

# Optimal Capacitor Placement and Sizing using Backtracking Search Algorithm

S.S.Chauhan<sup>1</sup> Neetu Agrawal<sup>2</sup>  
<sup>1,2</sup>VITM Gwalior

**Abstract**— This paper presents solutions for complex power system optimization problems using a recent swarm intelligence motivated algorithm namely, Backtracking Search Algorithm (BSA) algorithm. The BSA is applied to solve the complex power system problems. Here, BSA applied to solve capacitor placement and sizing problem. The problem is considered to reduce losses in transmission and distribution part of the power. The comparison of results with other significant methods for above problems shows that BSA may play the vital role in solving power system optimization problems.

**Key words:** Power System Optimization, Backtracking Search Algorithm, Capacitor Placement and Sizing Problem

## I. INTRODUCTION

The optimal capacitor placement supplies the part of reactive power demand which helps in reducing the energy losses, peak demand losses and improves the voltage profile, power factor (pf) and system stability. Therefore, specific size capacitors are required to be placed at specific places in the distribution network to achieve the optimum reactive power. To achieve this objective while maintaining the optimal economy, optimal placement of capacitor with proper sizing is known as optimal capacitor placement and sizing problem. This problem has been solved by conventional [19] or non-conventional strategies [23, 17]. This problem has also been solved by applying fuzzy approximate reasoning [24], genetic algorithm (GA) [25], hybrid differential evolution (DE) algorithm [2], artificial bee colony (ABC) algorithm [21], and particle swarm optimization (PSO) algorithm [3] etc

Global Optimization which can be defined as the process of searching for the global optimum in an optimization problem is a hotspot in applied mathematics. However, the classical methods are difficult to find global optimum for the problems as the dimension size of search space increased.

Optimization algorithms aim to find the best values for a system's parameter under various conditions. The first step in solving an optimization problem is determining the objective function may have a non-linear, complex or non-differentiable form. Optimization problems are usually designed in a way that defines the global optimum of an objective function as the global minimum. The process of searching for the global optimum for an optimization problem is called global optimization. Desirable features for optimization algorithms include the ability to reach a problem's global minimum value quickly with a small number of control parameters and low computational cost, as well as robustness and ease of application to different problem models. When the objective function for an optimization problem is non-linear and non-differentiable, evolutionary algorithm (EA) techniques are typical used to find the global optimum.

The most commonly used EA optimization techniques are based on swarm intelligence and evolution Theory. Swarm-intelligence optimization algorithms generally use simplified mathematical models of complex social behaviors of living creatures. However, recent swarm-intelligence based optimization algorithms have also been inspired by natural events involving gravitational physics, behavior of birds and various branches of art such as music.

Unlike classical optimization techniques, EAs do not guarantee finding the optimum parameter values for a problem. However, also unlike classical optimization algorithms, EAs are sufficiently flexible to solve different types of problems. EAs must have global exploration and local exploitation abilities. A global exploration ability means that the optimization algorithm effectively uses the entire search, while a local exploitation ability means that the optimization algorithm searches for the best solution near solution it has already discovered.

EAs are population-based stochastic search mechanism that search for near-optimal solutions to a problem. An EA tries to evolve an individual into one with better fitness value through a 'trial individual'. To generate a trial individual, the EA chooses existing individuals as raw genetic material and combines these using various genetic operators. If the trial individual has a better fitness value than the original individual, the trial individual replaces it in the next-generation populations. EAs radically differ from one another based on their strategies for generating trial individuals. Because these strategies have a considerable effect on their problem-solving success and speed, ongoing efforts are aimed at developing EAs with faster and more successful problem-solving processes.

BSA is a new EA, BSA's unique mechanism for generating a trial individual enables it to solve numerical optimization problems successfully and rapidly. BSA uses three basic genetic operators- selection, mutation and crossover- to generate trial individual. BSA has a random mutation strategy that uses only one direction individual for each target individual in contrast with many genetic algorithms such as DE and its derivative jDE, JADE and SaDE. BSA randomly chooses the direction individual from individuals of a randomly chosen previous generation. BSA uses a non-uniform crossover strategy that is more complex than the crossover strategies used in many genetic algorithms.

