Fibonacci Cordial Labeling of Some Special Graphs

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Abstract— Fibonacci cordial labeling was introduced in this paper. We prove that the graph such as wheel graph w_n bistar Bn,n are Fibonacci cordial graph. If the induced function f*from the edge set E of graph G to the set {0, 1} defined by f*(uv) = (f (u) + f (v)) (mod2) satisfies the condition |e_f(0) – e_f(1)| ≤ 1, where e_f(0) is the number of edges with label 0and e_f (1) is the number of edges with label 1. In a graph G An injective function f from vertex set V of a graph G to the set {F0, F1, F2, . . . , Fn}, where Fj is the j^th Fibonaccinumber (j = 0, 1, . . . , n), is said to be Fibonacci cordial labeling .A graph which admits Fibonacci cordial Labeling is called Fibonacci cordial graph. In this paper we discuss Fibonacci cordial labeling of different graphs.

Key words: Fibonacci Cordial Labeling

I. INTRODUCTION

All graphs in this paper are finite, simple, connected and undirected. The symbols V (G) and E(G) will denote the vertex set and edge set of the graph G. For standard terminology and notations we follow Gross and Yellon[1]. We will give brief summary of definitions which are useful for the present investigation. Path, cycleare Fibonacci divisor cordial graph proved by Sridevi, Nagarajan, Nellaimurugan and Navaneethakrishnan. Every year Gallian published and updated his a dynamic survey of graph labeling .In this paper we investigate a new concept of Fibonacci cordial labeling. We have derived different graph families satisfying the conditions of Fibonacci cordial labeling. We have also discussed Fibonacci cordial labeling in context of different graph operations.

A. *Definition 1.1.*

A function f: V (G) → {0, 1} is called a binary vertex labeling of a graph
G and f (v) is called label of the vertex v of G under f.
For an edge e= uv, the induced edge labeling f*: E (G) → {0, 1} is given by
$F^*(e) = |f(u) - f(v)|$.

1) *Notations*

v_f (0): number of vertices with label 0.
$v_f(1)$: number of vertices with label 1.
$e_f(0)$: number of edges with label 0.
$e_f(1)$: number of edges with label 1.

B. *Definition 1.2.*

A binary vertex labeling of a graph G is called cordial labeling if $|v_f(0) -$
v_f (1)| ≤ 1 and $|e_f(0)$-$e_f(1)|$ ≤ 1. A graph G is cordial if it admits cordial labeling.
Cahit [1] introduced the concept of cordial labeling.

C. *Definition 1.3.*

Fibonacci numbers can be defined by the linear recurrence relation F_n =
Fn−1 + Fn−2, n ≥ 2, where F0 = 0, F1 = 1. This generates the infinite sequence of integers
beginning 0,1,1,2,3,5,8,13,21,34,55,89,144,. . . .

D. *Definition 1.4.*

An injective function f: V (G) → {F_0, F1 …F_n}, where F_j is the j^{th}
Fibonacci number (j = 0, 1, .. n), is said to be Fibonacci cordial labeling if the induced
Function f* : E (G) → {0, 1} defined by f*(uv) = (f (u) + f (v))(mod2) satisfies the
Condition $|e_f$ (0) – e_f (1)| ≤ 1. A graph which admits Fibonacci cordial labeling is called
Fibonacci cordial graph.

E. *Definition 1.6.*

Wheel graph is a graph formed by connecting a single vertex to all vertices of a cycle. A wheel graph with *n* vertices can also be defined as the 1-skeleton of an (*n*-1)- gonal pyramid. W_n is denote a wheel graph with *n*+1 vertices (n ≥ 3), which is formed by connecting a single vertex to all vertices of a cycle of length *n*.

F. *Definition 1.5.*

Bistar $B_{n,n}$, is the graph obtained from two copies of $K_{1,n}$ by joining the Apex vertices by an edge.

