Second Order Moment in the PSF of an Amplitude Apodised Optical System

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Abstract— In the present paper, the Second order moment and Ripple factor of an optical system has been studied with an amplitude apodisation filter. It has been found that for a given percentage of light flux within the diffraction pattern, the value of the encircled radius increases gradually with apodisation parameter.

Key words: Fourier Optics, Apodisation, Second Order Moment

I. INTRODUCTION

The encircled energy factor measures the fraction of the total energy in the PSF, which lies within a circle of specified radius with its center at the origin of the co-ordinate system defining the focused Gaussian plane. It is well-known that the image of a point source of light obtained with a diffraction-limited system is not a point. There is a spread of light flux over a considerable region of space in the focused plane and the actual nature of this spread is controlled by various factors viz., the size and the shape of aperture, aberrations and the type of the non-uniformity of transmission. The spread of this light flux at and near the focus of an optical system can manifest in two important ways in decreasing the value of the central intensity in the Airy pattern and increasing the geometrical size of the ideal point image. Important of this was first realized by LOMMEL[1] who developed a mathematical theory of light distribution, in three-dimensions at and near the focus of an optical system with a circular aperture. LOMMEL[1] was also successful in verifying some of his theoretical results experimentally.

Subsequent to the works of LOMMEL[1], a few more studies were reported in this direction. Thus to mention a few of these studies, intensity distributions near the edges of the geometrical shadow were studied by STRUVE [2]. The effects of amount of defocus from the distribution of intensity at a point away from the Gaussian focal plane were considered by SCHWARTZCHILD [3]. The conclusions drawn from these studies, led RAYLEIGH [4] to point the important of the encircled energy factor as an image quality assessment parameter. Strongly enough, no interest was shown at all by any investigator on this topic for over a period of nearly four decades in spite of the important work by Rayleigh on encircled energy factor. We come across only one related work by MONDAL[5] during this long period of time, who had studied the pattern of the diffracted field at and away from the focus and established a few more general features of the far-field pattern of the diffraction-limited systems.

II. MATHEMATICAL EXPRESSION FOR SECOND ORDER MOMENT

This parameter for the image quality evaluation has been defined by ASAKURA, [6] as

$$\Delta = \int_0^\infty \frac{|A_1(0,z)|^2_\beta}{|A_1(0,z)|^2_A} z^3 \, dz \qquad (1)$$

KUSAKAWA [7] have obtained an alternative expression for the same in terms of the pupil function f(r) of the optical system and its first derivative f' (r). Thus,

$$\Delta = \frac{\int_0^1 \{ f^1 (r) \}^2 \, r \, dr}{[\int_0^1 f (r) \, r \, dr]^2}$$

For our sinusoidal filters, the above expression becomes,

$$\Delta = \frac{\int_0^1 [\frac{d}{dr} \{ \frac{1+\beta Cos\pi r^2}{1+\beta} \}]^2 \, r \, dr}{[\int_0^1 \{ \frac{1+\beta Cos\pi r^2}{1+\beta} \} r \, dr]^2} \qquad (2)$$

It is observed from the Fig.1 that smaller is the value of the Second Order Moment, higher is the light flux concentrations near the diffraction head and consequently, image quality is better than that when its value is more. The second order moment is, therefore, an indirect measure of the flux concentration in and around the central region of the diffraction pattern. The computed values of the second order moment of various values of the apodisation parameter have been tabulated in the table 1. There is, a remarkable departure from normal behavior for β = 1.00, when the Second Order Moment progressively increases to very high values at an extra-ordinary faster rate.

Δ	SOM
0.0	0.000000
0.1	0.007291
0.2	0.032493
0.3	0.081964
0.4	0.164497
0.5	0.292439
0.6	0.483419
0.7	0.763109
0.8	1.16975
0.9	1.76188
1.0	2.63195

Table 1: Values of SOM for various β

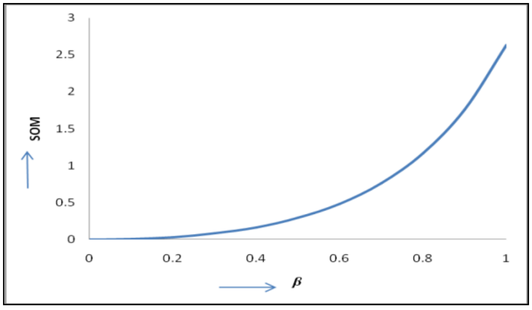

Fig. 1: Variation of SOM with β.

III. RIPPLE FACTOR

The term ‘Ripple Factor’ (RF) has been borrowed from the domain of digital signal processing where it implies.

$$\boxed{\text{RF} = \frac{[\text{Maximum side-lobe amplitude}] \times 100\%}{\text{Main-lobe amplitude}}} \qquad (3)$$

The most important objective of employing an amplitude apodisation filter in optical systems is to suppress, to the extent possible, the secondary side-lobes in a normal diffraction pattern. It is, therefore, very important that the performance of an apodiser should be assessed in terms of the relative peak intensity of the first secondary maximum just outside the central Airy spot. In that contest an estimate of the RF for the various values of the apodisation parameter should be considered as a very useful figure of merit for the evaluation of image quality of an optical system operating with a given specific class of amplitude apodisation filters. With the above consideration in mind, we have tabulated the values of RF in table- 2.

