Modelling and Analysis of Torsional Vibration Characteristics of Mechanical Gearbox using FEA

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Abstract— In this paper a mathematical model of mechanical gearbox is developed for analysis of torsional vibration characteristics. In this work Eigen value method is used to find the natural frequencies for mechanical gearbox system. The gear system converted into an equivalent system by taking different gear ratios. The various mode shapes are also shown to illustrate the state of the equivalent system at natural frequencies. The model is created in solidworks software & then analyse using finite element analysis or finite element method. Then the result was compared to find the suitable conclusion.

Key words: Mode Shape, Finite Element Method, Torsional Vibration, Natural Frequency, Eigen Value

I. Introduction

There are many models for gear dynamics in literature. The objects of these models vary from noise control to stability analysis. Mathematical models developed for the dynamic analysis of gears range from simple single degree-of freedom (SDOF) models to non-linear rotor dynamic models. Then the results will be compared with the FEA results for verification.

Most of the machines used in industries are rotary in nature and are subjected to torsional vibrations. Torsional vibration are more dangerous and also harder to detect .These vibration can damage the machine. The amplitude of vibration can increase significantly if the system speed is close to natural frequency of the system hence it is important to find out natural frequencies of the system. Due to excessive strain at speeds near the natural frequencies causes excessive stress which causes component failures. It can also lead to premature fatigue failures. Rotating systems with rotors or gears are common in engineering thus finite element method are commonly used to detect the natural frequency of the machine. [3]

II. Finite Element Method (FEM)-

If the inertial of the shafts connecting the rotors is neglected, then the finite element method reduces to representation of the equations of motion for rotor in the form of an Eigen value problem.

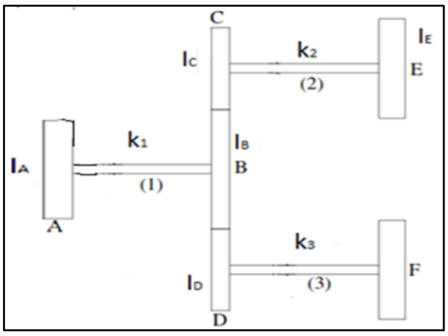

Fig. 1: Gearbox System[4]

These systems can be reduced to an equivalent form with one to one gear by multiplying all the inertias and stiffness of the branches by the squares of their speed ratios.

A. Mathematical Model of the System

In this branched power system we have six gears and three shafts of different parameters. Here we have to find the natural frequency of this system.

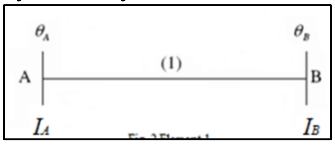

Fig. 2: Element 1

Equation of motion of element (1)

$$\therefore \begin{bmatrix} I_A & 0 \\ 0 & I_B \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_A \\ \ddot{\theta}_B \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \end{Bmatrix} = \begin{Bmatrix} -T_A \\ T_B \end{Bmatrix} \quad 3.1$$

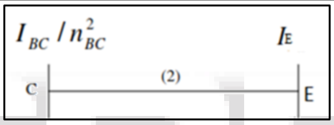

Fig. 3: Element 2

$$\begin{bmatrix} I_C/n_{BC}^2 & 0 \\ 0 & I_E \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_B \\ \ddot{\theta}_E \end{Bmatrix} + \begin{bmatrix} k_2/n_{BC}^2 & k_2/n_{BC} \\ k_2/n_{BC} & k_2 \end{bmatrix} \begin{Bmatrix} \theta_B \\ \theta_E \end{Bmatrix} = \begin{Bmatrix} -T_C/n_{BC} \\ T_E \end{Bmatrix} \quad 3.2$$

Gear ratio is defined as

$$nBC = \omega B/ \omega C; \qquad \theta C = - \theta B / nBC$$

To find stiffness matrix we will consider Potential energy equation.

$$U = 1/2\ k2(\theta E - \theta C)2 = 1/2\ k2\ (\theta E + \theta B / nBC)\ 2$$

$$U\ \theta B = (k2/ n2BC)\ \theta B + (k2/ nBC)\ \theta E;$$

$$U\ \theta E = (k2/ nBC)\ \theta B + k2\ \theta E$$

$$\therefore k_2' = \begin{bmatrix} k_2/n_{BC}^2 & k_2/n_{BC} \\ k_2/n_{BC} & k_2 \end{bmatrix}$$

Since

TBC = - TC / nBC

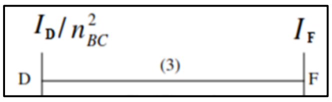

Fig. 4: Element 3

Similarly for this element, by considering Potential energy.