II. MAIN RESULT

A. *Theorem: 1*

Wheel w_n is Fibonacci cordial for n ≥ 3, n ∈ N .

1) *Proof*

Let $v_1, v_2 \dots v_n$ be successive rim vertices and v_0 be the apex vertex of w_n.
We define labeling f: U (w_n) → {F_0, F_1, F_2. . . F_{n+1}}, we consider the following two
Cases.
1) Case 1: n ≡ 0(mod3).
f (v_0) = F_1, f (v_i) = F_{i+1}, 1 ≤ i ≤ n.
2) Case 2: n does not congruent to 0(mod3).
f (v_0) = F_1, f (v_1) = F_0, f (v_i) = F_{i+1}, 2 ≤ i ≤ n .
Then in each case we have $e_f(0) = e_f(1)$ = n.
Hence wheel $W_n$is Fibonacci cordial for n ≥ 3, n ∈N.
3) Example 2.2. Fibonacci cordial labeling of wheel graph W_6, W_9, W_{12}

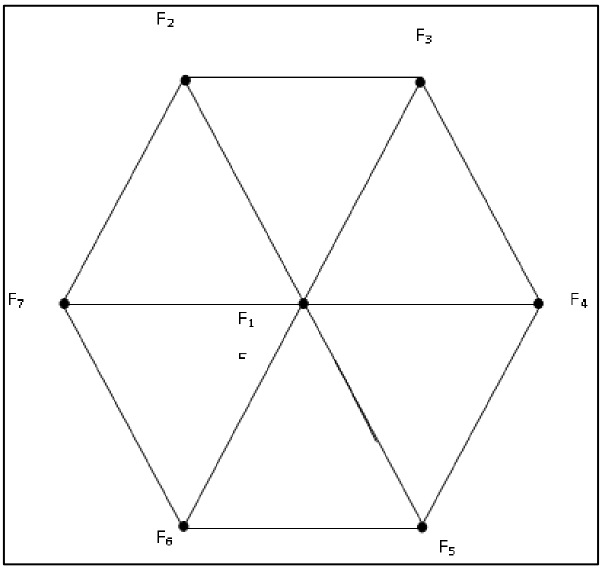

Fig. 1: Fibonacci Cordial Labeling of W_6

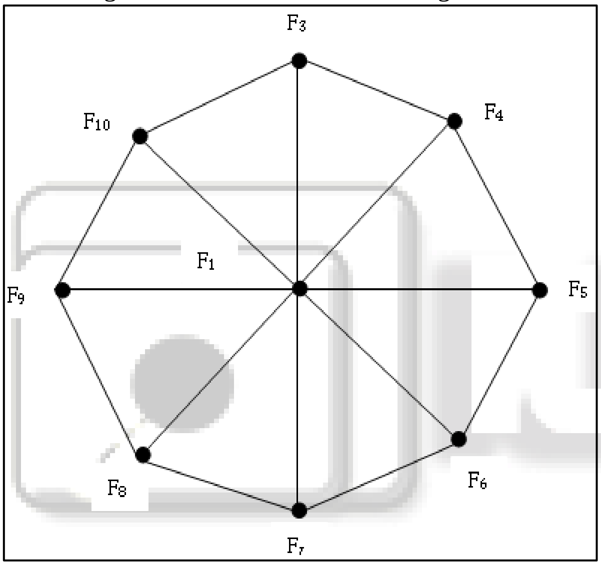

Fig. 2: Fibonacci Cordial Labeling of W_{12}

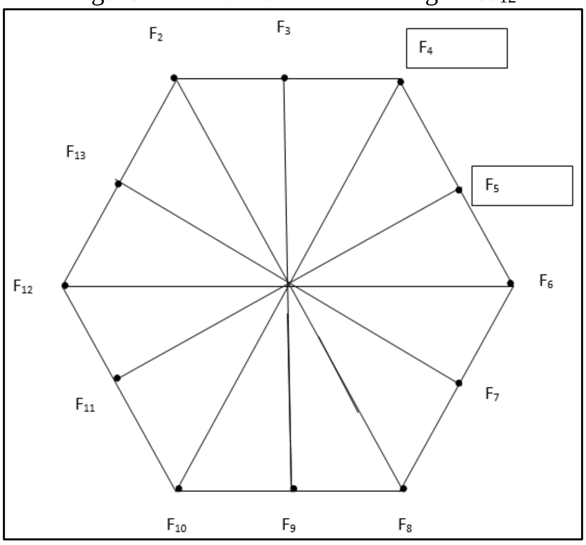

Fig. 3: Fibonacci Cordial Labeling of W_{12}

B. *Theorem 2.4.*

Bistar $B_{n,n}$ is a Fibonacci cordial, for all n.

1) *Proof.*

Let v_0, u_0 be the apex vertices of Bn,n.Let $v_1, v_2. . .v_n$ be the pendant vertices

Adjacent to the vertex v_0 and $u_1, u_2. . .u_n$ be the pendant vertices adjacent to the vertex

