Design Optimization of Axi-Symmetric Grain with Truncated Cone Bore and Aft End Has Cylinder Cut-off and Section of End Burnind Grain in Solid Propellant Rocket Motor

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Abstract— In the present research work, Axi-symmetric grain with truncated cone bore and aft end has cylinder cut-off and section of end burning grain is used for optimization. The aim of this research work is to compare the result of developed optimization tool with solid rocket motor test results Ref. [1], computation results Ref. [2], and predesigned grain using its objectives and constraints. Here, Genetic Algorithm is used for optimization of grain. Genetic algorithm is able to finding the best solution in design space. Within the geometric constraints, firstly the design variables are selected randomly and grain designs are developed in the amount of population numbers. Genetic Algorithm does not require any initial guesses as in case of Complex Algorithm. In the present study objective function is evaluated in the number of population size at each generation; therefore, objective function is calculated 30 times at each generation. The objective function value of the found geometry after 40 generation is 6.2% and propellant mass is reduced by 16.3% as compared to the actual rocket motor test results. Objective function of the actual grain geometry is calculated as 11.2% for the given objective thrust –time curve. Therefore, the genetic algorithm, having a better curve fit with the objective thrust-time curve than actual grain geometry.
Key words: Axi-Symmetric Grain, Solid Propellant Rocket Motor

I. INTRODUCTION

The optimization techniques are generally discussed in the main titles of linear and non linear programming. Non-linear programming deals with the problem of optimizing an objective function in the presence of equality and inequality constraints. If all the functions, are linear the problem is called linear programming, otherwise it is called nonlinear programming problem [3]. Optimization methods are classified into two groups: gradient-based and derivative free methods. In gradient based method, algorithms take a starting point and calculate the objective function. Conjugate Gradient Method, Davdon-Fletcher-Powell (DEF) Method, Broyden-Fletcher-Goldfrab-Shanno (BFGS) Method and Sequential Quadratic Programming are gradient based methods. The details of these methods are given in Ref. [4]. Major drawbacks of these methods are dealing with noisy problems or problems containing many local minima. Given a response surface with local minima, the algorithms will generally converge to the nearest local minima. Derivative –free optimization methods are typically developed to solve optimization problems whose gradient computation of objective function is unavailable. The simplest derivative free method is the one referred to as Direct Search Method. These methods sample the objective function at a finite number of points at each iteration and decide which action to take next solely based on those function values and without any explicit or implicit derivative approximation or model building. Hooke and Jeeves Pattern Search Method, Simulated Anneling Genetic Algorithms and Box Complex Methods are some examples of Direct Search Methods. Genetic algorithms are derivative-free, heuristic, global search methods based on principles of natural selection and genetics. In 1975, Johan Holland led to the development of genetic algorithms with his work about the investigation of the mechanism of natural adaption.

II. LITERATURE REVIEW

The study of Billheimer [5] in 1968 was one of the first attempts at using an automated procedure to design a SRM. The physical modeling used in the study was limited because of the computational resources available at that time; however this paper really acknowledged the importance of automating the design process for SRMs. Woltosz [6] tried to find optimal solid grain geometry by using a pattern search technique; Woltosz determined five critical design dimensions which maximize the total impulse-to motor weight ratio. This same pattern search technique was also used by Foster and Sforzini [7] in order to find the optimum values of five primary igniter design parameters. Sforzini [8] developed a computer program, called Solid Rocket Motor Design and Optimization program (SRMDOP), by utilizing the same pattern search technique used by Woltosz for the SRM design. Design and try to find the set of design parameters that will give the predicted thrust-time trace that most nearly matches a desired thrust-time trace [9].

Acik [10] developed a design and optimization tool for 2-D grain configurations by using complex method. In her study; star, slot, tubular and slotted-tube grains were employed. For the ballistic performance evaluation; an internal ballistic solver that uses 0-Dquasi-stedy model for the flow in combustion chamber and steady 1-D isentropic flow equations in the nozzle were utilized.

For the design optimization of SRMs Genetic Algorithm, Complex, Hybrid method, ±3 Sigma method etc. are used. But in the present research work Genetic algorithm is used for design optimization of Solid Rocket Motor.

