

# An Analytical Analysis of Mathematical Modeling & Simulation of Hydro Electric Plant: A Review

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**Abstract**— In the analysis paper, the thing to planning the tiny electricity plants are consistently analyzed and is addressed within the aim of enhancing the economic benefits of the investment. As even a little electricity station want an enormous capital investment, this reality signifies the analysis of the plant economic aspects. This text presents associate acceptable and optimized empirical system model that describing the hydro rotary engine potency. The plant model is intended by taking account of operational performance efficiency and construction characteristics like designing of machinery of the plant. Hydro Geographical Characteristics for the broader range of sites has been analyzed and also the plant model that will involve the vital physical parameters of the hydro plant has been developed. Drawback is developed because the mathematical programming problem, conjointly solved by exploitation programming techniques. The optimization do coated big selection of plant web site characteristics with three kinds of out there hydro plant rotary engine. The analysis article introduced associate empirical short-cut style technique for determination of optimum nominal flow-rate for the plant and estimation of expected per cost of electricity generated.

**Key words:** Small Hydro Plant, Modeling, Simulation

## I. INTRODUCTION

The term hydropower refers to the generation of shaft power from P.E. (Potential Energy) of water falling kind a high altitude. The facility might then be used for the direct mechanical functions or, additional often, for generating the electricity power. The Hydropower is that the most significant natural resources for electricity power generation in industrial investments. Although, the electricity generation is thought to be a mature technology, there are still prospects for improvement. Whereas some components of hydropower, like rotary engine potency and value, have reached associate extreme, identical cannot be aforesaid for the system itself. New rotary engine styles and transmission technology (which is of nice importance to hydropower development) with reference to regional characteristics, still advance. Moreover, environmental issues are driving changes in construction, design, improvement and operation of electricity plants [1, 2]. The look of reliable and value effective tiny hydropower plants capable of large-scale power production may be a requirement for the effective use of hydropower as another resource. During this sense, the look of a little electricity plant or equivalently the determination of sort and energy load of the actual hydro turbines ought to maximize the energy output beside the life-time of the machines. Altogether cases, the look objective is closely associated with the full annual output of the general hydro rotary engine operation in power terms. Obviously, given the facility curve of the hydro rotary engine to be used and regional flow period applied mathematics knowledge additionally because the topology of the

positioning, we will estimate the full annual energy output of the tiny electricity plant to be put in. during this case, each the sort and size of the hydro rotary engine, expressed in terms of its nominal got to be determined underneath an explicit economic setting. The determination of the optimum plant characteristics should be supported specific style objectives. During this case, the look drawback could also be developed as a mathematical programming drawback, involving associate objective perform representing the investment potency that is expressed by the profit expected per unit of capital endowed.

Mathematical Modeling may be a technical procedure for expressing in an exceedingly simple means the optimum results of an in depth style drawback through empirical equations involving the corresponding style variables. During this means, all different model variables area unit directly computed through the model equations. The parameters of the empirical equations area unit evaluated by road model equations to the corresponding style drawback optimum results computed mistreatment the total method model. Short-cut style is very imp measuring device for preliminary choice of other style state of affairs for pleased policies of investment at a regional level. During this means, short-cut evaluations of optimum styles sure sites area unit an essential tool for assessing regional coming up with methods at a national or international level. Moreover, it's very vital for crucial the means that energy sources might presumably penetrate the energy market by associate applicable grant policy. This work addresses the matter of tiny electricity plant road style in terms of increasing the economic edges of investment. Mathematical model of hydro turbines was developed taking into thought their performance with relevancy construction and operation. Associated empirical model was used for estimating the general rotary engine potency. The target perform to be maximized was the investment potency. The hydro geographical characteristics of a website are analyzed in terms of serious physical parameters and sculpturesque fittingly. Improvement was meted out for a large vary of site characteristics expressed by the corresponding model parameters and for 3 differing kinds of economic hydro turbines. Associated empirical short-cut model equation was introduced for crucial the optimum nominal of the hydro turbines. The regions of relevancy for all turbines concerned, make up my mind as a perform of model parameters. From the engineering purpose of read, such associated analysis can directly function an analysis tool for expressly crucial the immovableness of exploiting hydropower at an explicit region. All different model variables area unit directly computed through the model equations. The parameters of the empirical equations area unit evaluated by fitting the short-cut model equations to the corresponding style drawback optimum results computed mistreatment the total method model. Road style is very vital