The detailed description of this section may be categorized as follows: Section 3.2 describes capacitor sizing and optimal allocation problem. BSA for optimal placement and sizing of capacitor is explained in Section-3.3. Finally, in Section 3.4 conclusion of the work is presented.

## II. CAPACITOR SIZING AND OPTIMAL PLACEMENT PROBLEM

The placement of capacitors in the distribution network is mainly needed, for improving power transfer capability, for

properly serving to reactive loads, for the smooth working of power transformers, and for the secure as-well-as stable transmission system in different network configurations. Further, these capacitors improve voltage profile and maintain contractual obligations for electrical equipment. The capacitors also help in reducing the energy consumption of voltage dependent sources as well as technical losses [18]. The capacitors have been widely installed by utilities, to provide reactive power compensation, to enhance the efficiency of the power distribution, and to achieve deferral of construction [37]. Economically, we can say that the capacitors installation in distribution network help in increasing, generation capacity, transmission capacity, and distribution substation capacity. Subsequently, it helps in increasing revenue generation. But the placement of capacitors exactly at required optimal position in a distribution system is a challenging task or can say a difficult problem for the distribution engineers [38]. The objective of this problem is to minimize the energy losses while considering the capacitor installation costs [39]. In other words, the goal is to achieve the optimal placement and sizing of capacitors with the system constraints in the distribution network. The problem is defined as follows [20]:--

The total loss in a distribution system having n number of branches is given by equation 1.

$$PL_t = \sum_{i=1}^n [I_i^2] R_i \quad (1)$$

Here,  $I_i$  and  $R_i$  are current magnitude and resistances respectively for the  $i^{th}$  branch. The branch current obtained from load flow solution has two components; active ( $I_a$ ) and reactive ( $I_r$ ). In active and reactive branch currents, the associated losses are given by equations 3.2 and 3.3 respectively.

$$PL_a = \sum_{i=1}^n [I_{ai}^2] R_i \quad (2)$$

$$PL_r = \sum_{i=1}^n [I_{ri}^2] R_i \quad (3)$$

In loss minimization technique of the capacitor placement, a single capacitor is repetitively placed by varying its size for determining a sequence of nodes in view of loss minimization of the distribution system. The concept of loss minimization by a singly located capacitor can be extended for multiple capacitors [19].

Let us consider the following [22]:

$m$  = number of capacitor buses.

$I_c$  =  $m$  dimensional vector consisting of capacitor currents.

$\alpha_j$  = set of branches from the source bus to the  $j^{th}$  capacitor bus ( $j = 1, 2, \dots, m$ ).

$D$  = a matrix of dimension  $n \times m$ .

The elements of  $D$  are considered as

$D_{ij} = 1$ ; if branch  $i \in \alpha$

$D_{ij} = 0$ ; otherwise

When the capacitors are placed in the system, the new reactive component of branch currents is given by equation 4.

$$[I_r^{new}] = [I_r] + [D][I_c] \quad (4)$$

The loss associated with the new reactive currents in the compensated system is shown as equation 3.5.

$$P_{Lr}^{com} = \sum_{i=0}^n (I_{ri} + D_{ij} I_{cj})^2 R_i \quad (5)$$

The loss saving ( $S$ ) is obtained by placing the optimal size capacitors in the distribution network. The loss saving is calculated by taking the difference of the equations 3 and 5 and is shown as follows:

$$S = -\sum_{i=1}^n [(2I_{ri} \sum_{j=1}^m D_{ij} I_{cj} + (\sum_{j=1}^m D_{ij} I_{cj})^2] R_i \quad (6)$$

For achieving the maximum loss saving, optimal capacitor currents can be obtained from the equation 7.

