A Parametric Study on Effect of Confining Pressure on Stress-Strain Relation in Concrete

Asadullah¹ M. G. Shaikh²
¹Research Student ²Associate Professor
¹,²Department of Applied Mechanics
¹,²Government Engineering College Aurangabad

Abstract—In structural concrete, it is a challenge to determine the exact physical properties and behavior under various stress combination. The concrete, heterogeneous in nature, shows not only elastic deformation but also inelastic and time dependent deformation. Different constitutive models proposed in literature are reviewed in this paper. Different loadings for unconfined and confined concrete are studied. It is seen that in the descending branches of stress-strain curve, the mixes without silica fume and three with silica fume strengths varied between 60 to 130 MPa. An analytical model for the stress-strain curve but experimental results shows that effect of time element must be taken into consideration for calculating failure load. A realistic continuous function for the relation of stress and strain in flexure, based upon the concrete properties $f_c$, $\varepsilon_0$ and $\varepsilon_u$, which should apply to a wider range of concrete properties than those approaches based primarily on $f_c$. [2]

The objective of the study is to draw the stress-strain relation of both unconfined and confined concretes by using empirical relation based on the compressive strength and to study the elastoplastic behavior of concrete under different loading condition.

I. INTRODUCTION

Concrete is a structural material that is used widely in many major construction projects such as tall building, offshore platforms, reactor vessels and nuclear containment structures.

The application of finite element analysis on these structures has become increasingly important because it is not possible to obtain the deformational and failure behavior by conventional procedures, and experimental studied on these structures are very expensive. The rapid development in computer technology and numerical techniques has provided structural engineering with a powerful tool in the form of finite element method for the analysis of concrete structures. In recent years good progress has been achieved in the area of constitutive modeling of concrete materials. Various predictive models have been proposed and used for concrete materials. A common approach to describe concrete behavior is based on the principals of continuum mechanics, neglecting the microstructure of concrete materials. During loading concrete suffers not only elastic deformations, but also inelastic and time-dependent deformations caused by microstructural changes. This inelastic deformation is primarily due to microcracking and internal friction sliding. Many of the micro cracks in concrete are caused by segregation, shrinkage, or thermal expansion in the mortar and therefore exist even before any load has been applied. Some of the micro cracks can be developed during loading because of the differences in stiffness between aggregates and mortar. Therefore, the aggregate-mortar interface constitutes the weakest link in the composite system. This is the primary reason for the low tensile strength of concrete materials. The purpose of this research work is to summarize some of the key facets of the experimental behavior of plain concrete. This is essential in the generalized development of various constitutive models for concrete. Desayi and Krishnan [1] simplified equation proves that initial tangent modulus is twice the secant modulus at maximum stress. Ref.(1) also does not take into account the effect of rate of stressing (rate of straining) on the stress-strain curve but experimental results shows that effect of time element must be taken into consideration for calculating failure load. A realistic continuous function for the relation of stress and strain in flexure, based upon the concrete properties $f_c$, $\varepsilon_0$ and $\varepsilon_u$, which should apply to a wider range of concrete properties than those approaches based primarily on $f_c$. [2]

II. LITERATURE SURVEY

Stress-strain relations proposed by various researchers are given below:

A. Desayi and Krishnan [1]

A simple equation is proposed for the stress-strain curve of concrete in compression. The equation is found to represent it well not only up to the maximum stress but also beyond, and may conveniently be adopted in the computation of ultimate resisting moment of reinforced concrete sections.

$$f = \frac{E \varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2}$$  (1)

Where $f$, $\varepsilon$ are stress and strain tensor $E$ = a constant (same as initial tangent modulus) such that $E = \frac{2f_0}{\varepsilon_0}$ $\varepsilon_0$ = maximum strain at failure

B. Smith and Young [2]

Describes a stress block for ultimate load analysis based upon the stress strain relation of 6 x 12-in. cylinders. The stress-strain relation from a cylinder, which includes a decrease in stress beyond the ultimate, is described by a single continuous function. The function is used to compute the total compressive force in the compression zone, position of neutral axis, and ultimate moment.

$$f = f_c \left[1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2\right]$$  (2)

$f_c$ = compressive strength of 6 x 12-in. concrete cylinder.