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II. MATHEMATICAL MODELLING OF HYDRO TURBINES

The power obtained by a hydro rotary engine operated in an exceedingly little electricity plant is proportional to the mechanical energy lost by the falling fluid and is given by the subsequent equation [1]

$$P = \mu g Q H \tag{1}$$

The rotary engine potency, Z, concerned within the calculation of water potential regenerate energy through higher than expression, represents the particular utilization of the out there mechanical energy of the system. Normally, a hydro-turbine is intended in order that it may be operated for a good vary of operating around a nominal operative purpose. The performance of the rotary engine is characterized by its nominal flow-rate, Q<sub>r</sub>, that's a definite indication of its size. For a particular style of rotary engine, its size and capability that are directly analogous to its diameter are proportionately associated with its nominal flow-rate. Therefore, the rotary engine nominal flow-rate may be an appropriate variable for size the rotary engine and every one relevant instrumentality of the plant. The rotary engine potency depends on the operating fluid flow-rate and actual rotary engine characteristics. Experimental knowledge for the rotary engine potency, within the case of three business hydro turbines studied during this work (i.e. FRANCIS, PELTON and AXIAL), are given in Fig. 1, as a operate of the quantitative relation of operating flow-rate to its corresponding nominal flowrate and also the rotary engine potency at the nominal flow-rate, η<sub>r</sub> [10]. All curves exhibit a most at a flow-rate representing the nominal performance of the hydro-turbine. Associate empirical expression for representing the rotary engine potency graphical record is proposed:

$$\frac{\eta}{\eta_r} = a\{Q|Q_r\}^2 + b\{Q|Q_r\} + c \tag{2}$$

In this expression, the rotary engine characteristics are the nominal rotary engine potency, η<sub>r</sub>, the nominal rotary engine rate, Q<sub>r</sub>, and three parameters expressing the form of the curves. It should be noted that the nominal power of the rotary engine is given by the subsequent expression:

$$P_r = \eta_{r,g} Q_r H \tag{3}$$

Excellent fits to actual experimental knowledge were detected once expression (2) was used as real rotary engine knowledge. The predictions of the empirical equation projected for the case of rotary engine potency experimental values are given in Fig. 1, indicating the excellence of the field and suggesting the sensible significance of equivalent. (2). the calculable rotary engine parameters of eq. (2). The nominal rotary engine potency is freelance of the nominal flow-rate for all turbines examined, and its corresponding values are given in Table one. Every hydro rotary engine is built to work between two extremes, a minimum and a most operating rate of flow. We tend to introduce two rotary engine characteristic parameters q<sub>min</sub> and q<sub>max</sub>, representing the fraction of its nominal rate of flow appreciate the lower and higher extreme operating flow rates, severally. These values are enclosed in Table one for every one in every of the business turbines studied. The minimum and most operating flow rates for the rotary engine are consequently given by the subsequent equations (4 & 5):