$$\frac{\delta S}{\delta I_{c1}} = 0, \frac{\delta S}{\delta I_{c2}} = 0, \dots, \frac{\delta S}{\delta I_{ck}} \quad (7)$$

After some mathematical manipulations, equation 3.7 can be expressed by a set of linear algebraic equations as shown in equation 8.

$$[A][I_c] = [B] \quad (8)$$

Where,  $A$  is a  $m \times m$  square matrix and  $B$  is a  $k$ -dimensional vector. The elements of  $A$  and  $B$  are given by equation 9, 10, and 11.

$$A_{jj} = \sum_{i \in \alpha_j} [R_i] \quad (9)$$

$$A_{jj} = \sum_{i \in (\alpha_j \text{ or } \alpha_m)} [R_i] \quad (10)$$

$$B_j = \sum_{i \in \alpha_j} [I_{ri} R_i] \quad (11)$$

Only the branch resistances and reactive currents in the original system are required to find the elements of  $A$  and  $B$ . The capacitor currents for the highest loss saving can be obtained from equation 3.8.

$$[I_c] = [A]^{-1} [B] \quad (12)$$

Once the capacitor currents are known, the optimal capacitor sizes can be written as  $Q_c$  in MVAR as equation 13

Here,  $V_m$  is the voltage magnitude vector of capacitor buses. The saving in the compensated system can be estimated from equation 6 using the value of  $I_c$  given by equation 12.

The objective function may be formulated using equation 6 as shown in equation 14.

$$\min_f(x_{location}, x_{size}) = S \quad (14)$$

### III. BACKTRACKING SEARCH ALGORITHM

BSA is a population-based iterative EA designed to be a global minimizer. BSA can be explained by dividing its functions into five processes as is done in other EAs: initialization, selection-I, mutation, crossover and selection-II.

A. Algorithm 1 presents BSA's general structure.

#### Algorithm 1. General Structure of BSA

```

1. Initialization
repeat
  2. Selection-I
  Generation of Trial-Population
  3. Mutation
  4. Crossover
end
5. Selection-II
until stopping conditions are met;
    
```

Fig. 1: General Structure of BSA

B. Initialization:

BSA initializes the population  $P$  with Eq. (15):

$$P_{i,j} \sim U(low_j, up_j) \quad (15)$$

for  $i = 1, 2, 3, \dots, N$  and  $j = 1, 2, 3, \dots, D$ , where  $N$  and  $D$  are the population size and the problem dimension, respectively,  $U$  is the uniform distribution and each  $P_i$  is a target individual in the population  $P$ .

### Selection-I

BSA's Selection-I stage determines the historical population oldP to be used for calculating the search direction. The initial historical population is determined using Eq. (16):

$$oldP_{i,j} \sim U(low_j, up_j) \quad 16$$

BSA has the option of redefining oldP at the beginning of each iteration through the 'if-then' rule in Eq. (17):

$$\text{If } a < b \text{ then oldP} := P|a, b \sim U(0,1) \quad 17$$

where := is the update operation. Eq. (17) ensures that BSA designates a population belonging to a randomly selected previous generation as the historical population and remembers this historical population until it is changed. Thus, BSA has a memory.

After oldP is determined, Eq. (18) is used to randomly change the order of the individuals in oldP:

$$oldP := \text{permuting}(oldP) \quad (18)$$

The permuting function used in Eq. (18) is a random shuffling function.

### Mutation:

BSA's mutation process generates the initial form of the trial population Mutant using Eq. (19).

$$\text{Mutant} = P + F \cdot (\text{oldP} - P) \quad (19)$$

In Eq. (19), F controls the amplitude of the search-direction matrix (oldP-P). Because the historical population is used in the calculation of the search-direction matrix, BSA generates a trial population, taking partial advantage of its experiences from previous generations. This paper uses the value  $F = 3 \cdot \text{rndn}$ , where rndn is the standard normal distribution.