III. METHODOLOGY

GENOP, firstly, reads the input parameters of the genetic algorithm which are the number of design variables ,upper and lower limits of the variables, bit number, population size, maximum generation number , cross over probability ,mutation probability, selection method, cross over method and replacement method .After the creating and evaluating the first population ,the iteration procedure is started .At each itertation,SELECT,CROSVR,MUTATEand EVAL subroutine are called until the iteration is equal to the maximum generation number. Finally the best solution in the last generation is given as output.

The basic idea of the algorithms is to mimic the evolution of a group of individuals of the same species. Since the individuals who adapt better to the requirements imposed by their environment will survive in the population, their genes will be passed more frequently to subsequent generations than others. This means, the average fitness of the population increase with time [11]

Once the design variables are encoded in a chromosomal manner and a fitness measure for discriminating good solutions from bad ones has been chosen, genetic algorithms start to evolve solutions by using Initialization, Evaluation, Selection, Recombination, Mutation and Replacement [2].

IV. RESULT AND DISCUSSION

6 design variables and their dimensionless values for Axi-symmetric end burning grain with truncated cone bore and cylinder cut-off are given in Table 1.Design variables other than grain length and nozzle throat diameter are the ones defining the cross-section geometry of the grain.

Grain Length, L/D_t	15.4
Outer Diameter D_{out}/D_t	4.5
Web thickness w/w	1
Cone Radius, R_1/w	0.3
Bone radius, r_1/w	0.12
Nozzle Throat Diameter, D_t/D_t	1

Table 1: Non-Dimensional Geometric parameters of the Grain

Design constraints are taken from the Ref. [13]. The geometric bounds on the variables are defined as shown in Table 2. Values in Table 2 are non-dimensionalized with the Axi-symmetric End burning grain with truncated cone bore and cylinder cut-off grain parameters. Grain length and outer diameter bounds come from the grain design process. However, the bounds of the other parameters were chosen arbitrarily considering manufacturability.

Parameters	Lower Bound	Upper Bound
Grain Length, L/D_t	0.96	1.02
Outer Diameter, D_{out}/D_t	0.97	1.03
Web Thickness, w/w	1	1
Cone Radius, R_1/w	0.3	1.2
Bore radius, r_1/w	0.64	2.45
Nozzle Throat Diameter, D_t/D_t	1	1

Table 2: Non dimensional Geometric Bounds of the Grain optimization

Maximum chamber pressure is constrained to be 5.5MPa and propellant mass is constrained to be 12 kg. The propellant and nozzle properties of the actual motor are used as the input of the internal ballistics.

Considering the mission requirements of a Solid Propellant Rocket system, the grain design requirements can be simplified to the thrust –time history of the rocket motor. In this case study, the design requirements of Truncated End burning Rocket Motor like burning time and average thrust level are defined as an objective thrust-time curve. Using the objective thrust-time curve, the optimization problem becomes finding the grain design whose ballistic performance fits the objective curve best within the given design constraints. Therefore, the objective function can be defined as follows:

$$f(x) = \frac{\sqrt{(F_{des\ i}(x) - F_{obj\ i})^2}}{N_{tb} F_{obj\ ave.}} 100 \qquad (1)$$

The objective function is typically summation of differences between the (F_{obj}) and computed thrust values at specified times during motor operation divided by average desired thrust (F_{des}) thrust values at specified times during motor operation divided by average desired thrust (F_{objave}) and total number of time data (N_{tb}).

Finally, the optimization problem can be defined as:

Minimize f(x) Subject to

$l_i \le x_i \le u_i \qquad i = 1, \ldots\ldots, 6$

$p_c(x) \le 5.5 \ (MPa)$

$m_p(x) \le 12 \ (kg)$

Where I is a vector containing the lower bounds and u is a vector containing the upper bounds on design variables.

After defining the design constraint and objective function, the following studies are done in order to find genetic algorithm parameters of the optimization tool giving the best solution. Considering the number of function evaluations and computation time, the maximum number of generation is taken as 40. Population number as 20 is use. The bit number is set to 8, which is sufficient for the precision of variables.