$$Q_{min} = q_{min} Q_r \tag{4}$$

$$Q_{max} = q_{max} Q_r \tag{5}$$

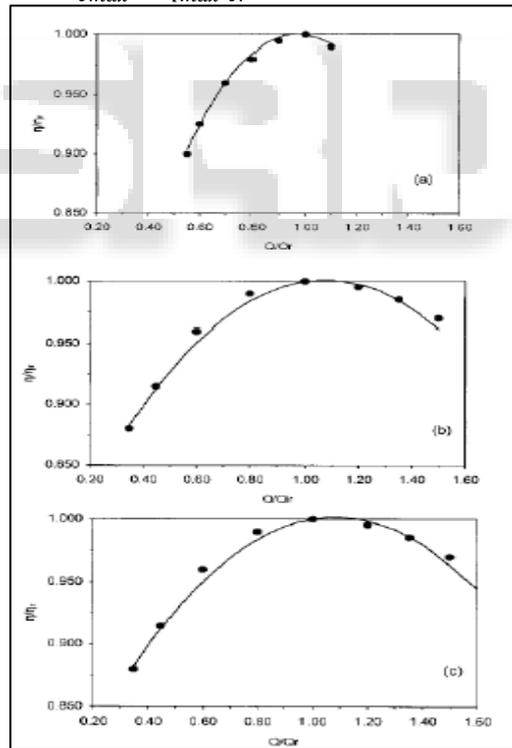


Fig. 1: Hydro Turbines Efficiency Curves

The water flow rate through the turbine, Q, is determined by the following relationship as a function of the available water flow rate, Q<sub>a</sub>.

$$Q = \begin{cases} 0 & Q' < Q_{min} \\ Q^* & Q_{min} < Q' < Q_{max} \\ Q_{min} & Q_{max} < Q' \end{cases} \tag{6}$$

The offered hydraulic head concerned in eq. (1) may be calculable by subtracting the friction losses through the penstock from the offered vertical fall of water [11]:

$$H - H_o \left[ 1 - \lambda \left( \frac{Q}{Q_r} \right)^2 \right] \quad (7)$$

The friction constant,  $\lambda$ , concerned in eq. (7) depends on the penstock size, configuration and topology of the region within the sense that the impact of piping network and also the dam layout is embodied during this parameter. Eqs. (1), (2) and (7) will currently be combined to precise the particular power regenerate as a function of rotary engine operating flowrate and accessible vertical fall of water. The nominal power of the plant is given by:

$$P_r = \eta_r g Q_r H_r \quad (8)$$

The potential of a stream is characterized by the accessible vertical fall ( $H_0$ ) and its flow that's sometimes expressed by the rate of flow period curve. The flow period curve provides the fundamental quantity throughout that the flow of the stream is larger than a particular worth (cumulative distribution of stream flow on associate annual basis). Natural watercourse flow is extremely variable, a characteristic that has vital implications for the planning of electricity plants and their incorporation into the electrical generation system. Most rivers exhibit pronounced seasonal variation in their flow. In some cases, the typical three month high flow could also be over 10 times bigger than the typical three month low flow, whereas in others, it's less than double. So as to supply a generalized model for predicting stream flow period curve the subsequent expression is suggested:

$$Q^* = \frac{Q_{max}^* + (Q_{min}^* K - Q_{min}^*)}{1 + (K-1)t} \quad (9)$$

Where notation  $K$  can be express as below:

$$K = \frac{Q_{max}^* - Q_{50}^*}{Q_{50}^* - Q_{min}^*}$$

Eq. (10) describes the rate of flow period curve of a stream by utilizing solely three parameters; the annual highest stream rate,  $Q_{max}$ , the annual lowest stream rate,  $Q_{min}$ , and also the stream rate such as the mid-year purpose of the flow period curve,  $Q$  at 50. All these characteristic parameters of the stream period curve together with the stream period curve itself are given in Fig. 2.

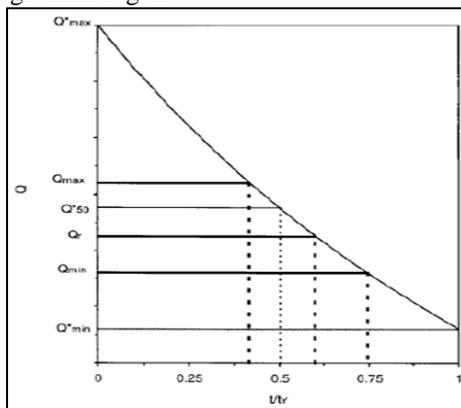


Fig. 2: Placement of Hydro Turbine on the site stream duration curve

The primary two parameters indicate the poles of the flow period curve, whereas the third one, its form (curvature). Clearly this parameter describes the sharpness of differences due to the season in stream rate. For example the method that a hydro rotary engine may be placed on the flow period curve, Fig. 2 conjointly includes the characteristic variables of a hydro rotary engine placed haphazardly. So as to explain a lot

