Thermodynamic Modeling and Analysis of Regenerator of Stirling Refrigerator

Monil Jain[1] Sharad Choudhary[2]
[1,2]Department of Mechanical Engineering
[1,2]IET-Devi Ahilya University, Indore

Abstract— A mathematical modelling of a Stirling refrigerator is developed here. Regenerator is also analysed through mathematical modelling. In these modelling Beta type Stirling refrigerator is considered with air as the Working fluid and will be useful in optimizing the mechanical design of these machines. Two pistons cyclically compress and expand air while moving sinusoidal in same chamber connected by a regenerator, thus creating a temperature difference across the system. A complete non-linear mathematical model of the machine, including air Thermodynamics and heat transfer from the walls, as well as heat transfer and fluid Resistance in the regenerator is developed. Effect of air velocity on the air temperature at any position within the regenerator is studied.

Key words: Stirling Refrigerator, Thermodynamic Modeling

I. Introduction

In recent years, alternative energy sources have garnered considerable interest.

The need for energy permeates nearly all aspects of daily life. A low-cost energy source available when traditional energy sources are absent or depleted is valuable. This work analyzes a specific application of energy harvesting that converts a mechanical energy input into thermal energy, or thermo compressive energy harvesting .This application of energy is used in Stirling Refrigerator. This type of refrigerator used mechanical energy to drive heat flow from low temperature to high temperature. Stirling refrigerators have limited use in household applications as it works in very small temperature limits. Smaller Stirling machines are mostly used to cool computer parts. These Stirling model consist of an expansion space and compression space filled with the working fluid connected by a regenerator. The expansion space has a cold heat exchanger to exchange heat between the working fluid and the low temperature source. The compression space has a warm heat exchanger to exchange heat between the working fluid and the high temperature sink. The spaces are compressed or expanded by two pistons i.e. power piston and displacer piston. Even though the operation of the Stirling refrigerator differs from that of the ideal Stirling cycle due to discontinuity in the process temperature continuously varies.

Firstly, Robert Stirling [1] gives the idea of the Stirling air engine and the regenerator. Thombare and Verma [2] provided a thorough review of the work done on Stirling cycle-based machines. Although the review focused on engines, the analysis of the departure of Stirling machines from the ideal Stirling cycle remains relevant. Wu [3] and Kaushik and Kumar [4] used a finite time thermodynamic analysis of Stirling machines. By assuming the ideal cycle partially or completely, any analysis of the refrigerator deviates from the actual physics occurring during operation. Finkelstein T. [5] first analyzed the non-isothermal working space (finite heat transfer in working space by means of heat transfer coefficient). Later the researcher Patrick K. McFarlane [6] gives both a mathematical and physical model of a Stirling refrigerator is developed for an Air filled Alpha Stirling Refrigerator.

II. System Description and Cycle

The ideal Stirling-cycle refrigerator or heat-pump is, in effect, identical to a Stirling-cycle engine except that the heat absorbing end of the machine now becomes the cold region, and the heat rejecting end of the machine becomes the hot region. The thermodynamic processes for a refrigerator /heat-pump are illustrated using a simplified beta-configuration machine in shown on pressure-volume and Temperature-entropy diagrams in. Because refrigerator/heat-pumps tend to have a smaller temperature difference between hot and cold regimes than an engine, the pressure-volume and temperature-entropy diagrams appear somewhat squatter in comparison. It should be noted that for the ideal Stirling Cycle the heat- The Stirling cycle consist of four processes:

1) Isothermal expansion.
2) Isochoric displacement
3) Isothermal compression.
4) Isochoric displacement.

A. Isothermal Expansion

The low-pressure working gas expands isothermally at cold end temperature, hence absorbing heat from the cold space (via the heat absorbing heat-exchanger) and doing work to the power-piston.

B. Isochoric Displacement

The displacer-piston transfers all the working gas isochoric through the regenerator to the hot end of the machine. Heat is delivered to the gas as it passes through the regenerator, thus raising the temperature of the gas to that of the hot space. As the temperature rises, the gas pressure increases significantly.

C. Isothermal Compression

The power-piston does work to the gas and compresses it isothermally at hot end temperature, hence rejecting heat to the hot space (via the heat rejecting heat-exchanger). Because the gas is at high pressure, more work is required for compression than was obtained from the gas during expansion. The cycle therefore has a network input.

D. Isochoric Displacement

The displacer piston transfers all the working gas isochoric through the regenerator to the cold end of the machine. Heat is absorbed from the gas as it passes through the regenerator, thus lowering the temperature of the gas to that of the cold space. As the temperature reduces, the gas pressure drops significantly, and the system returns to its initial Conditions.