N_b	8	Selection Method	Roulette-Wheel
N_p	20	Cross –over Method	Two point
N_{gen}	40	Replacement Method	Best Alive
P_c	0.8		
P_m	0.03		

Table 3: Optimization parameters of grain

In the genetic algorithm, objective function is evaluated in the number of population size at each generation; therefore in this study, objective function is calculated 30 times at each generation.

Average and best objective function is evaluated in the number of population size at each generation of genetic algorithm is given in Figure 1. The objective function value of the found geometry after 40 generation 6.2% and propellant mass is reduced by 16.3% as compared to the actual rocket motor test results. Objective function of the actual grain geometry is calculated as 11.2% for the given objective thrust –time curve. Therefore, the genetic algorithm, having a better curve fit with the objective thrust-time curve than actual grain.

Parameters	Actual grain geometry	Genetic Algorithm
Grain Length, L/D_t	1	1
Outer Diameter, D_{out}/D_t	1	0.95
Web Thickness, w/w	1	1.2

Cone Radius,R_1/w	1	1.11
Bore radius,r_1/w	1	0.78
Nozzle Throat Diameter, D_t/D_t	1	0.95
Objective Function(F(x))	11.2	6.2
Propellant mass, $m_p/m_{p,axisym}$	1	0.8
Max. Chamber Pressure, $p_{max}/p_{max.axi\text{-}sym}$	1	1.2

Table 4: Non-Dimensional Results for the Grain

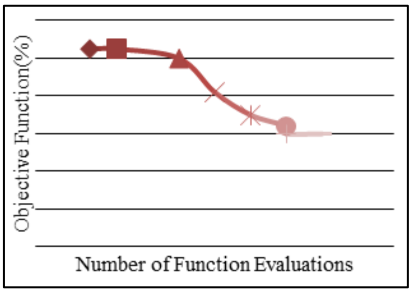


Fig. 1: Fitness value Evaluation of Genetic Algorithm

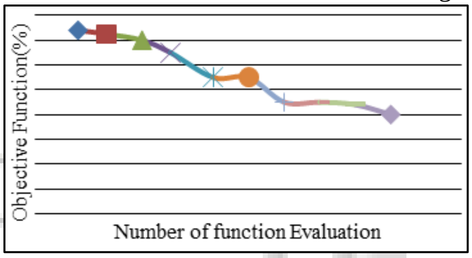
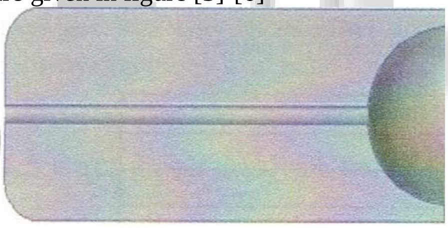
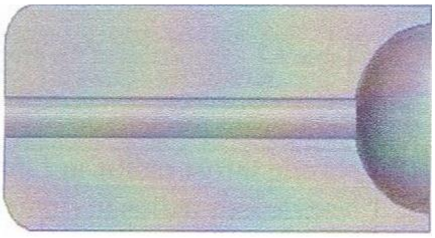


Fig. 2: Fitness Evaluation of actual grain

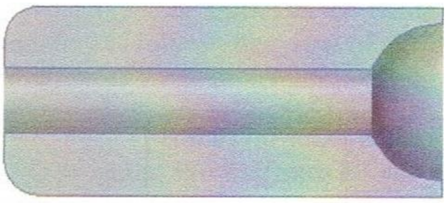
Optimized geometries of the grain during Burnback analysis are given in figure [3]-[6]



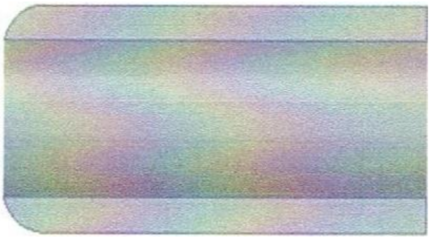
0 mm web increment



5 mm web increment



10 mm web increment



15 mm web increment

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