### C. Crossover

BSA's crossover process generates the final form of the trial population T. The initial value of the trial population is Mutant, as set in the mutation process. Trial individuals with better fitness values for the optimization problem are used to evolve the target population individuals. BSA's crossover process has two steps. The first step calculates a binary integer-valued matrix (map) of size N.D that indicates the individuals of T to be manipulated by using the relevant individuals of P. If  $\text{map}_{nm}=1$ , where T is updated with  $T_{nm} := P_{nm}$ .

Algorithm 2 shows BSA's unique crossover strategy.

#### Algorithm 2. Crossover Strategy of BSA

```

Input: Mutant, mixrate, N and D.
Output: T: Trial-Population.
0 map(1:N, 1:D) = 1 // Initial-map is an N-by-D matrix of ones.
1 if a < b | a, b ~ U(0, 1) then
2   for i from 1 to N do
3     | mapi, u(1: [mixrate · rnd · D]) = 0 | u = permuting({1, 2, 3, ..., D})
4   end
5 else
6   for i from 1 to N do, mapi, randi(D) = 0, end
7 end
8 T := Mutant // Initial T
9 for i from 1 to N do
10  for j from 1 to D do
11  | if mapi, j = 1 then Ti, j := Pi, j
12  end
13 end

```

Fig. 2.2: Crossover Strategy of BSA

Two predefined strategies are randomly used to define BSA's map. The first strategy uses mix rate (Algorithm 2, lines 2–4). The second strategy allows only

one randomly chosen individual to mutate in each trial (Algorithm 2, line 6). BSA's crossover process is more complex than the processes used in DE. Some individuals of the trial population obtained at the end of BSA's crossover process can overflow the allowed search-space limits as a result of BSA's mutation strategy. The individuals beyond the search-space limits are regenerated using Algorithm 3.

#### Algorithm 3. Boundary Control Mechanism of BSA

```

Input: T, Search space limits (i.e., lowj, upj)
Output: T
for i from 1 to N do
  for j from 1 to D do
    if (Ti, j < lowj) or (Ti, j > upj) then
      | Ti, j = rnd · (upj - lowj) + lowj
    end
  end
end
end

```

Fig. 2.3: Boundary Control Mechanism of BSA

### D. Selection: II

In BSA's Selection-II stage, the T<sub>i</sub>s that have better fitness values than the corresponding P<sub>i</sub>s are used to update the P<sub>i</sub>s based on a greedy selection. If the best individual of P (P<sub>best</sub>) has a better fitness value than the global minimum value obtained so far by BSA, the global minimizer is updated to be P<sub>best</sub>, and the global minimum value is updated to be the fitness value of P<sub>best</sub>. The structure of BSA is quite simple; thus, it is easily adapted to different numerical optimization problems.

### E. Stopping criteria

The predetermined criteria used to stop the algorithms' searches are as follows.

- If the absolute value of the objective function is less than 1016, stop.
- If the algorithm has failed to find a better solution than the existing solution during the last 200,000 function evaluations stop.
- If the number of function evaluations reaches 2,000,000, stop.
- If the maximum number of generations has been reached, stop.

## IV. BSA FOR OPTIMAL PLACEMENT AND SIZING OF CAPACITORS

In this chapter, the BSA is applied to solve the optimal placement and sizing problem of capacitors in the distribution network. First, in BSA, the solutions are generated randomly in a given range i.e. capacitors of given value are placed at random nodes in the distribution system. Here, each solution represents the size and location of capacitors in the distribution network, for example in the 3-capacitor problem; a solution will be of six dimensions of which first three will represent the size of the capacitors while remaining three will show the locations of the capacitors. Here, it should be noted that the locations of the capacitors are represented by discrete values while size by continues values. Therefore, the first three real values are converted into discrete values by rounding off in the nearby integer value. In this way, a mixed representation of the solution is prepared. As the capacitor placement and the sizing problem is non-separable and multimodal in nature,

