

# Analysis of State Estimation of Electrical Power System-A Review

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**Abstract**— The control centers of utilities receive abundant raw measurements from power system networks. The measurements may be lost while being transmitted to the control centers due to defects in sensors or loss of communication channels. State estimation is a key energy management system function, responsible for estimating the state of the power system. In this paper the basic concepts and recent trends of the power system state estimation are described. The merits and demerits of each method are also reviewed.

**Key words:** Energy management system power system, State estimation

## I. INTRODUCTION

As the power system grows larger and more complex, real time and monitoring and control becomes very significant in power system. The Energy Management System (EMS) functions are responsible for this task of monitoring and control. State estimation forms the backbone of the energy management system by providing a database of the real time state of the system for using in other EMS functions [1]. State estimator (SE) is the key element of energy management system (EMS) in control center, which provides the snapshot of the system to all other on-line applications, such as real-time power-flow, security assessment, and so on. Traditional SE is to get the estimation of state variables according to redundant measurements provided by SCADA, which is solved by weighted-least-square (WLS) method. With the increasing use of synchronized phasor measurement unit (PMU) in recent years, it gains great interests to improve SE with phasor measurements due to their synchronized characteristic and high data transmission speed [2-5]. State estimation (SE) is an essential function in energy management system for system security monitoring and the control of power systems. SE is based on mathematical relation between system state variables and actual measurements. Least squares (WLS) method is generally used to solve the state estimation problem [6]. For the conventional state estimation, the measurement equations for bus voltage magnitude, real power, and reactive power are non-linear. The solution must be obtained through an iterative algorithm as in the Newton-Raphson power-flow procedure. Recently, phasor measurements are applied to the power system SE [7-8]. State estimation was introduced by Gauss and Legendre (around 1800). The basic idea was to "fine-tune" state variables by minimizing the sum of the residual squares.

This is the well-known least squares (LS) method, which has become the cornerstone of classical statistics. The reasons for its popularity are easy to understand: At the time of its invention there was no computers, and the fact that the LS estimator could be computed explicitly from the data (by means of some matrix algebra) made it the only feasible approach. Even now, most statistical packages still use the same technique because of tradition and computational speed.

## II. CONVENTIONAL STATE ESTIMATION MODEL

In traditional SE, the measurement equation is given below.

$$z = h(x) + v \quad (2.1)$$

Where  $z$  is the measurement vector (m-vector) provided by SCADA which is composed of voltage magnitude, active and reactive power-flow and injection measurements;  $x$  is the state vector (n-vector, and  $n < m$ );  $h(x)$  is the non-linear vector measurement function, and  $v$  is the measurement error vector (m-vector). All the measurement errors are assumed to be independent and follow the normal distribution with zero mean and  $\sigma^2$  variance.

In order to find the optimal solution of SE, the objective function is to minimize the following weighted square errors.

$$J(x) = [z - h(x)]^T R^{-1} [z - h(x)] \quad (2.2)$$

Where  $R$  is the measurement covariance matrix ( $m \times m$  matrix) and  $R^{-1}$  is called weight matrix. It is clear that  $R^{-1}$  is a diagonal matrix and its diagonal element  $R^{-1}_{ii}$  equals  $1/\sigma_i^2$ .

The iterative solution based on the first order optimality condition for  $J(x)$  can be formulated as follows:

$$\begin{aligned} \Delta z(x^k) &= z - h(x^k) \\ \Delta x^k &= [H^T R^{-1} H]^{-1} H^T R^{-1} \Delta z(x^k) \quad (2.3) \\ x^{k+1} &= x^k + \Delta x^k \quad (2.4) \end{aligned}$$

Where  $H$  is  $m \times n$  Jacobian matrix,  $H^T R^{-1} H$  ( $n \times n$  matrix) is called the gain matrix or information matrix. For the power-flow measurement, its linearized measurement equation is:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial U} \\ \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial U} \end{bmatrix} \begin{bmatrix} \Delta \theta \\ \Delta U \end{bmatrix} \quad (2.5)$$

It should be noted that the structures of four Jacobian sub-matrices are same with that of node-branch incidence matrix.

For the injection measurement, its linearized measurement equation has the same form of (5), and the structure of the corresponding Jacobian sub-matrix is same with that of node admittance matrix.