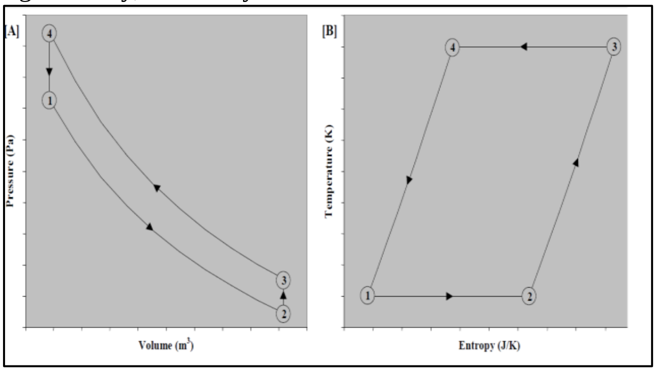

Fig. 1: Pressure-volume and temperature-entropy process diagram of Stirling Refrigerator

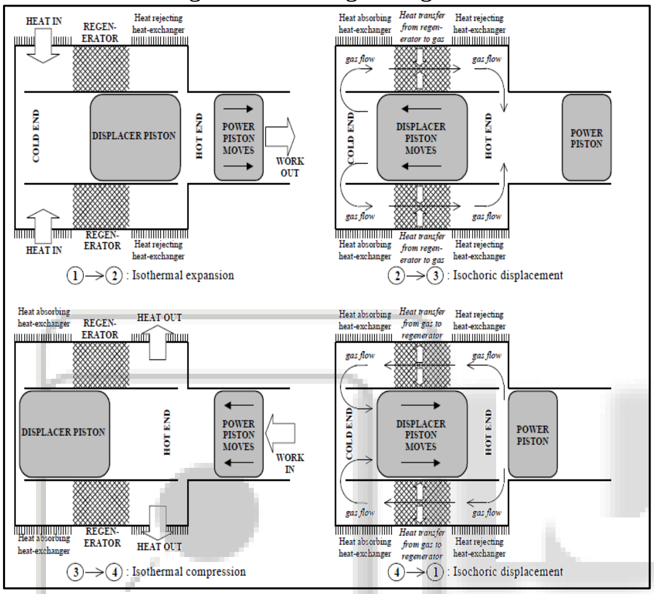

Fig. 2: Thermodynamic processes in the ideal Stirling-cycle refrigerator/heat-pump as shown on a simplified Beta-configuration machine.

III. METHEDOLOGY

A. *Regenerator Analysis through Mathematical Modelling*

1) *Regenerator*

Regenerator is basically a device which storing additional during one part of the cycle and discharging in the other part of the cycle. The regenerator's role is simple and it made of compact structure to limit the dead volume. Normally regenerator is made of porous media and it used to place between heating and cooling heat exchanger.

B. *Nomenclature Used For Regenerator*

T_r	Air temperature in the regenerator
c_p	Specific heat of air
h_r	Convective heat transfer coefficient
D_r	Diameter of regenerator
ρ	Density of air
U	Velocity of air in regenerator
L_r	Length of regenerator.
T_w	Wall Temperature of regenerator

Table 1:

Consider the regenerator as a metal tube of length Lr, and diameter Dr as control volume.

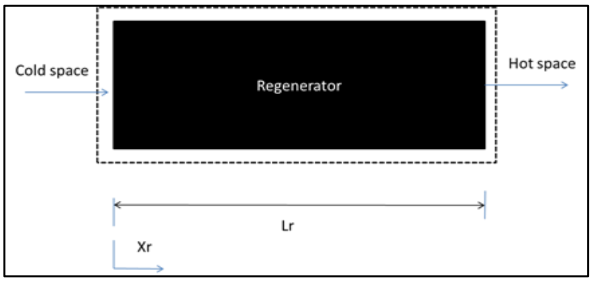

Fig. 3:

Here we can say that air temperature is the function of position (Xr) and time (T) throughout the regenerator as the air proceeds from one end position to other end.

$$\text{Air temp.} \rightarrow T_r = f(x_r, t)$$

From mathematics of total differential we can write,

$$dT_r = \frac{\delta T_r}{\delta x_r} dx_r + \frac{\delta T_r}{\delta t} dt$$

Now, divide by, dt, on both sides.

$$\frac{dT_r}{dt} = \frac{\delta T_r}{\delta x_r}\frac{dx_r}{dt} + \frac{\delta T}{\delta t}$$

$$\text{here, } \frac{dx_r}{dt} = u$$

$$\frac{dT_r}{dt} = u\frac{\delta T_r}{\delta x_r} + \frac{\delta T}{\delta t} \longrightarrow \boxed{1}$$

Now, from control volume, energy balance in regenerator,

$$mc_p \frac{dT_r}{dt_r} = h_r \pi D_r L_r (t_w - t_r) \longrightarrow \boxed{2}$$