BSA is applied to solve it. In this chapter, the loss minimization is carried out by providing the optimal size and location of the capacitors in the network. For testing, the performance of BSA, it is applied on IEEE-14, 30 and 33 test bus radial distribution system. The BSA is used to update the bus data of the considered bus systems iteratively for reducing the system losses. The reported results are compared with GA and SMO (The parameter settings of GA and SMO are same as their legitimate research papers [156, 5]) as shown in Table 1, Table 2, Table 3, Table 4, Table 5, and Table 9. The better results are represented by bold values. From these tables, it is clear that the size of the capacitor is determined in MVAR (i.e.  $10^6$  VAR) while power loss is measured in Mega Watts (i.e.  $10^6$  Watts). So, a little difference in power loss and capacitor size affects the performance significantly. The results show that the loss occurred using BSA is minimum among all the considered cases and algorithms. Therefore, the BSA may be used for solving the capacitor placement and sizing problem of the distribution system.

Algorithm	Optimal placement of capacitor on bus	Size of capacitors (MVAR)	Loss
GA	[6, 13, 9]	[14.64, 10.47, 41.77]	13.28
SMO	[14, 5, 9]	[10, 27.78, 21]	13.28
BSA	[13, 6, 9]	[10, 25.46, 22.56]	13.272

Table 1: Optimal placement and sizing of capacitor for IEEE 14 bus 3 Capacitor Prob.

Algorithm	Optimal placement of capacitor on bus	Size of capacitors (MVAR)	Loss
GA	[6,3,21,20,13]	[17.56, 20.39, 17.35,13,41.2]	17.48
SMO	[17,4,3,24,21]	[10,26.78,10,11.29,10]	17.35
BSA	[17,4,3,24,21]	[10,25.09,10,12.32,10]	17.343

Table 2: Optimal placement and sizing of capacitor for IEEE 14 bus 5 Capacitor Prob.

Algorithm	Optimal placement of capacitor on bus	Size of capacitors (MVAR)	Loss
GA	[6, 22, 24]	[40.20, 10.47, 13.12]	17.53
SMO	[22, 7, 4]	[22, 10, 41]	17.48
BSA	[3, 24, 21]	[35.08, 10.50, 16.05]	17.383

Table 3: Optimal placement and sizing of capacitor for IEEE 30 bus 3 Capacitor Prob.

Algorithm	Optimal placement of capacitor on bus	Size of capacitors (MVAR)	Loss
GA	[6,3,21,20,13]	[17.56,20.39,17.35,13,41.2]	17.48
SMO	[17,4,3,24,21]	[10,26.78,10,11.29,10]	17.35
BSA	[17,4,3,24,21]	[10, 25.09,11,11.32,10]	17.336

Table 4: Optimal placement and sizing of capacitor for IEEE 30 bus 5 Capacitor Prob.

Algorithm	Optimal placement of capacitor on bus	Size of capacitors (MVAR)	Loss
GA	[12, 28, 13]	[45.97,43.73,45.80]	165.840
SMO	[29, 13, 12]	[43.89,44,43.44]	165.42
BSA	[29, 13, 12]	[43.89,44,23,43.47]	165.416

Table 5: Optimal placement and sizing of capacitor for IEEE 33 bus 3 Capacitor Prob.

Algorithm	Optimal placement of capacitor on bus	Size of capacitors (MVAR)	Loss
GA	[30,12,13,22,29]	[49.84,40.39,46,47.68,47.175]	158.05
SMO	[29,12,11,13,25]	[45.05,45,45.69,48,44.03]	157.67
BSA	[25,11,13,12,29]	[48,47,47,48,47]	157.23

Table 6: Optimal placement and sizing of capacitor for IEEE 33 bus 5 Capacitor Prob.

## V. CONCLUSION

In this work, a complex real-world optimization problem, optimal placement and sizing of capacitors in distributed network is solved with IEEE 14, 30, and 33 bus test system using BSA. Results have been compared with those of GA and SMO. It is observed that BSA obtains minimum distribution and transmission losses while maintaining the minimum cost. This work may further be extended to an unbalanced radial system as a future research perspective.

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