$\varepsilon_0$ = variable strain in the compression block. $\varepsilon_u$ = concrete strain corresponding to $f_c$ as determined from test of cylinder.

$f$ = variable stress in stress block.

C. Attard and Setunge [3]

Experimentally determined full stress-strain curves from standard triaxial tests are presented. Confining pressures of between 1 to 20 MPa (145 to 2900 psi) were applied. Five mixes were studied using three types of aggregates, two of the mixes without silica fume and three with silica fume. The uniaxial compressive strengths varied between 60 to 130 MPa. An analytical model for the full stress-strain
relationship for confined and uniaxially loaded concrete is empirically developed and is shown to be applicable to a broad range of concrete strengths between 20 and 130 MPa.

\[ Y = AX + BX^2 + CXY + DX^2 \]  

(3)

Where A, B, C and D are material constant

X, Y refers to stress and strain non-dimensional with respect to the corresponding values at peak stress.

D. Carreira and Chu [4]

A general form of the serpentine curve is proposed to represent the complete stress-strain relationship of plain concrete in compression. The parameters that define the relationship are physically significant and can be estimated from empirical relationships or determined experimentally. Proposed equations fit a wide range of testing conditions and concretes for both the ascending and descending branches of the stress-strain diagram in compression

\[ \frac{f_c}{f'_c} = \frac{\beta (\frac{\varepsilon_c}{f_c})}{\beta - 1 + (\frac{\varepsilon_c}{f_c})} \]  

(4)

Where stress corresponding to the strain \( \varepsilon_c \), \( f_c \) point of maximum stress, \( \varepsilon_c \) strain corresponding to maximum stress \( f_c \), \( \beta \) is a parameter depends on the shape of the stress-strain diagram.

E. Carreira and Chu [5]

A stress-strain relationship represents the overall behavior of reinforced concrete in tension, which includes the combined effects of cracking and slippage at cracks along with the reinforcement is proposed. The serpentine curve previously used for the compression stress-strain relationship is also used in tension with parameters that are physically significant. These parameters can be determined experimentally from reinforced concrete prismatic specimens or estimated from proposed empirical relationships

\[ f_t = \frac{\beta (\frac{\varepsilon_t}{f_t})}{\beta - 1 + (\frac{\varepsilon_t}{f_t})} \]  

(5)

\( f_t \) is the stress corresponding to the strain \( \varepsilon_t \), \( f_t \) = the strain corresponding to the maximum stress \( f_t \), \( \beta \) a parameter that depends on the shape of the stress-strain diagram

F. Gribniak and el. at [6]

Research was dedicated to investigation of finite element size effect on deformational behavior of reinforced concrete members. Post-cracking behavior of reinforced concrete beams assuming different finite element sizes has been investigated by commercial finite element software ATENA. Softening behavior of tensile concrete has been modelled by three different approaches based on: a stress-crack width relationship with constant fracture energy; a stress-strain relationship with a constant ultimate strain; a stress-strain relationship with an ultimate strain adjusted according to the finite element size.

\[ \varepsilon_c^\sigma = \frac{\sigma_c^\sigma}{E_c} \]  

(6)

Where \( E_c \) is the secant modulus of concrete; \( \sigma_c^\sigma \) is the principal stress in concrete

G. Gerstle [7]

An octahedral representation of the multiaxial stress-strain relations for concrete assuming isotropic, nonlinear behavior is outlined in which concrete behavior is represented by variable tangent bulk and shear moduli. A simplified formulation of the volumetric and deviatoric relations is suggested for biaxial stress states, leading to linearly varying tangent moduli, depending only on readily available strength and stiffness characteristics of concrete

\[ t_0 = t_0(1 - e^{-(2G_0/t_0)}) \]  

(7)

In which \( t_0 \) is the octahedral shear strength, and \( G_0 \) is the initial shear modulus. Differentiating, we obtain the tangent shear modulus

\[ 2G = \frac{t_0}{\gamma_0} = 2G_0e^{-(2G_0/t_0)} \gamma_0 \]

\[ G = G_0 \left( 1 - \frac{t_0}{t_0} \right) \]