It is observed from the table that for a particular value of y, the value of RF decrease as the value of β is decreased from β = 0 to 1.0. Beyond this value of β =1.0, the RF value is again found to increase indicating thereby the optimum value of the apodisation parameter β for the purpose of suppressing the first secondary maximum to the extent possible, must be in the neighborhood of 1.0. A very insignificant minor departure from the above value of β may be noticeable in a very small number of cases of high value of the defocusing parameter y. It is, therefore, concluded that information of RF value is very helpful in understanding the amount of separation of the first diffraction maximum.

y	β=1.0	β=0.9	β=0.8	β=0.7	β=0.6	β=0.5	β=0.4	β=0.3	β=0.2	β=0.1	β=0.0
0.000	0.320	0.371	0.441	0.548	0.711	0.880	1.000	1.163	1.250	1.609	1.700
0.314	5.800	5.987	6.607	7.632	8.409	9.400	10.000	10.923	11.600	12.761	13.213
0.628	6.000	6.207	6.970	7.895	8.716	9.600	10.316	11.077	11.733	22.093	13.353
0.942	6.143	6.618	7.154	8.098	9.131	9.869	10.538	11.281	12.297	12.791	14.127
1.256	6.439	6.927	7.620	8.563	9.537	10.348	11.045	11.965	12.714	13.512	14.213
1.570	6.950	7.617	8.271	9.366	10.029	10.785	11.693	12.577	13.446	14.185	14.830
1.884	7.835	8.348	9.272	10.009	10.846	11.835	12.683	13.481	14.280	14.950	15.596
2.198	2.521	2.755	3.327	3.787	11.917	12.758	13.771	14.477	15.198	15.794	16.375
2.512	2.648	3.021	3.465	4.058	4.731	14.008	14.666	15.550	16.189	16.704	17.214
2.826	2.792	3.207	3.739	4.389	5.048	5.603	15.877	16.686	17.237	17.890	18.299
3.140	3.080	3.413	3.922	4.631	5.272	5.942	6.398	18.168	18.565	19.116	19.650
3.454	3.263	3.779	4.319	5.043	5.662	6.321	6.889	7.269	20.396	20.830	21.252
3.768	1.425	1.702	4.803	5.506	6.099	6.743	7.294	7.784	22.304	22.605	22.902
4.082	1.549	1.840	5.162	5.841	6.585	7.210	7.739	8.208	8.656	24.433	24.589
4.396	1.621	1.937	2.482	6.395	7.123	7.724	8.227	8.870	9.346	9.846	26.633
4.710	1.798	2.136	2.683	3.150	7.748	8.561	9.064	9.705	10.133	10.594	11.132
5.024	1.905	2.362	2.911	3.371	8.545	9.348	9.806	10.422	11.000	11.587	11.913
5.338	2.122	2.506	3.046	3.626	4.201	10.201	10.858	11.424	11.953	12.497	12.932
5.652	0.000	2.782	3.320	3.950	4.666	11.122	12.036	12.534	13.248	13.711	14.058
5.966	0.000	1.461	3.645	4.314	5.038	5.636	13.014	13.763	14.425	14.809	15.300
6.280	0.000	0.000	4.051	4.722	5.453	6.044	6.689	15.124	15.714	16.291	16.671

Table 2: Values of RF for various y and β

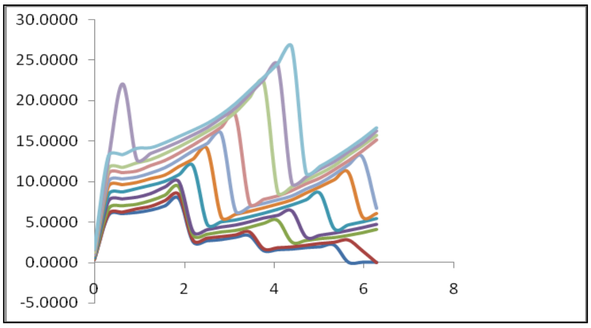

Fig. 2: Variations of RF with β.

IV. RESULTS AND DISCUSSIONS

The Second Order Moment progressively increases to high values at an extra-ordinary faster rate with the increase in the value of β.

The value of Ripple Factor is very high for β = 0 i.e., for diffraction limited system. For non-zero values of β,0.25 ≤β≤1.00, the values of the Ripple Factor are very low and within these low range of values it decreases at a slow rate.

For the resolution of Binary Stars with unequal intensities and very close to each other like, for example, the Binary star SIRIUS and its companion, the application of the Rayleigh criterion of resolution is totally invalid, because of the involvement of a large number of variable parameters.

REFERENCES

[1] LOMMEL, E., Abh.BayerAkad., vol.15, Abth.2, 1885.
[2] STRUVE, H., mem. Akad.Sci., st.peterburg, vol.34, 1886.
[3] SCHWARTZSCHILD, K. (1898), Sitzb. MunchenAlcad. Wiss.Math.Phys. K1.28, 271.
[4] RAYALU, Ch. A.D., and MONDAL, P.K. (1977 a), Atti. Fond. G. Ranchi, 32, 597.
[5] P.K.Mondal, “Fourier Analytical Treatment of Optical Images a review” Ind.Jour.Opti. (2009).
[6] ASAKURA, T., and UENO, T.(1976), Nouv. Rev. Opt., 7, 199.
[7] KUSAKAWA, T. (1971), Jap. J. Appl. Phys., 10, 320.