Y then $(G_1,E)\subseteq Y\cap(H_1,E)$, and $(G_2,E)\subseteq Y\cap(H_2,E)$ $[Y\cap(H_1,E)]\cap[Y\cap(H_2,E)]=\phi$ where $Y\cap(H_1,E)$

and $Y \cap (H_2, E)$ are supra soft gpr- open sets in X. Therefore, (X, μ, E) is supra soft gpr- normal space.

IV. SUPRA SOFT GPR -IRRESOLUTE FUNCTIONS

A. Definition 4.1

Let (X, τ_1, E) and (Y, τ_2, K) be soft supra topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2 respectively. The supra soft function $f_{pu}: S_E(X) \to S_K(Y)$ is called

- Supra soft gpr -open set, if the image of each supra soft gpr-open set in X is supra soft gpr- open in Y.
- Supra soft gpr-irresolute, if the inverse image of supra soft gpr-open set in X is supra soft gpr- open in Y.
- Supra soft gpr -irresolute open, if the image of each supra soft gpr -open set in X is supra soft gpr- closed in Y.

B. Theorem 4.2

Let (X, τ_1, E) and (Y, τ_2, K) be soft supra topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2 respectively and $f_{pu}: S_E(X) \to S_K(Y)$ be a supra soft function which is bijective and supra soft gpr -irresolute open set. If (X, τ_1, E) is supra soft gpr -T_0- space, then (Y, τ_2, K) is also a supra soft gpr -T_0- space.

1) Proof

Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then there exist $x_1, x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist supra soft gpr - open sets (F,E) and (G,E) in X such that, either $x_1 \in (F,E)$ and $x_2 \notin (F,E)$, or $x_2 \in (G,E)$ and $x_1 \notin (G,E)$. So, either $x_1 \in F_E(e)$ and $x_2 \notin F_E(e)$, or $x_2 \in G_E(e)$, and $x_1 \notin G_E(e)$, for each e ∈ E. This implies that, either $y_1 = u(x_1) \in u[F_E(e)]$ and $y_2 = u(x_2) \notin u[F_E(e)]$ or $y_2 = u(x_2) \in u[G_E(e)]$ and $y_1 = u(x_1) \notin u[G_E(e)]$ for each e ∈ E. Hence, either $y_1 \in f_{pu}(F,E)$ and $y_2 \notin f_{pu}(F,E)$ or $y_2 \in f_{pu}(G,E)$ and $y_1 \notin f_{pu}(G,E)$. Since f_{pu} is supra soft gpr -irresolute open function, then f_{pu} (F,E) and f_{pu} (G,E) are supra soft gpr -open sets in Y. Hence, (Y, τ_2, K) is also a supra soft gpr -T_0-space.

C. Theorem 4.3

Let (X, τ_1, E) and (Y, τ_2, K) be soft supra topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2 respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a supra soft function which is bijective and supra soft gpr -irresolute open set. If (X, τ_1, E) is supra soft gpr -T_1-space, then (Y, τ_2, K) is also a supra soft gpr -T_1-space.

1) Proof

It is similar to the proof of theorem 4.2.

D. Theorem 4.4

Let (X, τ_1, E) and (Y, τ_2, K) be soft supra topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2 respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a supra soft function which is bijective and supra soft gpr -irresolute open set. If (X, τ_1, E) is supra soft gpr -T_2-space, then (Y, τ_2, K) is also a supra soft gpr -T_2-space.

1) Proof

Let $y_1, y_2 \in Y$ such that $y_1 \neq y_2$. Since f_{pu} is surjective, then there exist $x_1, x_2 \in X$ such that $u(x_1) = y_1$, $u(x_2) = y_2$ and $x_1 \neq x_2$. By hypothesis, there exist supra soft gpr - open sets (F,E) and (G,E) in X such that, either $x_1 \in (F,E), x_2 \in (G,E)$ and $(F,E) \cap (G,E) = \varphi_E$.

So $x_1 \in F_E(e)$, $x_2 \in G_E(e)$ and $F_E(e) \cap G_E(e) = \varphi$ for each $e \in E$. This implies that, $y_1 = u(x_1) \in u[F_E(e)]$, $y_2 = u(x_2) \in u[G_E(e)]$ for each $e \in E$. Hence $y_1 \in f_{pu}(F,E)$, $y_2 \in f_{pu}(G,E)$ and $f_{pu}(F,E) \cap f_{pu}(G,E) = f_{pu}[(F,E) \cap (G,E)] = f_{pu}[\varphi_E] = \varphi_K$ from theorem 2.12. Since f_{pu} is supra soft gpr - irresolute open function, then f_{pu} (F, E) and f_{pu} (G, E) are supra soft gpr -open sets in Y. Hence, (Y, τ_2, K) is also a supra soft gpr -T_2-space.

E. Theorem 4.5

Let (X, τ_1, E) and (Y, τ_2, K) be soft supra topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2 respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a supra soft function which is bijective and supra soft gpr -irresolute open set. If (X, τ_1, E) is supra soft gpr -regular-space, then (Y, τ_2, K) is also a supra soft gpr -regular-space.

1) Proof

Let (G, K) be a supra soft gpr -closed set in Y and y ∈ Y such that $y \notin (G,E)$. Since f_{pu} is surjective and supra soft gpr-irresolute, then $\exists x \in X$ such that $u(x) = y$ and $f_{pu}^{-1}(G,E)$ is supra soft gpr -closed set in X such that $x \notin f_{pu}^{-1}(G,E)$. By hypothesis, there exist supra soft open-sets (F, E) and (H, E) in X such that $x \in (F,E)$ $f_{pu}^{-1}(G,E) \subseteq (H,E)$ and $(F,E) \cap (H,E) = \varphi_E$. It follows that, $x \in F_E(e)$ for each e∈ E and $(G,K) = f_{pu}[f_{pu}^{-1}(G,K)] \subseteq f_{pu}(H,E)$. from Theorem 2.12.