$$\begin{bmatrix} I_D/n_{Bd}^2 & 0 \\ 0 & I_F \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_B \\ \ddot{\theta}_F \end{Bmatrix}$$

$$\begin{bmatrix} k_3/n_{BD}^2 & k_2/n_{BD} \\ k_3/n_{BD} & k_3 \end{bmatrix} \begin{Bmatrix} \theta_B \\ \theta_F \end{Bmatrix} \begin{Bmatrix} -T_D/n_{BD} \\ T_F \end{Bmatrix}$$

Since

TB - TC / nBC - TD / nBC = 0

∴The assembled equation of motion of complete system is given by

$$\begin{bmatrix} I_A & 0 & 0 & 0 \\ 0 & (I_B+I_C/n_{BC}^2+I_D/n_{BD}^2) & 0 & 0 \\ 0 & 0 & I_E & 0 \\ 0 & 0 & 0 & I_F \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_A \\ \ddot{\theta}_B \\ \ddot{\theta}_E \\ \ddot{\theta}_F \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2/n_{BC}^2+k_3/n_{BD}^2 & k_2/n_{BC} & k_3/n_{BD} \\ 0 & k_2/n_{BC} & k_2 & 0 \\ 0 & k_3/n_{BD} & 0 & k_3 \end{bmatrix} \begin{Bmatrix} \theta_A \\ \theta_B \\ \theta_E \\ \theta_F \end{Bmatrix} = \begin{Bmatrix} T_A \\ 0 \\ T_E \\ T_F \end{Bmatrix} \quad 3.4$$

Now for eigen value problem

$$\left| K-\omega 2M \right| = 0$$

$$k= \begin{bmatrix} k_1 & -k_1 & 0 & 0 \\ -k_1 & k_1+k_2/n_{BC}^2+k_3/n_{BD}^2 & k_2/n_{BC} & k_3/n_{BD} \\ 0 & k_2/n_{BC} & k_2 & 0 \\ 0 & k_3/n_{BD} & 0 & k_3 \end{bmatrix}$$

$$M= \begin{bmatrix} I_A & 0 & 0 & 0 \\ 0 & (I_B+I_C/n_{BC}^2+I_D/n_{BD}^2) & 0 & 0 \\ 0 & 0 & I_E & 0 \\ 0 & 0 & 0 & I_F \end{bmatrix}$$

By solving this eigen value equation we get value of natural frequency

Let us take an example which is designed in soidwork software to solve the eigen value problem

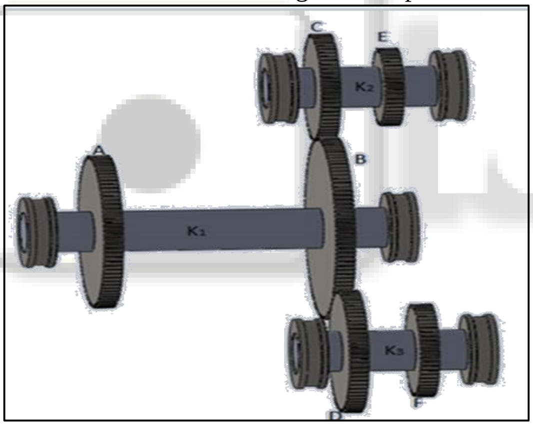

Fig. 5: Gearbox Model

IA=.15312 kg-m2 ; IB=.31918 kg-m2 ; IC=.00359 kg-m2
ID=.0062 kg-m2 ; IE=.00874 kg-m2 ; IF=.00189 kg-m2
TA=100 ; TB=120 ; TC=70 ; TD=60 ; TE=50 ; TF=40
D1=D2=D3=.080 m ; L1=500m ; L2=L3=250 m
Modulus of rigidity = 7.69×1010 N/m2
K1=CJ/L =(π×(.08)^4)/((32)(.500)) =6.17×105 Nm/rad ;
K2=K3 =(π×(.08)^4)/((32)(.250)) =1.223×105Nm/rad
Gear ratio : nBC = 120/70 and nDC= 120/60
by using these values and above equations we get the value of natural frequencies
ω1=2.28×103 rad/sec ; ω2=2.54 ×103 rad/sec ;
ω3=2.79×103 rad/sec; ω4= 3.74×103 rad/sec