For the voltage magnitude measurement, its linearized measurement equation is expressed as:

$$\Delta U_i = I \times \Delta U \quad (2.6)$$

Where  $U_i$  is the residual vector of voltage measurements (suppose it is a 1-vector, and  $1 < m$ ),  $\Delta U$  is the correction of voltage magnitude of state vector (suppose it is a j-vector, and  $j < (n/2+1)$ ), and  $I$  is the Jacobian sub-matrix ( $1 \times j$  matrix) for voltage measurements. It can be seen that only when the voltage magnitude measurement of bus  $i$  is available

(i.e.  $U_i(i) \neq 0$ ), there is a non-zero element (1) appearing at column i of corresponding row of matrix **I**.

### III. DECOMPOSITION TECHNIQUE

Let us assume that the entire network under consideration is decomposed in r non-overlapping observable sub networks. Each sub-network consists of internal buses, which are adjacent to branches belonging only to this sub-network, and boundary buses. These buses are connected to other sub-networks by tie-lines. To obtain decoupled state estimation equations of these sub-networks, constraints of measurement placement are as follows:

- 1) There is no injection current measurement at the boundary bus.
- 2) There is no line current measurement on the tie-line.
- 3) Each sub-network has at least one bus bar voltage measuring device

Based on the above conditions, parameters of the tie-lines do not appear in the estimation equations and the measurement model of the power system state estimation can be written as,

$$\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_r \end{bmatrix} = \begin{bmatrix} [H_1] & [0] \\ & [H_2] \\ & & \ddots \\ & & & [H_r] \\ [0] & & & & [H_r] \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_r \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_r \end{bmatrix} \quad (3.1)$$

The measurement placement obtained from the minimum condition number criteria can be rearranged to minimize the number of placement sites by the following heuristic algorithm:

Step 1. Based on the placement position list obtained from the measurement placement algorithm, determine the bus where either an injection current or a busbar voltage measuring device is installed. These buses are called major buses. Other buses are called minor buses.

Step 2. If there is a line current measuring device on the branch connected to the major buses, the device is moved close to the major buses.

Step 3. From the branches with line current measuring devices which are not connected to major buses, determine the minor bus with the maximum connection number of those branches which have line current measurements. Then, the line current measuring device on the connected branches is moved close to the selected bus. Note that this minor bus will not be considered again in the next iteration.

Step 4. Repeat Step 3 until the maximum connection number of line current measuring branches equals to one.

*Advantage of Decomposition technique:*

Since the problem size is smaller, this decomposition improves the computational time required to obtain types and positions of the available measurement compared to the previously proposed method with considering the whole network. From the numerical experimental, the decomposition technique provides a promising strategy to solve the measurement placement problem of a large power system state estimation.

### IV. HYBRID STATE ESTIMATOR MODE [10]

- 1) The procedure of the proposed hybrid SE is illustrated below.
- 2) Set the initial values of the state variables to  $1.0 \angle 0^\circ$  as flat start or the previous estimated values.
- 3) Set the iterative counter  $l=0$ .
- 4) (3) The branch current phasor measurements in polar coordinates are transformed into the real part vector  $I_{ij,r}$  and the imaginary part vector  $I_{ij,i}$  in rectangular coordinates.
- 5) Jacobian sub-matrix  $H(x^{(l)})$  of conventional measurements is formed.
- 6) Calculate the residuals vector  $z-h(x^{(l)})$  of conventional measurements.
- 7) The Jacobian sub-matrix  $H_l(x^{(l)})$  of branch current phasor measurements is obtained.
- 8) Calculate the values of branch current phasor measurement functions to get the corresponding residuals vector  $I_{ij,r}$  and  $\Delta I_{ij,i}$ .
- 9) Calculate the residuals vector  $\theta_p$  and  $U_p$  of the voltage phasor measurements.
- 10) The Jacobian matrix  $H_m(x^{(l)})$  and residuals vector  $\Delta z_m^{(l)}$  for including all measurements are formed.