u_0.

We define labeling function f: U (G) $\rightarrow \{F_0, F_1, F_2. . .F_{2n+2}\}$ as follows.

1) Case 1**:** $n \equiv 2(\text{mod}3)$.

$f(v_0) = F_1$, $f(v_1) = F_2$, $f(v_i) = F_{i+2}$, $2 \leq i \leq n$.

$f(u_0) = F_0$, $f(u_1) = F_3$, $f(u_i) = F_{n+i+1}$, $2 \leq i \leq n$.

Then we have $e_f(0) = n + 1$ and $e_f(1) = n$.

2) Case 2: n does not congruent to 2(mod3).

$f(v_0) = F_1$, $f(v_i) = F_{i+1}$, $1 \leq i \leq n$.

$f(u_0) = F_0$, $f(u_i) = F_{n+i+1}$, $1 \leq i \leq n$.

Then we have $e_f(0) = n + 1$ and $e_f(1) = n$.

Hence, $B_{n,n}$ is a Fibonacci cordial graph, for all n.

2) *Example: Fibonacci cordial labeling of $B_{5,5}$, $B_{6,6}$*

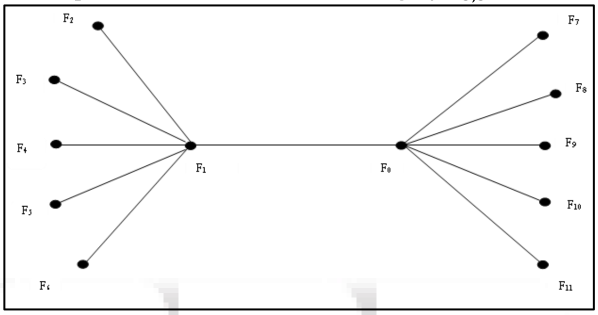

Fig. 4: Fibonacci cordial labeling of Bistar $B_{5,5}$

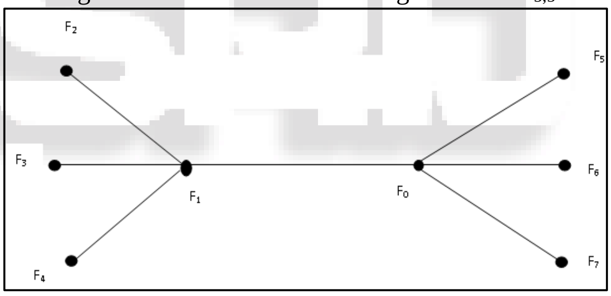

Fig. 5: Fibonacci Cordial Labeling of Bistar $B_{3,3}$

III. Conclusion

We have presented the Fibonacci cordial labeling of certain classes of graphs such as wheel graph w_n, bistar graph B_(n,n) .It is very interesting to investigate graph families satisfying the condition of Fibonacci cordial labeling.

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