of expeditiously the location characteristic parameters, we have a tendency to introduce the subsequent variables, expressing the corresponding rate period curve parameters as a fraction of the utmost annual rate worth of the stream:

$$q_{min}^* = \frac{Q_{min}^*}{Q_{max}^*} \quad (10)$$

$$q_{50}^* = \frac{Q_{50}^*}{Q_{max}^*} \quad (11)$$

The annual energy obtained by the operation of the hydroelectric plant is calculated by integrating Eq. (1) for the entire year:

$$E = \int_0^{365} P dt \quad (12)$$

The installation cost of the plant is given by the following equation as a function of the turbine nominal power and the vertical free fall of the site [10]:

$$C_{\varphi} = c_{\varphi} P_r^{\alpha} H_o^{\beta} \quad (13)$$

The operational cost of the plant is proportional to the installed plant capacity and is given by the following equation [10]:

$$C_T = e C_{CP} + t_{OP} C_{OP} \quad (14)$$

As a result the unit cost of energy produced is the ratio of total annual cost and annual energy recovered:

$$C_E = \frac{C_T}{E} \quad (15)$$

The expected profits from the operation of the hydroelectric plant is therefore given below:

$$S = E(C_E - C_E)$$

The investment efficiency is expressed as the ratio of expected profits per invested capital:

$$PI = \frac{S}{C_{CP}} \quad (16)$$

Equations On top of represent the mathematical model of the whole electricity plant. During this case, the planning objective is to maximize the investment potency from the operation of the plant. Given the sort of hydro rotary engine and site hydro geographical characteristics and topology, there's just one style variable to be computed by suggests that of increasing the target function; the nominal rate of the hydro rotary engine.

### III. SHORT CUT DESIGN OF HYDROELECTRIC PLANTS

On the idea of the on top of, the planning strategy for tiny electricity plants will currently be clearly expressed. Given the kind of hydro rotary engine and site hydro geographical characteristics (i.e. stream period curve parameters) and topology (i.e. offered vertical fall of water) the nominal rate of flow at that the hydro rotary engine ought to operate should be determined by means that of optimizing acceptable techno economic criteria underneath specific operational and environmental constraints.

As a consequence of the on top of, the determination of the optimum plant configuration should be supported specific style objectives. In apply, the illustration of the planning drawback for tiny electricity plants ought to focus to corresponding mathematical figures obtained through associate adequate mathematical model as antecedently

developed. Altogether cases, the planning procedure ought to involve associate objective operate representing the economic advantages from the operation of such a plant or its potency in terms of energy handiness towards regional demand. Bound various objective operate varieties could also be taken into thought concerning the advantages expected:

- Maximization of the investment potency. This case suits to style issues confronted by individual power manufacturing industries (either personal or municipal) that have invested with or conceive to invest during this field, in countries wherever legislation permits thus. In alternative words, this objective refers to the direct economic advantages expected from such associate investment underneath a particular competitive economic surroundings, therefore deciding the practicability of exploiting this sort of renewable energy supply.
- Maximization of the number of energy annually made from the offered hydro resources. This case suits to style issues sometimes confronted for regions wherever no alternative sources of energy are technically exploitable, and also the objective is to use the very best potential energy potential of a section so as to hide the local demand, presumptuous that the utilization of hydropower remains profitable compared to the value of electricity offered in remote national regions as a result of inflated transportation cost.