$$H_m(x^{(l)}) = \begin{bmatrix} H(x^{(l)}) \\ H_U(x^{(l)}) \\ H_I(x^{(l)}) \end{bmatrix} \quad \Delta z_m^{(l)} = \begin{bmatrix} \Delta z^{(l)} \\ \Delta \theta_p^{(l)} \\ \Delta U_p^{(l)} \\ \Delta I_{ij,r}^{(l)} \\ \Delta I_{ij,i}^{(l)} \end{bmatrix}$$

- 11) The hybrid covariance matrix  $R_m$  is built up based.
- 12) The linear Equation is solved to obtain the correction vector  $x^i$  of state variables and  $\max |\Delta x_i|$ .

$$\begin{aligned} & [(H_m^T(x^{(l)})^{-1} R_m^{-1} H_m(x^{(l)})] \\ & = H_m^T(x^{(l)}) R_m^{-1} \Delta z_m^{(l)} \end{aligned}$$

- 13) To update  $x(l) = x(l-1) + x^i$  and judge whether  $\max |x_i|$  is less than the given convergence tolerance. If yes, the iterative process converges and  $x(l)$  is the estimated state vector; otherwise, iterative counter  $l$  plus 1 and go to step (4) until the stop condition holds.

As compared with other state estimators with or without phasor measurements, the proposed method can get better performance in terms of estimation accuracy and the convergence speed is also acceptable.

### V. DYNAMIC STATE ESTIMATION

An easy way of following the changes in a power system on a real time basis is by using the state estimation techniques on a continuous basis with a certain time step. But these techniques, though computationally very efficient, do not use any physical modeling of the time varying nature of the power system, and hence may not be as accurate as desirable. Dynamic state estimation uses the present (and sometimes previous) state of the power system along with the knowledge of the system's physical model, to predict the state vector for the next time instant. This prediction feature of the DSE provides vital advantages in system operation, control, and decision making. It allows the operator more time to act in cases of emergency, helps in detection of anomalies, bad data etc [11].

The main steps involved in DSE algorithms to achieve an optimized estimate of the state vector are presented in the following subsections. Throughout the discussion, 'k' is used to suggest the present instant of time and 'k+1' to suggest the next instant of time

#### A. Mathematical Modeling:

This is the first step in DSE and involves the identification of correct mathematical model for the time behavior of the power system. The general mathematical model used for a dynamic system is given by [1], [12]:

$$x_{k+1} = f(x_k, u_k, w_k, k)$$

Where, 'k' is the time sample, 'x' is the state vector, 'u' is the control actions, 'w' represents the uncertainties in the model and 'f' represents the nonlinear function. But such a model is extremely complex, costly and impractical. Hence, certain assumptions are made to ease the implementation (some have already been mentioned earlier). They are:

- The system is quasi static and hence changes extremely slowly
- Time frames considered are small enough, for the usage of linear models to describe the transition of states between consecutive instants of time
- The uncertainties are described using white Gaussian noise with zero mean and constant covariance Q.

Considering these assumptions, we can obtain a generic linear model for the DSE as:

$$x_{k+1} = F_k x_k + G_k u_k + w_k$$

Where  $F_k$  is the function representing the state transition between two instants of time,  $G_k$  is associated with the trend behavior of the state trajectory and  $w_k$  is the white Gaussian noise with zero mean and covariance Q [1], [12].

### VI. ADVANTAGES OF DYNAMIC STATE ESTIMATION

The ability of predicting the state vector one step ahead is a very important advantage of DSE. Some of the advantages of that includes [1], [13]:

- It allows security analysis to be carried out in advance and hence allows the operator to have more time during emergencies.
- It helps to identify and reject bad data and hence improves the estimator performance.
- In cases where pseudo measurements are to be used, DSE readily provides high quality values and hence avoids ill conditioning.
- DSE can be used for data validation as the states are predicted one time stamp before.
- Similarly, with the help of the predicted state vector we can identify sudden changes in the system, topological errors and other anomalies.

### VII. CONCLUSION

Real time monitoring and control of power systems is extremely important for an efficient and reliable operation of a power system. State estimation forms the back bone for the real time monitoring and control functions. Since power system changes continuously, the operator has to be extremely alert in taking decisions on real time, especially in cases of emergency. Decomposition technique requires less

computational time as compared to conventional state estimation method. But the dynamic state estimation technique provides best result among all these techniques.

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