Here, t_w = regenerator wall temperature

Now from equation (1),

$$mc_p \left[u\frac{\delta T_r}{\delta x_r} + \frac{\delta T_r}{\delta t} \right] = h_r \pi D_r L_r (t_w - t_r) \longrightarrow \boxed{3}$$

Now,

$$m = \frac{\pi}{4} * D_r^{\,2} * L_r * \rho$$

$$\frac{\pi}{4} D_r^{\,2} L_r \rho c_p u \frac{\delta T_r}{\delta x_r} + \frac{\pi}{4} D_r^{\,2} L_r \rho c_p \frac{\delta T_r}{\delta t} = h_r \pi D_r L_r (t_w - t_r) \xrightarrow{\text{On}} \boxed{4}$$

Simplify we get,

$$u\frac{\delta T_r}{\delta x_r} + \frac{\delta T}{\delta t} = \frac{4h_r}{\rho c_p D_r}(t_w - t_r)$$

Now substitute,

$$H = \frac{4h_r}{\rho c_p D_r}$$

By using solution to partial differential equation as:

$$u\frac{\delta T_r}{\delta x_r}+\frac{\delta T}{\delta t}=H(t_w-t_r) \longrightarrow \boxed{5}$$

$$\left(\frac{I.V}{Cofficient}\right)_1=\left(\frac{I.V}{Cofficient}\right)_2=\left(\frac{D.V}{Constant}\right)_3$$

Then,

$$\frac{dt}{1}=\frac{dX_r}{u}=\frac{dT_r}{H(t_w-t_r)}$$

On Simplify this equation we get,

$$\frac{dT_r}{dx_r}=\frac{H}{u}(t_w-t_r) \longrightarrow \boxed{6}$$

From above Equation we draw a conclusion that air temperature Tr within the regenerator depends on u i.e. velocity of air at any position.

For calculation purpose we consider some physical parameters of conventional Stirling machine as below:

Piston diameter D	50mm
Regenerator pipe diameter Dr	5mm
Regenerator pipe length Lr	100mm
Convection coefficients Hr	100 W/(m2 K)
Frequency of piston motion ω	1.5 rad/s
Density of air ρ	1.2Kg/m3
Specific heat c_p	1005 J/Kg.K

Table 2:

IV. Conclusion and Results

The graph shown below between, difference of air temperature and wall temperature v/s length of the regenerator at velocity u= 5m/s. It is concluded from above graph that as air temperature within the regenerator is inversely proportional on velocity and directly proportional to the position of the working fluid in the regenerator. The various graphs can be drawn, switching to different velocities of air within the regenerator.

The differential equation for Tr (6) used depends on u. If the flow has not reached a location Xr along the regenerator, the temperature at that point only depends on the heat transfer from the regenerator walls. If the flow has reached a location Xr, the temperature at that point depends on the flow from the chamber.

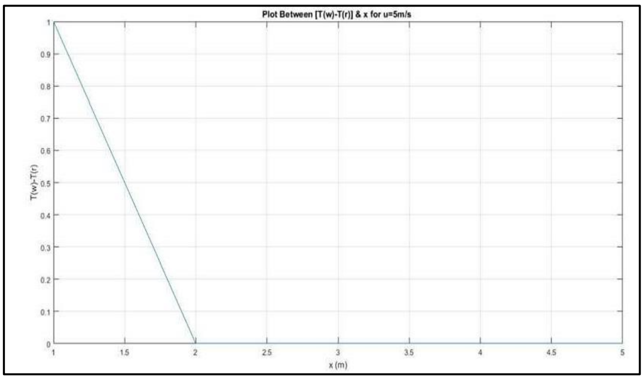

Fig. 3: Graph between instantaneous temperature difference and position in the regenerator at any air velocity of 5 m/s

References

[1] Robert S.,(1816), Patent no. 4081, Stirling air engine and the heat regenerator.
[2] Thombare D.G. and Verma S.K.(2008), “Technological development in the Stirling cycle engines”, Renewable and sustainable energy (12).
[3] Wongwises S. and Mongkut’s k.,(2006), “Thermodynamic analysis of a Stirling engine including dead volumes of hot space and regenerator”, Renewable energy 24(77).
[4] Kaushik S.C. and Kumar,(2000) “Finite time thermodynamics analysis of endoreversible Stirling heat engine regenerative losses”. Applied energy (25)
[5] Finkelstein T.,(1960) “Generalized thermodynamic analysis of Stirling cycle engines”. SAE paper .1184.
[6] Patrick K.M.,(2014) ,“Mathematical Model And Experimental Design Of A n Air Filled Alpha Stirling Refrigerator” , Department of aerospace and mechanical engineering of the university of Notre ,dame.
[7] Bachelier C.,(2009), “Stirling engines A technological overview”, Department of energy technology Royal Institute of technology Stockholm, Sweden
[8] Martini WR.,(1979), “Stirling design manual”, New jersey (26).