III. MODE SHAPES

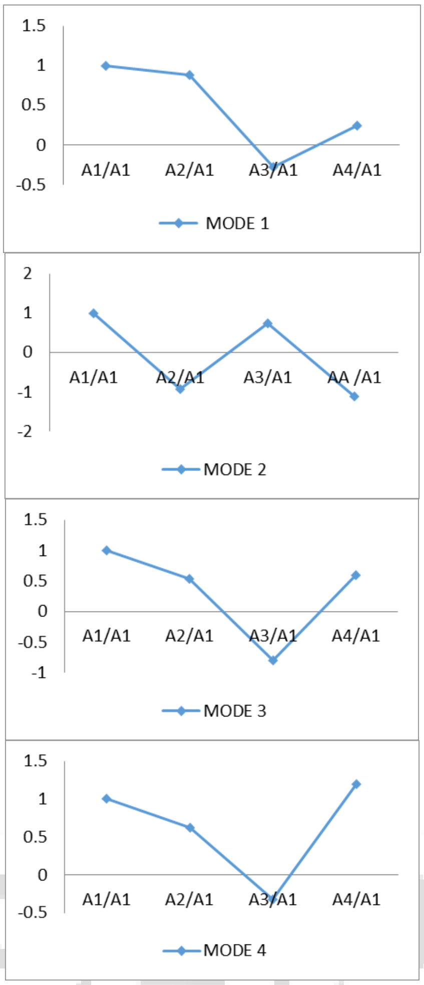

Fig. 5.1: Graphs of Modes Shapes

Now we can solve the same problem with ANSYS software

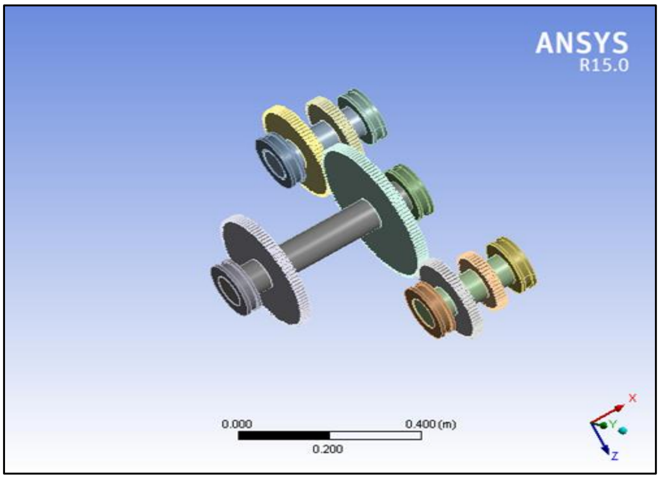

Fig. 6: ANSYS software output

IV. ANSYS RESULT

Object Name	Total Deformation 13	Total Deformation 14	Total Deformation 15	Total Deformation 16	Total Deformation 17	Total Deformation 18	Directional Deformation	Directional Deformation 2
State	Solved							
Scope								
Scoping Method	Geometry Selection							

Geometry	All Bodies						
Definition							
Type	Total Deformation						Directional Deformation
Mode	1.	2.	3.	4.	5.	6.	1.
Identifier							
Suppressed	No						
Orientation							X Axis
Coordinate System							Global Coordinate System
Results							
Minimum	0. m						-1.3427e-002 m
Maximum	0.34655 m	0.31075 m	0.29249 m	0.43917 m	0.35188 m	0.40249 m	1.013e-002 m
Minimum Occurs On	Part 10						Part 5
Maximum Occurs On	Part 7	Part 5					
Minimum Value Over Time							
Minimum	0. m						-1.3427e-002 m
Maximum	0. m						-1.3427e-002 m
Maximum Value Over Time							
Minimum	0.34655 m	0.31075 m	0.29249 m	0.43917 m	0.35188 m	0.40249 m	1.013e-002 m
Maximum	0.34655 m	0.31075 m	0.29249 m	0.43917 m	0.35188 m	0.40249 m	1.013e-002 m
Information							
Frequency	373.85 Hz	564.56 Hz	599.8 Hz	698.43 Hz	795.74 Hz	865.79 Hz	373.85 Hz

Table 1: Ansys Result

V. Conclusion

Natural frequencies (Hz)	F1	F2	F3	F4
Finite element method	599.8			373.63
Eigen value method	595	444	404	363
difference b/w FEM and Eigen value method	4.8			10.63
% Error	0.80			2.85

Table 1: Conclusion

Total deformation	Maximum	Minimum
FEM	1.013	-1.134
Eigen value method	1.05	-1.136
Diff. b/w FEM and Eigen value method	0.037	-0.002
% Error	3.65	0.17

Table 2: Conclusion

Natural frequencies for the branched system been calculated and the corresponding mode shapes been plotted. The difference between the result obtained by FEM and Eigen value method is shown in table I and table 2.

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