So, $y = u(x) \in u[F_E(e)]$ for each e∈ E and $(G,K) \subseteq f_{pu}(H,E)$. Hence, $y \in f_{pu}(F,E)$ and $G \subseteq f_{pu}(H,E)$ and $f_{pu}(F,E) \cap f_{pu}(H,E) = f_{pu}[(F,E) \cap (H,E)] = f_{pu}[\varphi_E] = \varphi_K$ from Theorem 2.12. Since f_{pu} is supra soft gpr -irresolute open function. Then, $f_{pu}(F,E), f_{pu}(H,E)$ are supra soft gpr –open-sets in Y. Thus, (Y, τ_2, K) is also a supra soft gpr -regular space.

F. Theorem 4.6

Let (X, τ_1, E) and (Y, τ_2, K) be soft supra topological spaces, μ_1 and μ_2 be associated supra soft topologies with τ_1 and τ_2 respectively. Let $f_{pu}: S_E(X) \to S_K(Y)$ be a supra soft function which is bijective and supra soft gpr -irresolute open set. If (X, τ_1, E) is supra soft gpr -normal-space, then (Y, τ_2, K) is also a supra soft gpr -normal-space.

1) Proof

Let (F,E), (G,E) be supra soft gpr -closed sets in Y such that $(F,E) \cap (G,E) = \varphi$. Since f_{pu} is supra soft gpr -irresolute , then $f_{pu}^{-1}(F,E)$ and $f_{pu}^{-1}(G,E)$ are supra soft gpr -closed set in X such that $f_{pu}^{-1}(F,E) \cap f_{pu}^{-1}(G,E) = f_{pu}^{-1}[(F,E) \cap (G,E)] = f_{pu}^{-1}[\varphi_K] = \varphi_E$ from Theorem 2.12. By hypothesis, there exist supra soft gpr-open soft sets (K,E) and (H,E) in X such that

$f_{pu}^{-1}(F,E) \subseteq K, f_{pu}^{-1}(G,E) \subseteq (H,E)$ and $(F,E) \cap (H,E) = \varphi$. It follows that, $(F,E) = f_{pu}[f_{pu}^{-1}(F,E)] \subseteq f_{pu}(K,E)$, $(G,E) = f_{pu}[f_{pu}^{-1}(G,E)] \subseteq f_{pu}(H,E)$ from Theorem 2.12 and $f_{pu}(K,E) \cap f_{pu}(H,E) = f_{pu}[(K,E) \cap (H,E)] = f_{pu}[\varphi_E] = \varphi_k$ from Theorem 2.12. Since f_{pu} is supra soft gpr -irresolute open function. Then, $f_{pu}(K,E), f_{pu}(H,E)$ are supra soft gpr -open sets in Y. Thus, (Y, τ_2, K) is also a supra soft gpr -normal space.

Reference

[1] M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, "On some new operations in soft set theory", Computers and Mathematics with Applications 57(2009) 1547-1553.
[2] A. M. Abd El-latif, "Soft supra strongly generalized closed sets", Intelligent and Fuzzy System, 31 (3) (2016), 1311-1317.
[3] A. M. Abd El-latif and Serkan Karatas, " Supra open soft sets and associated soft separation axioms", Submitted for publication.
[4] Banu Pazar Varol and Halis Aygun, "On soft Hausdorff spaces", Annals of Fuzzy Mathematics and Informatics 5 (2013) 15-24.
[5] S. A. El-Sheikh and A. M. Abd El-latif, "Decompositions of some types of supra soft sets and soft continuity", International Journal of Mathematics Trends and Technology, 9(1)(2014) 37–56.
[6] D. N. Georgiou, A. C. Megaritis, V. I. Petropoulos: "On Soft Topological Spaces", Applied Mathematics and Information Sciences 7(2) (2013) 1889-1901.
[7] D. V. Kovkov, V. M. Kolbanov, D. A. Molodtsov, "Soft sets theory-based optimization", Journal of Computer and Systems Sciences International 46 (6) (2007)872-880.
[8] A. Kandil, O. A. E. Tantawy, S. A. El-Sheikh and A. M. Abd El-latif, "Supra generalized closed soft sets with respect to an soft ideal in supra soft topological spaces", Applied Mathematics and Information Sciences, 8(4)(2014) 1731–1740.
[9] D.Molodtsov,V. Y. Leonov, D. V. Kovkov, "Soft sets technique and its application", Nechetkie Sistemy i Myagkie Vychisleniya 1(1)(2006)8-39.
[10] D. A. Molodtsov, " Soft set theory firste results", Computers and Mathematics with Applications 37(1999)19-31.
[11] P. K. Maji,R. Biswas, A. R. Roy, "Soft set theory", Computers and Mathematics with Applications 45 (2003)555-562.
[12] W. K. Min, "A note on soft topological spaces", Computers and Mathematics with Applications 62 (2011) 3524-3528.
[13] D. Pei, D. Miao, "From soft sets to information systems", in: X. Hu, Q. Liu, A. Skowron, T.Y. Lin, R. R. Yager, B. Zhang (Eds.), Proceedings of Granular Computing, in: IEEE, vol.2, 2005, pp. 617-621.
[14] M. Shabir and M. Naz, "On soft topological spaces", Comput. Math. Appl. 61 (2011) 1786-1799.
[15] Vinnarasi S, Mohana K, "Supra soft generalized pre-regular closed and open sets in soft supra topological spaces", International Journal of Applied Research, 2017; 3(7): 1011-1017.