H. Belarbi and Hus [8]

Constitutive laws of concrete in tension and reinforcing bars stiffened by concrete are required in the formulation of the softened truss model theory for predicting the in-plane behavior of reinforced concrete membrane elements. These two constitutive laws were determined by testing 17 reinforced concrete panels under pure tension in a universal panel tester. The test panels were reinforced by deformed steel bars in the direction of the applied tensile stresses. Based on the test results, analytical expressions were derived for these two stress-strain relationships. It was shown that the concrete develops substantial tensile stresses even after extensive cracking. Expressions are given relating the average principal tensile stress in the concrete to the average principal tensile strain of the panel.

\[ \sigma_t = \frac{E_c}{\sqrt{f_c}} \]  

(8)

The average value of \( E_c \) was also found to be related to the cylinder compressive strength \( f_c \) as

\[ E_c = 47,000\sqrt{f_c} (\text{psi}) \]

III. System development

In this paper standard test results are taken for normal-loading, triaxial determined stress-strain relationships for laterally confined high strength concrete. Various mix parameters are studied with the uniaxial compressive strengths varies between 60 and 130 MPa. Mixes using three different crushed aggregates with and without silica fume are investigated. The triaxial experimental work was limited to low confining pressures between 1 and 20 MPa. Using available experimental data on normal-strength and high-strength concretes, an analytical model for the full stress-strain relationship for confined and uniaxial compression is proposed using mathematical model software.

Five concrete mixes a, b, c, d, and e were used. The concrete mix proportions are taken for Mixes a, b, c, and e having water-binder ratios of 0.26, 0.3, 0.35, and 0.45 respectively. Mixes a, b, and c with silica fume equivalent to 8 percent of the binder content, while Mix d has the same water-binder ratio without using silica fume. Mix e having normal-strength concrete with a design compressive strength of 45 MPa. A super plasticizer was used to obtain the required workability in all mixes except for normal concrete.

<table>
<thead>
<tr>
<th>Mixes</th>
<th>ID</th>
<th>Compressive strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>AB</td>
<td>109</td>
</tr>
<tr>
<td>AR</td>
<td>105</td>
<td></td>
</tr>
<tr>
<td>AH</td>
<td>111</td>
<td></td>
</tr>
</tbody>
</table>

All rights reserved by www.ijsrd.com
A Parametric Study on Effect of Confining Pressure on Stress-Strain Relation in Concrete (IJSRD/Vol. 4/Issue 05/2016/244)

Table 1: Concrete mix properties

<table>
<thead>
<tr>
<th>B</th>
<th>BR</th>
<th>93</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>BH</td>
<td>96</td>
</tr>
<tr>
<td>C</td>
<td>CR</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>CH</td>
<td>84</td>
</tr>
<tr>
<td>D</td>
<td>DB</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>DR</td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>DH</td>
<td>89</td>
</tr>
<tr>
<td>E</td>
<td>EH</td>
<td>45</td>
</tr>
</tbody>
</table>

Standard cylindrical specimens are taken 100 mm (3.94 in.) in diameter by 200 mm (7.87 in.) in height were prepared from each mix according to AS 1012. Akroyd [9] reported that at relatively low confining pressures the effect of pore water pressure on the triaxial test results of concrete is negligible. Akroyd [9] recommended the use of dry specimens to obtain uniform test results.

Specimens of all the mixes were tested at three confining pressures: 5, 10, and 15 MPa. Specimens of Mix cH were also tested at 5 and 15 MPa. Most of the specimens were tested at 90 days after casting. The Model was mounted to the loading platens of a 5000-kN testing machine with the compressive load applied between two spherically seated loading blocks.

The experimental stress-strain curves from the triaxial tests are given in Attard and Setunge [3] Fig. 1 through 3 show the test results for Mixes a, b, and c. The failure generally be shear failure Table 3 gives the values of the confined peak stress $f_c$, strain at the confined stress (peak) $\varepsilon_n$, elastic modulus $E_c$, uniaxial compressive strength $f_c$, and the uniaxial strain at the peak stress $\varepsilon_c$.