Throughout this paper we decide the primary risk for our objective operate. Characteristic economical figures regarding capital and operational price parts for a typical economic surroundings were taken into thought. Between these 2 cases, the previous evaluates associate optimum plant size that's fully completely different (smaller) than the latter. However, it may be shown that the optimum results of the field objective that's associate economically driven operate coincide with those of the second objective that's a strictly technical operate (independent of economics) once the cost of standard electricity approaches time. During this case the plant operates at the purpose of most energy recovery. The ability performance curve of the optimum plant size victimization

FRANCIS hydro rotary engine are going to be determined by maximization of the annual profits for the a typical web site involving a linear flow period curve delineated by most annual stream rate of 10,000 m<sup>3</sup>/h, zero level of minimum annual stream rate and offered vertical water fall of a hundred m, is given in Fig. 3. The ability curve is characterized by 2 completely different regions: a continuing power region that lasts for 257 days and a decreasing power region that lasts for the remaining 108 days of the year. The field region corresponds to days wherever stream rate is larger or adequate to the utmost hydro rotary engine rate. The second region corresponds to the world.

The power performance curve of the best plant size using FRANCIS hydro rotary engine, determined by the maximization of the annual profits for the a typical site involving a linear flow period curve delineate by most annual stream rate of flow of 10,000 m<sup>3</sup>/h, zero level of minimum annual stream rate of flow and accessible vertical water fall of 100 m, is given in Fig.. The facility curve is characterized by two completely different regions: a relentless power region that lasts for 257 days and a decreasing power region

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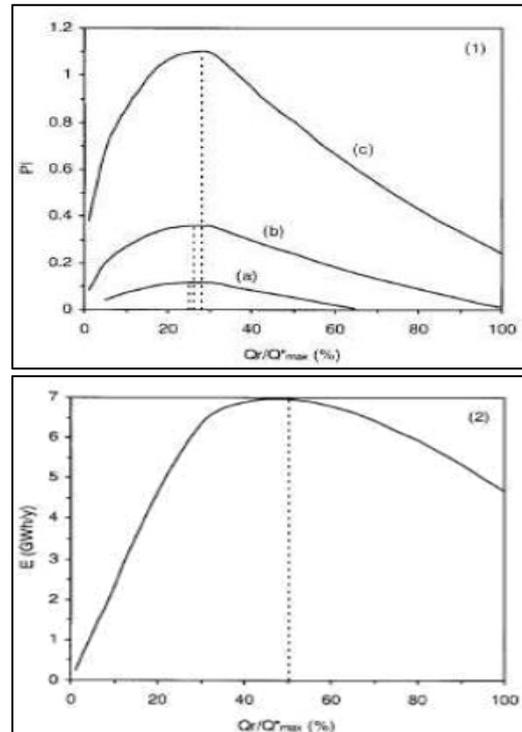


Fig. 3: Effect of Hydro Turbine nominal flow rate on process variables

The second region corresponds to days wherever the stream rate of flow is a smaller amount than the utmost hydro rotary engine rate of flow. During this case, the rotary engine operates for less than 55 days wherever the stream rate is larger or adequate to the minimum hydro rotary engine rate of flow. Obviously, for the remaining 53 days the plant is out of operation since the stream rate of flow is a smaller amount than the minimum hydro rotary engine rate of flow. The improvement procedure for the determination of the best rotary engine size delineate earlier was targeting the answer of a particular style downside that involving a predefined rotary engine kind and site hydro geographical characteristics. This procedure are often extended to incorporate a good vary of rotary engine varieties and stream particularities delineate by completely different values for the parameters of its flow length curve moreover as for various vertical water fall values. once the results of the improvement for every rotary engine and site combination square measure consistently compiled and given, associate empirical short-cut style equation are often assessed so the planning engineer will mechanically confirm the best size of every plant and afterwards evaluate its performance in terms of the recovered quantity of energy and also the cost of power created. A short-cut style empirical equation of the subsequent kind is proposed:

$$Q_r = \left[ \frac{7q_{90}^*}{1 + (7-1)q_{90}^*} \left( 1 - \frac{q_{\min}^*}{q_{\max}^*} + \frac{q_{\min}^*}{q_{\max}^*} \right) \right] Q_{\max}^* \quad (17)$$