### Table 2: Peak stress and strain under triaxial compression

<table>
<thead>
<tr>
<th>Mix</th>
<th>$f_c$, Mpa</th>
<th>$f_0$, Mpa</th>
<th>$f_1$, Mpa</th>
<th>$\varepsilon_0$</th>
<th>$\varepsilon_c$</th>
<th>$E_c$, Mpa</th>
</tr>
</thead>
<tbody>
<tr>
<td>aR</td>
<td>120</td>
<td>125</td>
<td>0.5</td>
<td>0.0026</td>
<td>0.0030</td>
<td>55,700</td>
</tr>
<tr>
<td></td>
<td></td>
<td>128</td>
<td>1.0</td>
<td>0.0029</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>165</td>
<td>5.0</td>
<td>0.0038</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>192</td>
<td>10</td>
<td>0.0053</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>220</td>
<td>15</td>
<td>0.0060</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>234</td>
<td>20</td>
<td>0.0080</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bR</td>
<td>120</td>
<td>168</td>
<td>5.0</td>
<td>0.0042</td>
<td>0.0028</td>
<td>52,800</td>
</tr>
<tr>
<td></td>
<td></td>
<td>187</td>
<td>10</td>
<td>0.0048</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>211</td>
<td>15</td>
<td>0.0057</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cR</td>
<td>110</td>
<td>150</td>
<td>5</td>
<td>0.0035</td>
<td>0.0028</td>
<td>55,400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>175</td>
<td>10</td>
<td>0.0044</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>192</td>
<td>15</td>
<td>0.0060</td>
<td></td>
<td></td>
</tr>
<tr>
<td>aB</td>
<td>132</td>
<td>180</td>
<td>5</td>
<td>0.0050</td>
<td>0.0034</td>
<td>49,300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>200</td>
<td>10</td>
<td>0.0058</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>222</td>
<td>15</td>
<td>0.0078</td>
<td></td>
<td></td>
</tr>
<tr>
<td>aH</td>
<td>118</td>
<td>154</td>
<td>5</td>
<td>0.0038</td>
<td>0.0028</td>
<td>49,400</td>
</tr>
<tr>
<td></td>
<td></td>
<td>173</td>
<td>10</td>
<td>0.0049</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>201</td>
<td>15</td>
<td>0.0062</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bH</td>
<td>110</td>
<td>153</td>
<td>5</td>
<td>0.0041</td>
<td>0.0028</td>
<td>57,800</td>
</tr>
<tr>
<td></td>
<td></td>
<td>164</td>
<td>10</td>
<td>0.0055</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>185</td>
<td>15</td>
<td>0.0059</td>
<td></td>
<td></td>
</tr>
<tr>
<td>cH</td>
<td>100</td>
<td>127</td>
<td>5</td>
<td>0.0039</td>
<td>0.0026</td>
<td>54,600</td>
</tr>
<tr>
<td></td>
<td></td>
<td>153</td>
<td>10</td>
<td>0.0052</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>169</td>
<td>15</td>
<td>0.0075</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Fig. 1: Comparison of model predictions against experimental results for Mix bR for confining pressures of 5, 10, and 15 MPa. [3]

Fig. 2: Comparison of model predictions against experimental results for Mix aH for confining pressures of 5, 10, and 15 MPa. [3]

Fig. 3: Comparison of model predictions against experimental results for Mix aR for confining pressures of 1, 5, 10, 15, and 20 MPa. [3]

All the stress-strain curves basically followed a similar pattern as for the confined and unconfined concrete as shown in Fig. 4. The initial linear ascending portion was followed by curvature up to the peak stress. For the same mix, the initial tangent modulus was approximately the same for all confining pressures with increasing in ascending direction the peak strength increased with increasing confining pressure. The descending curve becomes soft with increasing confining pressure and Inflection point is achieved at about 80 percent of the peak stress. Under reasonable confinement, the descending curve tended to reach a residual stress level at high strains. Presence of residual pressure shows he load carried by friction across the cracked shear plane.
A Parametric Study on Effect of Confining Pressure on Stress-Strain Relation in Concrete (IJSRD/Vol. 4/Issue 05/2016/244)

A. Stress–Strain Relationship Model

Sargin [10] suggested the following non dimensional mathematical form for the stress-strain curve of concrete applicable to the confined and uniaxial case.

\[ Y = AX + BX^2 \]

\[ 1 + CX + DX^2 \]

\[ Y = \frac{f}{f_0}, X = \frac{\varepsilon}{\varepsilon_0} \]

In the previous equation, \( f \) is the stress at strain \( \varepsilon \), while \( f_0 \) is the peak stress at strain \( \varepsilon_0 \). Two sets of the constants \( A, B, C, \) and \( D \) are required, one for the ascending portion and a second for the descending portion of the curve. The constants are determined from the different boundary conditions applied to the ascending and descending curves.

\[ A = \frac{E_t \varepsilon_0}{f_0} \]

\[ B = (A - 1)^2 + \frac{A^2(1 - \alpha)}{\alpha} \cdot \frac{f_{pl}}{f_0} \cdot \frac{1 - f_{pl}}{f_0} \]

\[ C = (A - 2) \]

\[ D = (B + 1) \]

in which \( E_t \) is the initial tangent modulus at zero stress and \( E_c \) is the secant modulus measured at a stress of \( f_{pl} \) (usually 0.45\( f_c \)). The initial tangent modulus can be assumed to vary linearly between 1.17\( E_c \) and \( E_c \) for 20 and 100 MPa concretes, respectively. Where \( \alpha = E_t/E_c \).

IV. RESULTS AND DISCUSSION

To validate the performance of the model (experimental) employed, experimental results are compared with proposed model (Attard et al 1996) for the specimen of aR, aB, aH, bR, bH, cR, and cH under triaxial compression is shown in fig.

Above fig. shows proposed model Attard et al 1996 for mix aR is higher than experimental result Exp.fr 20 triaxial test model. As the proposed model goes softer after peak stress point, it shows ductility increases.

![Fig. 6](image6.png)

Fig. 6: Comparison of model predictions against experimental for Mix aR120 for confining pressures of 5, 10, and 15 MPa.

Nature of Mix aB remains same as mix aR i.e. ductility goes on increases after peak stress point but other models like expfr results for 5, 10 and 15 shows serpentine nature after peak stress.

![Fig. 7](image7.png)

Fig. 7: Comparison of model predicted against experimental for Mix aH118 for confining pressures of 5, 10, and 15 MPa.

In this case, the proposed model as well as expfr15 shows same nature but the proposed model is higher results. i.e 36 % than expfr15.

![Fig. 8](image8.png)

Fig. 8: Comparison of model prediction against experimental for Mix bR 120 for confining pressures of 5, 10, and 15 MPa.
A Parametric Study on Effect of Confining Pressure on Stress-Strain Relation in Concrete
(IJSRD/Vol. 4/Issue 05/2016/244)

Fig. 9: Comparison of model predicted against experimental for Mix bH110 for confining pressures of 5, 10, and 15 MPa.

Nature of Mix bH remains same as mix aR i.e. ductility goes on increases after peak stress point but other models like expfr results for 5 and 10 shows serpentine nature after peak stress.

Fig. 10: Comparison of model predicted against experimental for Mix cR110 for confining pressures of 5, 10, and 15 MPa.

In this case, the proposed model as well as expfr15 shows same nature but the proposed model is higher results i.e 10% than expfr15

V. CONCLUSION

The proposed stress-strain relation of concrete well represents the behavior of both normal strength and high strength concrete in their unconfined and confined state.

The stress-strain relation is controlled by a few controlling parameter and empirical expression for these parameters based on $f'_c$ (compressive strength) are derived so that these relations be used in the absence of accurate experimental results.

To form stress-strain relationship for an analytical model the parameter required peak stress, elastic modulus, strain at peak stress and stress and strain at inflection point for uniaxial compressive strength on descending portion. To check the level of confinement, stress-strain at any point on descending curve is needed. It is assumed that inflection point coordinate calculated by extrapolation the triaxial results can better represented.

Based extra pollution inflection points coordinate, descending curve less brittle than by stander uniaxial compressive test.

REFERENCES