It involves just one parameter, g, and expresses the optimum hydro rotary engine nominal rate of flow in terms of investment potency maximization. The determination of this short cut model parameter was administered by on top of

equation to the best results of the total style downside for all turbines studied. The values of  $g$  for all turbines are given in Table three. The short-cut empirical model to the best plant nominal rate of flow for all turbines studied are given in Fig. Obviously, all were extraordinarily satisfactory and might be safely used for short-cut style functions within the case of little electricity plant style Equation. Another Four Figure Graph doesn't embody any dependence of the optimum plant size on the accessible vertical fall. For the whole vary of sites studied, PELTON and AXIAL hydro turbines involve smaller optimum installation than the one evaluated for FRANCIS. All hydro geographical model parameters ( $Q_{max}$ ,  $Q_{min}$  and  $Q_{50}$ ) have positive result on the optimum price of the plant nominal rate of flow. Between the location hydro geographical parameters, the most annual stream rate of flow has the best impact on the plant nominal rate of flow, whereas the mid-year annual stream rate of flow has the tiniest one. Clearly, for sites with high hydro potential, larger hydro turbines ought to be utilized to totally exploit hydropower during this case. Figs. expressing the Equation at the essence of style of little electricity plants with capacities up to one hundred MW. Given the kind of hydro rotary engine used and also the regional hydro geographical and topological characteristics (expressed by applicable model parameters), the engineer will mechanically appraise the plant optimum nominal size, the corresponding total quantity of energy recovered and an inexpensive estimation of the full plant value and profits expected at a preliminary style level. At this stage of style, the short-cut style equation for little. Electricity plants created could be a tool of nice significance for practicableness studies on such investments. The result of variation of hand-picked model parameters on best method variables for the case of FRANCIS hydro rotary engine and also the typical site of Figure is given is shown. best nominal rate of flow is greatly tormented by the form of the stream rate of flow length curve, whereas as indicated on top of the accessible vertical fall of water, and has no result in the least. The cost of electricity created is greatly tormented by the accessible vertical fall of water, whereas the form of the stream rate of flow length curve has the tiniest result. Moreover, the accessible vertical fall of water has the best impact on the full energy recovered whereas the minimum annual stream rate of flow has rock bottom one. The short-cut style model equation parameter,  $g$ , is usually tormented by the economic setting assumed. Additional specially, no impact of capital rotary engine value was discovered on this parameter. On the contrary, the cost of standard electricity had a powerful positive result on that, indicating that larger hydro turbines are favored as standard electricity value will increase. This result is given in Figure. For the FRANCIS hydro rotary engine. The results are directly associated with those obtained from the case study of Fig. 3. So as to see the vary of relevance of every hydro rotary engine, all hydro turbines were directly compared for a good vary of model parameters expressing topology and hydro geographic. Solely 2 parameters might recognize between the varied kinds of hydro turbines. The vary of the appliance for all hydro turbines is given. Clearly FRANCIS hydro rotary engine is inferior to each other's whereas PELTON is most well-liked for virtually most typical sensible cases.

#### IV. CONCLUSION

The design of tiny electricity plants may be properly analyzed and addressed by means of optimizing the expected advantages from such an investment within the field of renewable energy exploitation. Optimization may be distributed by developing the mathematical model of the hydro turbines, taking into consideration their construction characteristics and operational performance. The rotary engine potency may be with success shapely by suggests that of associate empirical equation. The plant model should conjointly involve the regional characteristics in terms of hydro geographical and topological model parameters. The planning drawback may be developed as a mathematical programming one, and may be resolved mistreatment acceptable programming techniques. Associate empirical short-cut style equation describes optimum size of the plant for a good vary of website characteristics and every one commercially out there hydro turbines studied. During this case, the optimum nominal rate, the quantity of energy recovered and an affordable estimation of the plant value may be mechanically determined.

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